Time to Synchronization for Metronomes

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The tendency towards synchronization pervades throughout nature. It is evident in the swarming behaviour of birds, fish and insects, and in crowds of people clapping or chanting. It also manifests itself in non-living oscillators, such as the synchronization of two pendulum clocks. The underlying mechanism for synchronization seems to be the transfer of information between oscillators via common medium. In the case of pendulums, this transfer medium can be a common surface through which impulse propagates. It is natural to expect that with an increasing number of pendulums in such a system, the time to synchronization should increase. After all, more oscillators means more information being transferred, and more information means more potential discrepancies, all of which correct themselves, under the tendency towards synchronization, through time. This experiment explores the effect of increasing numbers of oscillators on the time required for synchronization, in the context of metronomes standing on a common platform which is free in one direction parallel to the metronomes' swing. In addition, a simulation of coupled metronomes based on the work of Bennett et al. was extended to N metronomes and compared to the experimental data.

I. INTRODUCTION

The problem of synchronizing oscillators was first motivated by the problem of determining a ship's longitude when on extended voyages. Early attempts to solve this problem, by dead reckoning or sailing along known lines of latitude, were inaccurate, inefficient and highly dangerous. As a result of the losses incurred to various navies due to a lack of an accurate longitude-determination method, many prizes were offered throughout the 17th and 18th centuries for promising advancements towards solving the longitude problem. It was one of the most major problems of its day and most European scientists were involved in finding a solution. Among them were Christiaan Huygens, a Dutch mathematician credited with the invention of the pendulum clock. This invention showed promise with regard to solving the longitude problem, because the oscillation period for the pendulums was constant regardless of swing angle, and so these clocks had the potential to be used as accurate timekeepers out at sea. Because the earth revolves at a constant rate, the difference between the clock's time (set to the time at their port of departure) and the local time aboard the ship could be used to determine the ship's current longitude. To reliably employ such a system aboard a ship, it would have been necessary to employ a second clock for redundancy. This way, if one clock broad down or required maintenance, timekeeping would not be lost. When Huygens was confined to his room due to an illness, he idly watched two clocks which were mounted in the same case and noticed an interesting phenomenon. No matter what initial position of each pendulum was, the clocks would always eventually swing at the same frequency and 180 degrees out of phase. Even after being perturbed, the clocks would return to their anti-phase synchronicity within half an hour and remain there indefinitely. At first, Huygens hypothesized that the clocks were interacting through air currents passing between them, but quickly ruled this out after blocking the airflow between the pendulums. He eventually deduced that the interaction responsible for synchronization is carried through the supporting beam common to both clocks; that is, impulses from one pendulum were being transmitted through the beam and into the other pendulum. Huygens believed that this synchronization could be used to keep the clocks accurate out at sea, however the Royal Society deemed this the very grounds for the idea's rejection and Huygens' work was viewed as a setback. Huygens' observations began to be re-examined in the 20th century, and in 2001, a team at Georgia Tech (Bennett et al) examined Huygens' observations with an experiment consisting of two pendulum clocks mounted on a common wooden beam which itself was attached to a low-friction wheeled cart. The purpose of the experiment was to investigate the system's periodic and quasi-periodic in-phase, anti-phase, and beating death attractors (beating death occurred when one or both clocks ceased to function, and tended to occur when the mass of the common frame significantly exceeded the mass of the clocks) as certain parameters were varied: the mass ratio between platform and clocks, the damping, and the detuning (frequency difference). Interestingly, the team found that the anti-phase synchronization of Huygens' clocks depended on the system being in a specific mass ratio range. Huygens' clocks, intended for duty at sea, had to be secured from falling over due to the rolling motions of the ship. To this end, he put 100-lb weights into the bottom of the clocks, increasing the common-platform-to-clock mass ratio to within the range necessary for anti-phase synchronization. Thus, the Georgia Tech researchers found that Huygens' observations depended serendipitously on the practical considerations of his invention. Our experiment is based upon the work of Bennett et al and seeks to investigate the dynamics of a similar system, but with the damping, mass ratio and detuning fixed and instead varying the total number of coupled pendulums. In addition, the theoretical framework laid out by the team's paper provides a basis for our theoretical model.

II. METHODS

The experimental setup was a group of two to nine metronomes (Wittner's Super-Mini-Taktell Series 880) placed atop a poster-board platform, which itself was placed atop two empty 12 oz soda cans.

The metronomes were set to 200 beats-per-minute and all reflective parts near the base were blacked out with electrical tape and black marker so that it would contrast with the white dot we painted at the center of each metronome's pendulum bob. Similarly, the poster-board platform was spray painted black and a single white dot was painted on it. The experimental setup was placed against a black background. Each metronome was tested for synchronization against another metronome for consistency control.

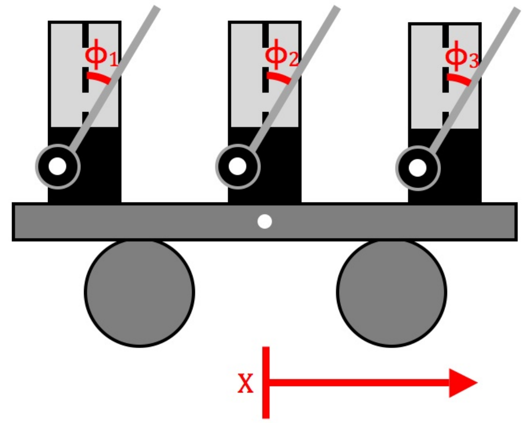

FIG. 1. Schematic for the experimental setup.

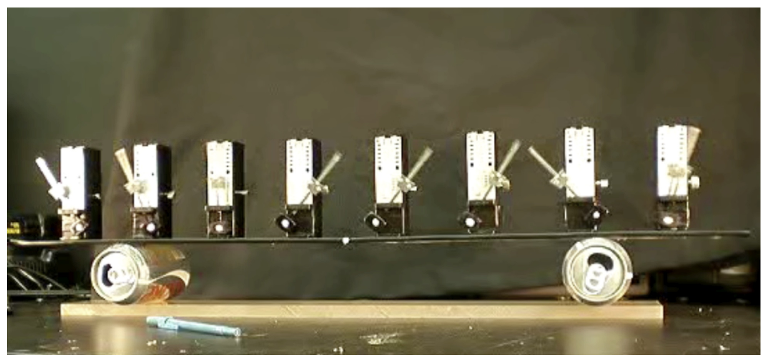

FIG. 2. Photo of the experimental setup in motion, 8 metronomes

The motion of the metronomes was captured by a camera running on a LabView-based motion tracking program which recorded the horizontal positions of each of the white tracking dots. The platform was held still while each metronome was non-systematically started. At the start of the

Nth metronome, the platform was released and the trial officially began. The system was then allowed to evolve until such time as synchronization was judged by a human observer to have been thoroughly established, or until the springs of the wound metronomes died down and their motions ceased.

Trials for $N = 2, 3, 4, 5, 6, and 9$ produced 20 usable sets of data, while $N = 4$ produced 19 and $N = 8$ produced 12. The data was in the form of a time-series of the horizontal positions of each tracking dot (in pixel-units as measured by the tracking camera). Because ideal synchronization is determined by matching frequencies and zero phase difference, a findpeaks function in MATLAB was applied to reduce the time-series into a vector of peak times. Only the peak times at one turning point (from a left-pointing velocity to a right-pointing velocity) were recorded in order to maintain consistency by throwing out instances of anti-phase synchronization.

Because the exact time of synchronization is difficult to pin down, both in theory and in practice, we chose to define synchronization as the time at which the peaks of all oscillator occur within some error region, known as the ϵ_{time} ball, and this condition is maintained for some r count.

The data was then trimmed such that each time series began after the release of the platform. Because the size of each peak vector could vary, one peak vector, v_{arb} was chosen. A reference peak is chosen from v_{arb} and compared to another metronome's peak within ϵ_{time} of the v_{arb} peak. Their positions in t were averaged and this became the new center for ϵ_{time}. This is repeated for each metronome's peak vector until no further peaks are found within ϵ_{time}. If the number of vectors that had a peak within ϵ_{time} is equal to N, then a count § is started and the procedure is repeated from the next peak time in v_{arb}. If, at any point, less than N peak times are found to occur, the count § is reset to 0. When $\S = r$, then the time of the first peak which counted toward § is taken as the time of synchronization t_{sync} for that particular trial.

In addition, a simulation was developed based on an extension of the simulation developed by Bennett et al /citebennett and Pantaleone /citepantaleone. The generalized equations of motion, extended to N metronomes are as follows:

$$\ddot{\phi}_j + b\dot{\phi}_j + \frac{g}{l}\sin\phi_j = -\frac{1}{l}\ddot{X}\cos\phi_j + F_j \quad (1)$$

$$(M+m)\ddot{X}+B\dot{X} = -ml\frac{d^2}{dt^2}(\sin\phi_1+\sin\phi_2+...+\sin\phi_N) \quad (2)$$

where ϕ_j is the angular displacement of the jth pendulum, b is the pivot damping coefficient, g is the acceleration due to gravity, l is the pendulum length, X is the linear displacement of the platform, F is the impulsive drive, M is the platform mass, m is the metronome bob mass, B is the platform friction coefficient, and the dots denote differentiation with respect to time.

Nondimensionalizing results in:

$$\frac{d^2\phi_j}{d\tau^2} + 2\tilde{\gamma}\frac{d\phi_j}{d\tau} + \sin\phi_j = -\frac{d^2Y}{d\tau^2}\cos\phi_j + \tilde{F}_j \quad (3)$$

$$\frac{d^2Y}{d\tau^2} + 2\Gamma\frac{dY}{d\tau} = -\mu\frac{d^2}{d\tau^2}(\sin\phi_1 + ... + \sin\phi_N) \quad (4)$$

in which the following dimensionless parameters are introduced:

$$\tau = t\sqrt{\frac{g}{l}} \quad (5)$$

$$\mu = \frac{m}{M + Nm} \quad (6)$$

$$\tilde{\gamma} = b\sqrt{\frac{l}{4g}} \quad (7)$$

$$\Gamma = \frac{B}{(M + Nm)}\sqrt{\frac{l}{4g}} \quad (8)$$

All of these parameters are well-defined and experimentally measurable, excepting the impulsive drive $\tilde{F}_j$ which is developed in Bennet et al[?] .

The mechanism which provides the kick to the metronome is modelled by

$$\left|\frac{d\phi}{d\tau}\right| \rightarrow \gamma\left|\frac{d\phi}{d\tau}\right| + c \quad (9)$$

in which the pendulum's angular velocity is slowed by a factor of γ and then a constant c is added.

The parameter values corresponding to our experimental setup were estimated as follows:

The platform mass is given by $M = 0.0655 + N(0.094 - m)$ which includes the mass of the metronome itself less the mass of its bob, which is estimated at $0.022kg$. The pivot damping coefficient b is estimated by measuring the decay time for an unwound metronome, which was determined to be $t_{decay} = 20s$, which gives $b = 2m/t_{decay} = 0.0022$. The platform damping coefficient B was estimated arbitrarily at $B = 0.001$ so as to allow the platform to roll relatively freely. γ and c were borrowed from Borrero-Echeverry[?] which in turn had been adjusted to match experimentally observed motions of the metronomes, that is, $\gamma = 0.97$ and $c = 0.025$. The pendulum length l was adjusted until the natural frequency of the simulated pendulums matched that of the actual pendulum, that is, $l = 0.1m$.

The equations of motion with the given parameters were solved by numerical integration using a computer for the cases $N = 2$ through $N = 9$. 100 simulations are run with random initial conditions for each N with constant total energy. The equations are integrated up to period $\tau = 2000$ and the time to synchronization was determined via the previously described synchronization condition. A strict synchronization test was also applied, in this case the peaks had to occur strictly within one time discretization.

III. RESULTS

Time to synchronization for both the experiment and the simulation are shown for two different values of parameter l. The blue line represents the results of applying the same synchronization criteria to the simulation results as was applied to the experimental data. The green line represents the results of applying to strict synchronization test to the simulation data, where all peak values of the simulated metronomes' angular displacement occur within a single time discretization.

The following figure shows proportion of trials which synchronized for each N number of metronomes.

The rapid drop-off in the experimental data after $N = 5$ can be attributed to the finite time window for synchronization inherent in the experiment due to the finite energy storage capacity of the metronome's winding spring. Based upon the results of the simulation, it is conceivable that, were the aforementioned restriction lifted, all systems of N would eventually synchronize.

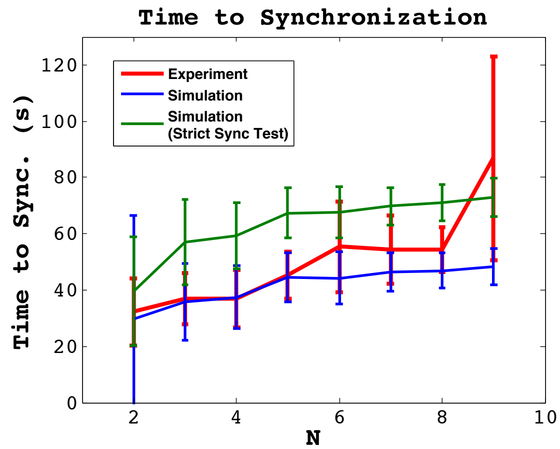

FIG. 3. Time to synchronization for $l = 10cm$. X-axis is N number of metronomes. Error bars indicate one standard deviation. Natural frequency of simulation approximately 3 times higher than that of experiment.

In the simulation, synchronization occurs in all but the $N = 2$ case. One possible explanation for this discrepancy is the thin basin of attraction for the antiphase state given our particular set of parameters. Wiesenfeld and Berrero-Echeverry demonstrated that varying the platform damping coefficient B can alter the size of the basins of attraction for in-phase and antiphase synchronization /cite-borrero. As the platform mass M is increased, the antiphase ceases to be an attractor for $N <= 3$.

IV. DISCUSSION

It is worth noting that the time of synchronization is difficult to define and must be

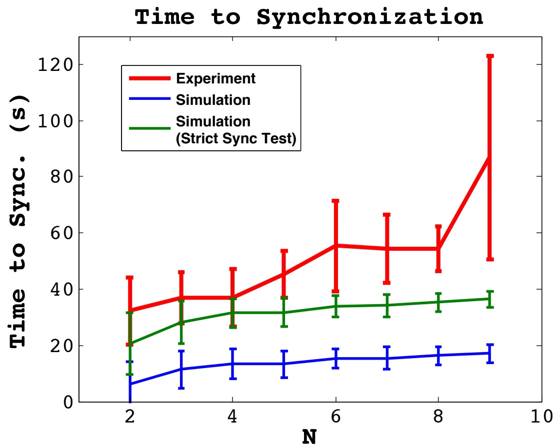

FIG. 4. Time to synchronization for $l = 2.5cm$. X-axis is N number of metronomes. Error bars indicate one standard deviation. Natural frequency of simulation and experiment approximately equal.

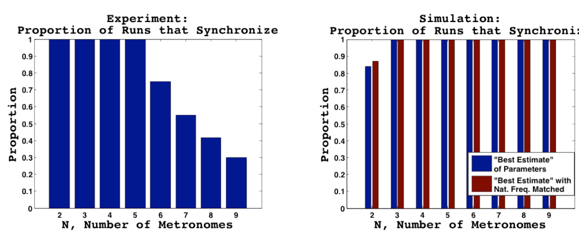

FIG. 5. Proportion of trials which achieved synchronization for simulation and experiment. Proportions for two simulations are given: one for $l = 10cm$ and one for $l = 2.5cm$.

done with some degree of arbitrary decision-making. In fact, the process of synchronization in the experiment was itself observed to proceed periodically - that is, the phase difference between the metronomes tended to overshoot and undershoot each other in rhythmic fashion as synchronization was approached. Even when synchronization was observed to occur, the actual phases of each metronome still tended to orbit around each other, making the characterization of 'in-sync' somewhat subjective. For future work, it would be particularly interesting to investigate and characterize this lead-up to synchronization. It is possible that analysing the time-series of pairwise phase-differences for each metronome for each N case would give further insight into the dynamics of synchronization.

V. CONCLUSION

We have experimentally investigated the time to synchronization for N coupled metronomes, as well as investigating such a system numerically via computer simulation after extending previously developed equations to the general $N >= 2$ case. The experimental results are in fairly strong agreement with the simulation results, at least in the case that $l = 10cm$. What disagreement between experiment and simulation exists may be attributed to such factors as inaccurate estimation of parameters or inadequate realism in the computer model.

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