Chua Circuits

Patrick Cheng,[1] Edward Coyle,[1] John Parker,[1] and Majid Sodagar[1]

School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA

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Chua circuits are the simplest circuit that meets the requirements to exhibit chaotic behavior and are cheap to construct with plenty of background research to check the accuracy of our results with. The initial design of the experiment was to synchronize multiple chaotic chua circuits acoustically to investigate ways of data masking for secure information transfer. This did not work out, and the experiment changed into the evaluation and cataloging of one of the chua circuits that we had constructed.

I. INTRODUCTION

Here is intro text

Chaos is defined in Strogatz book as aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions[1]. Chua circuits are one of the simplest circuits that can produce chaos, due to having a nonlinear element, three energy storage devices, and an active resistor. The circuit itself can take multiple forms, but the version that we used is shown in the following diagram.

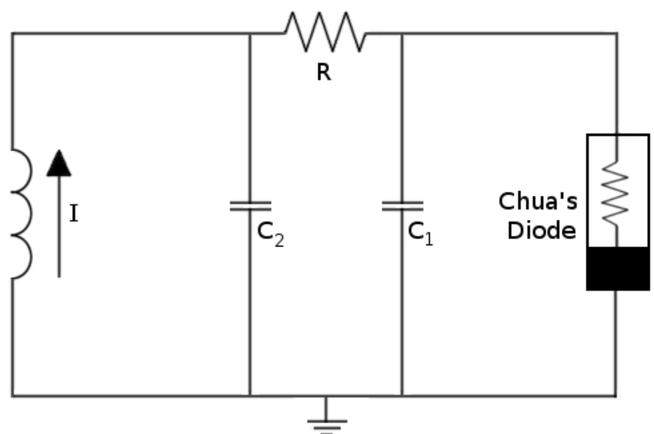

FIG. 1. Circuit diagram of a chua circuit

Research in chaos can relate to a very broad range of topics, but the original area of interest for this paper was in the investigation of masking data behind a chaotic signal as a way of ensuring secure data transmission. An article publish in 1993 details how chaotic signals would work well for transmission and, if intercepted, would yield no information[2]. This works due to the nature of how the information is encoded. The only way to access the information would be to feed the signal in the device that produced the signal, or an identical copy of one. If one can have two synchronized chaotic signals, one can freely transmits information between the two the knowledge that the only way any-one else could retrieve the information was if they had a synchronized circuit too. This has a large range of potential in this modern age where there are widespread privacy concerns and governments desiring to keep secrets from people and other governments.

The idea of creating a secure method of transmission of data between two points, led us to design the original experiment as a coupling of two chua circuits together acoustically[3]. The acoustic coupling was chosen because we had some previous work in the field, and the method of coupling allowed for a broad range of possible methods to alter exactly how the circuits coupled together. Some of the different changes we could make could range from a change in distance, change of the medium of propagation, or an alteration of the signal going into the acoustic coupling system. When this method of coupling failed, due to the piezoelectric crystal we had not producing a high enough pitch.

We tried other methods of coupling and none of them were a solution that we could fix within the restraints of the time period we had. Therefor, we changed the purpose of the experiment to the characterization of the nonlinear dynamics of the chua circuit and decided to compare that to computer simulations that many others have done, along similar experimental data. Such a task would also help us determine if there was a fault

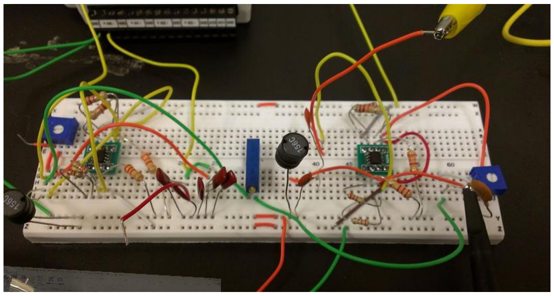

FIG. 2. Picture of circuit board as originally set up for coupling

in the circuits that we constructed, or if the fault lay in the coupling. That would pave the way for the continuation of our original experiment.

II. METHODS

The design for the construction of the chua circuits came from the website http://www.chuacircuits.com, which has a basic parts list of what we needed to construct a chua circuit. We chose to use a design that used an actual inductor and a design for the chua's diode that involved two op-amps. This will be shown in further detail later on. The last change that we made was to replace the resistor R, shown in figure 1, with a potentiometer, so we could scan through a large resistance range and see all the different behaviors of the circuit. The circuits were constructed on a breadboard in the lab, shown in Figure 2.

Once we moved onto testing a single chua circuit, we used voltmeters that had a high sample rate to feed the information into a computer, where it was interpreted by LabVIEW software and put into an easily manipulatable format. We measured the voltage at the two points on either side of the potentiometer. Along with this method of recording, we had an oscilloscope measuring the same variables, to give us realtime data with a much higher sampling rate than the hardware that we had.

We collected data at a range of different potentiometer readings, making sure to get one in each of the distinct patterns we saw on the oscilloscopes screen. We also took variable durations of data recording, although on average, each group of consecutive data was recorded in about 3 seconds. We ran a few very long sessions so that we would have better data to calculate the correlation dimension. Along with this, we made sure that while we were sweeping through all the resistance values, to go as slow as possible for one of the run throughs to see if there was any periodic behavior the would briefly emerge from the chaotic section, but we did not witness any.

III. RESULTS

The chua circuit yielded the results of a periodic orbit degenerating into chaos, although not through the case of period doubling. We found out this was due to the values of the resistance values for the chua diode, which resulted in a set of conditions in which period doubling would not occur. This is because, the curves for the diodes outputs, as shown in Figure 3, ***** are supposed to obey a specific relation where m0 ¡ -1 and -1 ¡ m1 ¡ 0, to achieve period doubling. In our case though, as shown in Figure 4, there is only a very small range in which m1, the red line, is greater than -1 while m0 is less than -1. It just so happens that in our circuit, the other parameters do not allow for such a thing to happen, and therefor, period doubling is found to be impossible on our current set up.

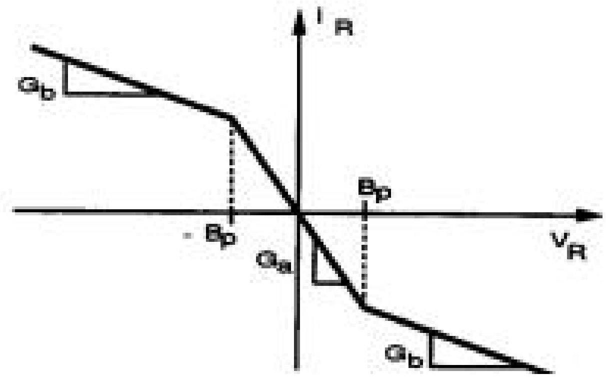

FIG. 3. : Picture of circuit board as originally set up for coupling

After that simple analysis, we calculated the Lyapunov exponent, a measure of how two nearby trajectories to diverge from one another, from the data. Since, each of the

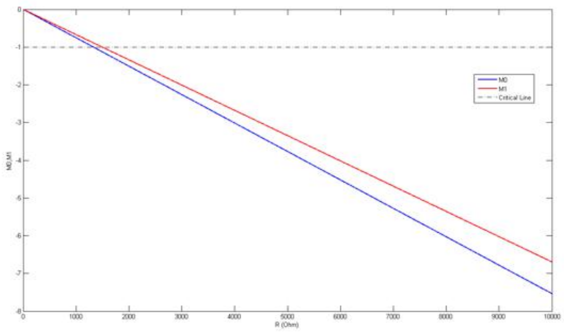

FIG. 4. Picture of circuit board as originally set up for coupling

$$|\delta \mathbf{Z}(t)| \approx e^{\lambda t} |\delta \mathbf{Z}_0|$$

FIG. 5. How to calculate the Lyapunov exponent

attractors in the chaos mode of the chua circuit, exhibit their own unique behavior, we calculated the Lyapunov exponent for both of them. To calculate such a thing, in each attractor, multiple close initial conditions were taken and the distance between each one of them was taken and averaged. From these values, an average Lyapunov exponent was calculated to be 1.33, with one side having 1.01 and the other 1.65 as their values. This allowed us to calculate the horizon time, the time it takes for a system to decent into chaos. For our circuit boar, the time for this to happen is 3.64 ms.

We also attempted to see if an iterative map of the system would produce anything

$$C(R) = \lim \left[\frac{1}{N^2} \sum_{i,j=1}^{N} H\left(R - | X_i - X_j |\right) \right]$$

interesting, but the out of the graphs produced, only the one for the periodic system seemed to have any discernible use or extractable data from it.

The final thing we did calculate with the data collected from the circuit was the correlation dimension. This is calculated by drawing multiple spheres of different radius and calculating how many points are within the radius. Using the following equation we calculated that the correlation dimension for the 2D projection of the complete attractor had a dimension of only 1.8. This is close to the literature's measurement of 1.9[4]. The measurement implies that the structure is not truly 2 dimensional, much like how many fractals are do not have an integer value number of dimensions.

IV. SIMULATIONS

The simulations were run in Matlab, solved mostly by Matlab's ode45 function. It used the model shown in Figure 6 for the circuit, except with an inductor swapped out on the left side to calculated the constants for the following equations.

This allowed for the creation of simulations of the chua circuit for any given value of

$$C1(dV1/dt) = (V2 - V1)/R - g(V1)$$

$$C2(dV2/dt) = -rI - V2$$

$$L(dI/dt) = -rI - V2$$

$$m1 = \frac{-1}{R6} - \frac{1}{R3}$$

$$m0 = \begin{cases} \frac{1}{R4} - \frac{1}{R3} \\ \frac{-1}{R6} + \frac{1}{R1} \end{cases}$$

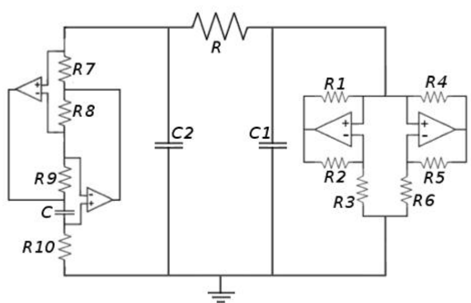

FIG. 6. Circuit diagram for the chua circuit used, except we used an inductor in place of the circuitry on the far left

the potentiometer R. We were able to simulate all of the give R values that we had taken measurements of, and we were able to find an almost identical evolution in patterns: periodic to screw attractor to chaos. There were some noticeable differences in the simulation, such as the values for the occurrences being off by 300 Ohms and the screw attractor being much less pronounced. We also did the

$$I(\tau) = -\sum_{h}^{j} \sum_{k}^{j} P_{h,k}(\tau) \ln \frac{P_{h,k}(\tau)}{P_k P_h}$$

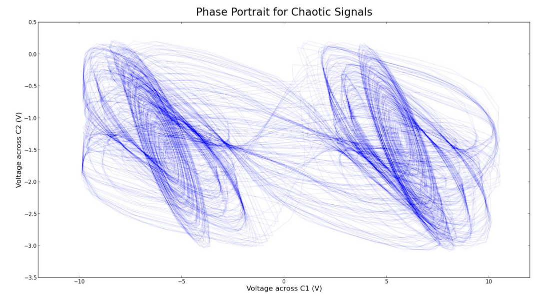

FIG. 7. Actual Data

same test for correlation dimensions with the simulated data and ended up with a value of 1.9.

A second type of simulation that we did was the scroll reconstruction. This method takes only one of the dimensions and then uses it, with a time lag, to run through the ODEs to extrapolate the entire figure. To calculate the proper time lag to us, the following equation is used to determine the points in time where there mutual information in minimal.

The following three graphs are the actual data, the simulated data, and the reconstructed data. They are all very similar to each other, but not so much to say that we have gotten any sort of extraordinary data.

V. CONCLUSION

Overall, the data we collected is in agreement with theory. It behaves in the way that

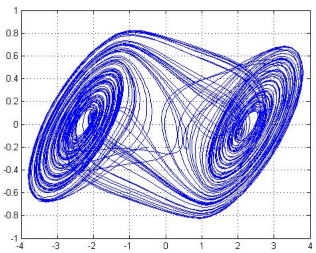

FIG. 8. Simulated Data

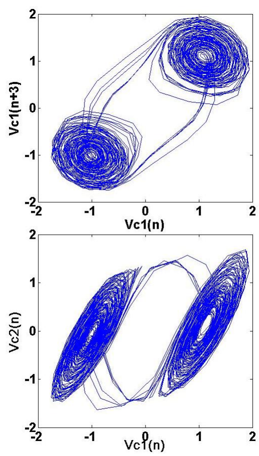

FIG. 9. Reconstructed Data

we expected too, and allowing for error, the bifurcations occurred close to where the simulations predicted. It went from a periodic signal, to a screw attractor, and then into chaos. While we showed that ours couldn't produce it, due to the values of the resistors we had, computer simulation showed that period doubling could occur. We also found the Lyapunov exponent of the Chua circuits to scrolls, the left being 1.01 and the right being 1.65. The correlation dimension was also calculated to be 1.8 in the experimental data and about 1.9, which is in agreement with the literature[4] in the simulated data. This shows we have a good understanding of how the circuit works and that given more time we could fully investigate our original question of the acoustic synchronization of multiple Chua circuits for the development of secure communication.

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