# Ferrofluid Dynamics of a Single-Peak under an Oscillating Magnetic Field

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*Nonlinear Dynamics and Chaos PHYS 4267*
*16 December 2011*

**Abstract**

A magnetic field applied normal to the surface of a ferrofluid is temporally varied so as to exhibit nonlinear behavior. The overall effect will result in hydrodynamic and magnetic nonlinearities, in which the magnetic coupling between the fluid and the external field results in a hysteresis effect that serves as the primary nonlinearity of interest. A period doubling route is observed with variance of amplitude, frequency, and voltage offset.

## I. Introduction

A ferrofluid consists of colloidal suspensions of magnetic monodomains in a non-magnetic carrier liquid[1]. Being in a liquid state distinguishes it from solid ferromagnets partly in that it exhibits a higher entropy of states, which competes with the ferromagnetic tendency of the material. In this sense, the behavior of the fluid is more closely referred to as super-paramagnetic[1].

When set under an external magnetic field, the magnetic monodomains will couple to the field to invoke ferrohydrodynamic behavior. In the case of the oscillatory field, the response of the fluid is similar to that shown in Faraday waves, though it also involves the additional feature of magnetic hysteresis. Since the fluid is partitioned into different magnetic monodomains, an application of an external magnetic field is likely to induce several peaks. To sufficiently isolate the effect of the magnetic interaction, a fluid container with a diameter of ~3mm and a depth of ~12mm is used. This way, the container is sufficiently small enough to isolate a single peak (which effectively acts as a single magnetic monodomain); this also helps to minimize the effect of the dynamics observed in Faraday waves.

## II. Experiment and Apparatus

--Equipment--

1. Solenoid Magnet
1. Magnet power supply
2. Function Generator
3. Ferrofluid: Ferrotec EFH1 Series
4. Test Tube container

The setup involves setting the ferrofluid under a solenoid magnet. The fluid is held in a test tube with a diameter slightly greater than ~3mm. The magnet is hooked to a function generator in series with an amplifier. The intended H-field variation is sinusoidal in character. Due to the field arrangement of the solenoid, the magnetic field is not homogenous. During the displacement of the fluid, the magnet field varies by about 0.01mT. The inhomogeneity also results in lateral forces (which should be symmetric about the peak).

## III. Measured Data

### Fluid Hysteresis

Hysteresis in general is a dependency of a system on either its internal state or its "history" of evolution. In ferromagnets, the hysteresis is a result of the exchange interaction between electrons, in which electrons tend to align with those nearby to form magnetic domains along the material. Because of this tendency, ferromagnets are able to retain their own magnetization even without the application of an external field.

Fig. 1 shows the height of the fluid in response to a triangle pulse of 1Hz. In view of the hysteresis that occurs during the displacement, the fluid appears to slowly move before sharply accelerating.

The relaxation time for the fluid (the time it takes for the spins to sufficiently realign to an equilibrium state after a change in the magnetic field) can occur via two processes. In the Néel mechanism of relaxation, the magnetic moments rotate while the particles themselves do not. In the Brownian mechanism of relaxation, the magnetic moments rotate with the rotation of the particles. Typically the overall relaxation time is orders of magnitudes less than a second. The net relaxation time τ of the fluid can be viewed as $\tau = \frac{\tau_N \tau_B}{\tau_N + \tau_B}$ [2], where $\tau_N$ and $\tau_B$ are the Néel and Brownian relaxation times respectively.

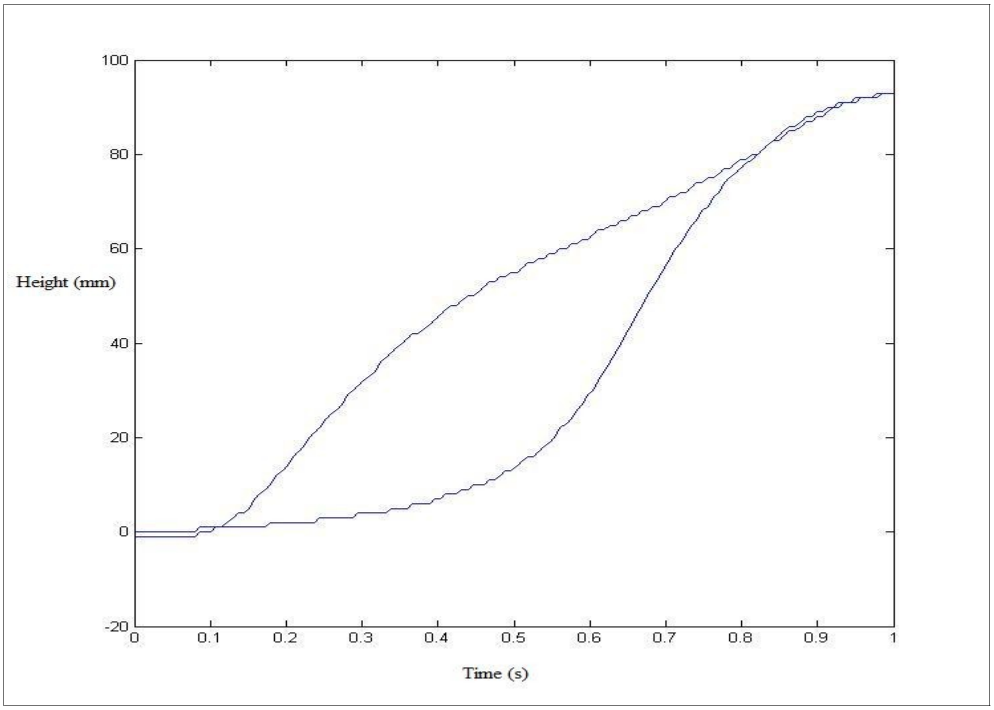

Figure 1: Height Hysteresis of Fluid during 1Hz Triangle Pulse

**Varying the Voltage Offset**

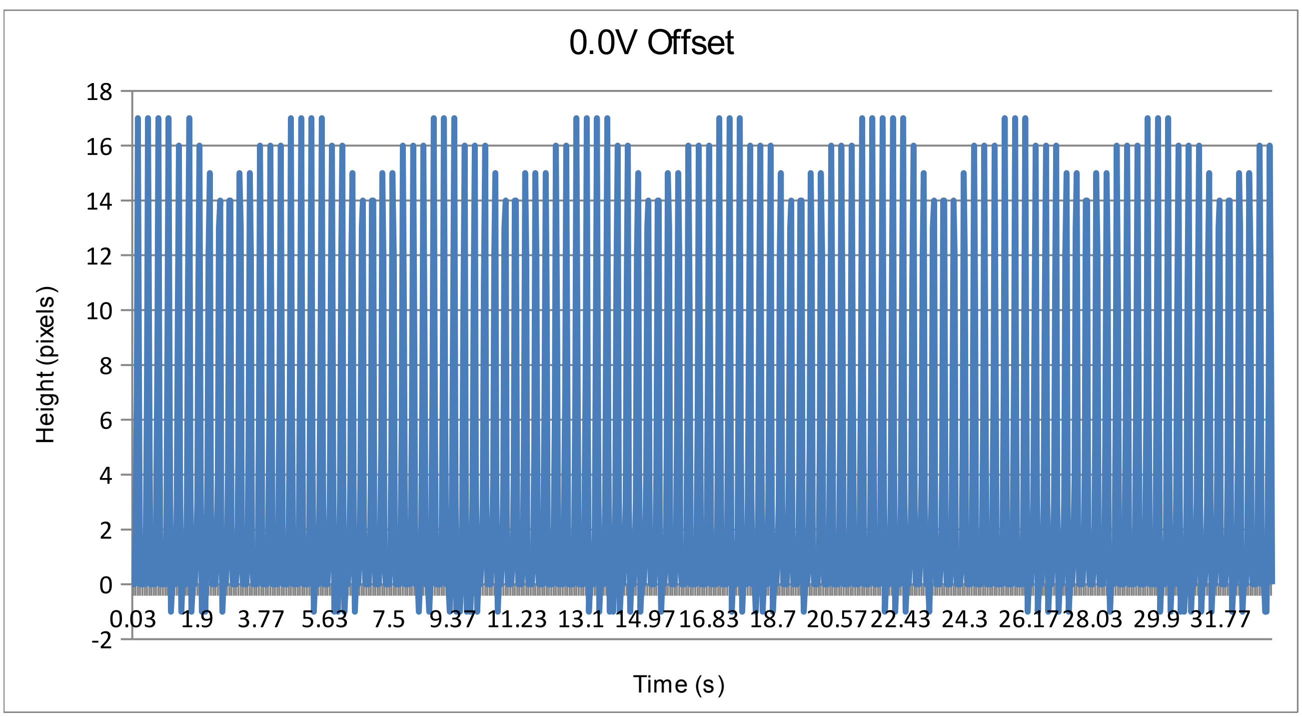

Figure 2: 0.0V Offset

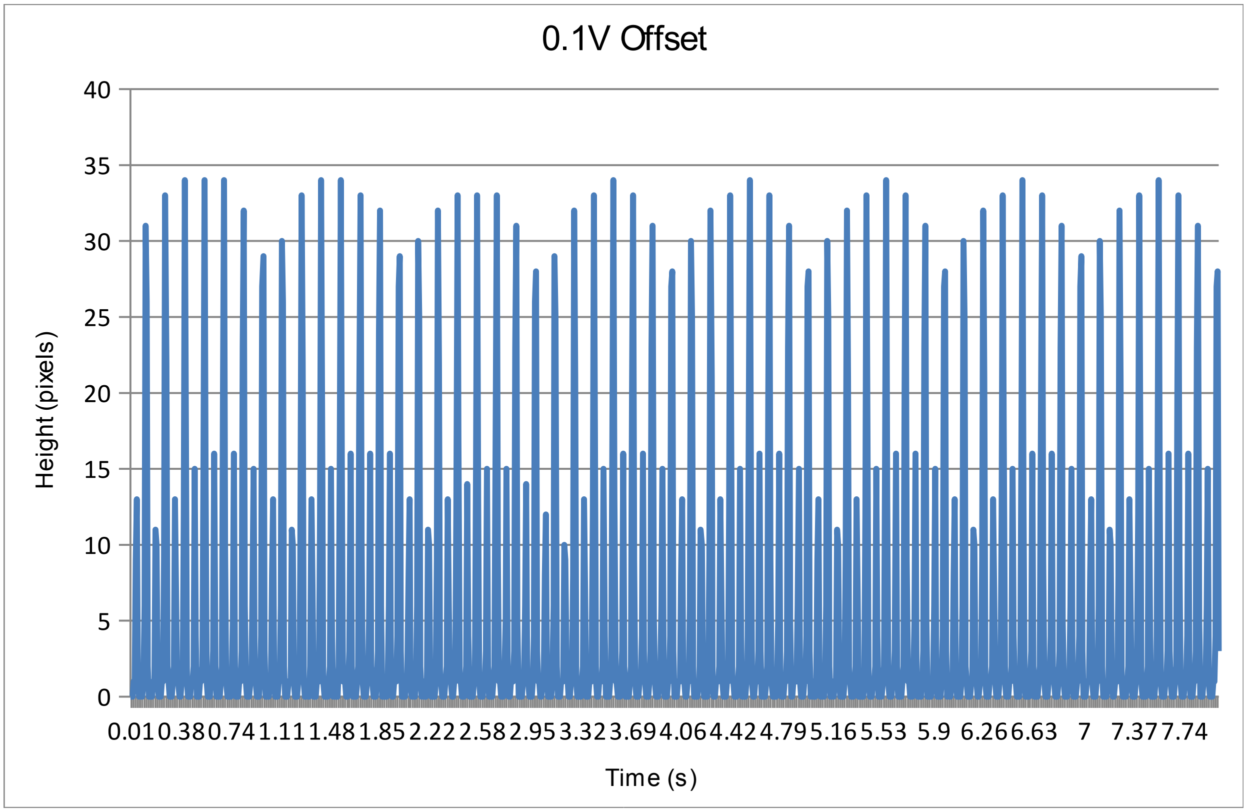


Figure 3: 0.1V Offset

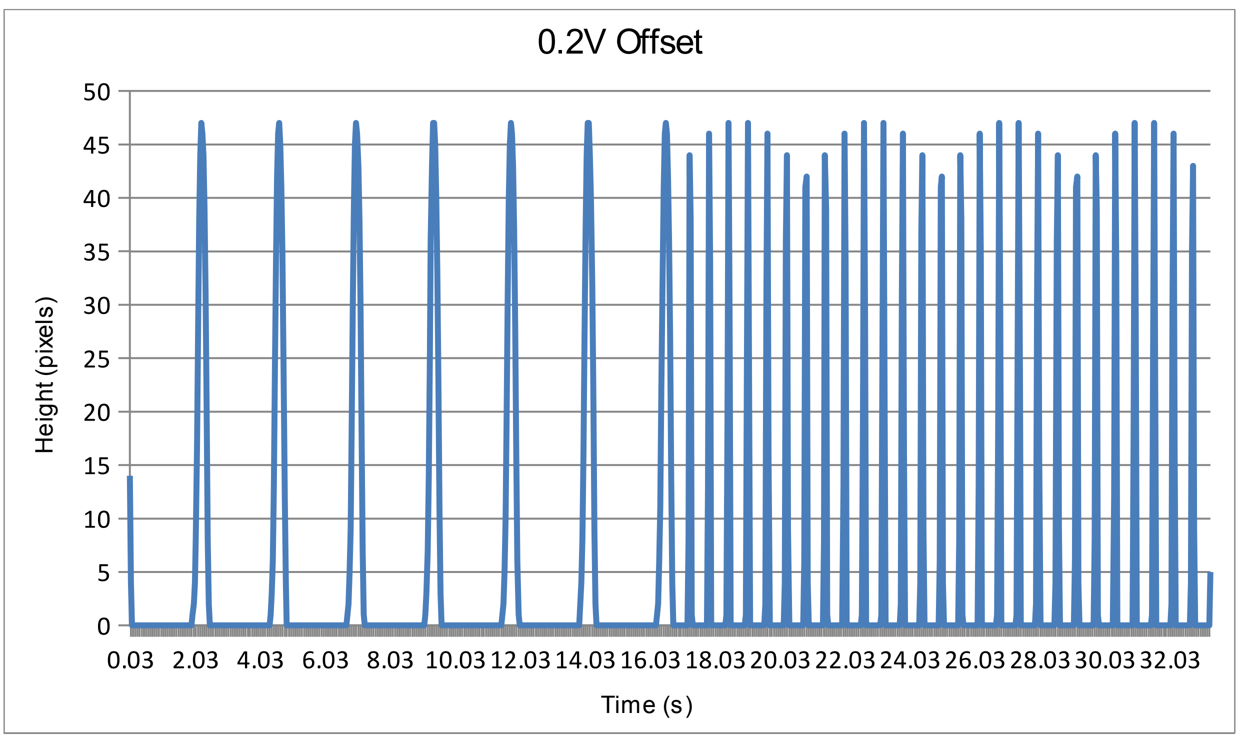


Figure 4: 0.2V Offset

The heights seem to fall under some enveloping curve. This envelope feature of the graphs is likely due to an instrumental effect. Thus, the peak heights should be about the same.

**Varying the Frequency and Amplitude while keeping the Voltage offset at 0.0V**
**The graphs depict the height (pixels) along the vertical axis vs. the time (seconds) along the horizontal axis.

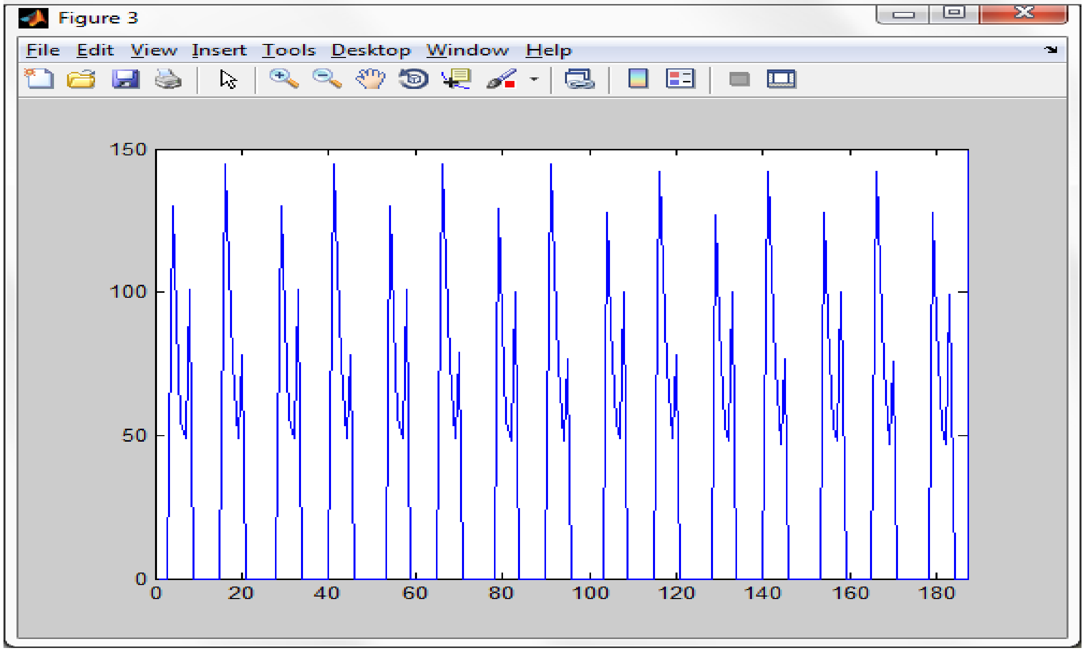

Figure 5: Height (pixels) vs. Time (s) 4.9V Amplitude, 5.0Hz Frequency

The following graphs show the variation in the oscillation behavior as the amplitude and frequency is modulated up from 4.8 V Amplitude, 10.0Hz Frequency

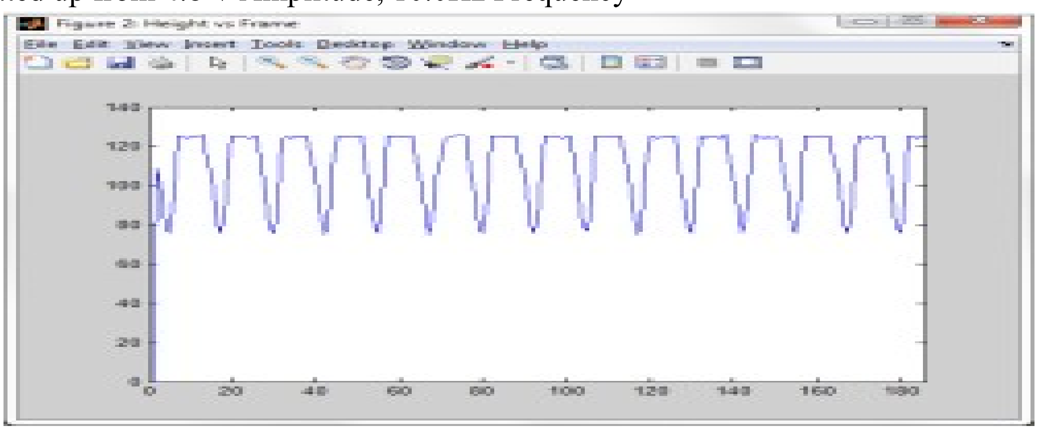

Figure 6: Height (pixels) vs. Time (s) (starting point)

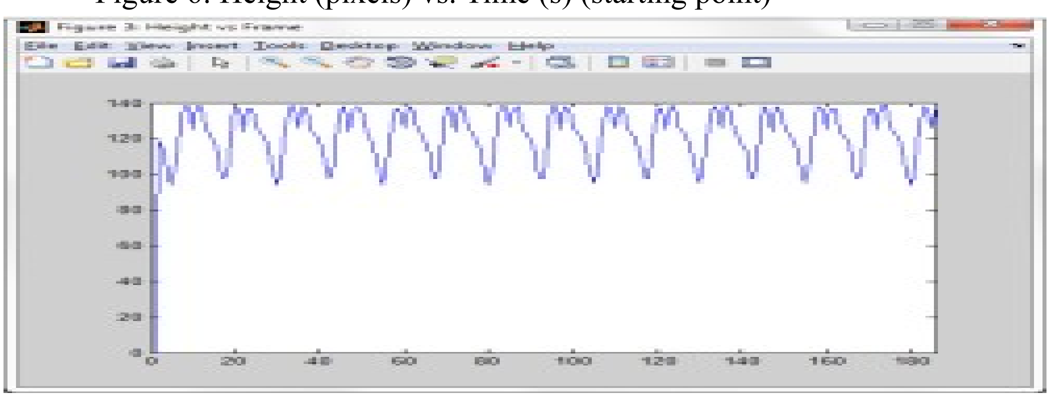

Figure 7: Height (pixels) vs. Time (s) (a bifurcation during the modulation)

As shown, varying the frequency, amplitude, and offset of oscillations gives way to period doubling bifurcations. Fig.5 shows a 4-period cycle at 4.9V Amplitude, 5.0H Frequency, and 0.0V offset. Fig.6 and 7 show a period doubling bifurcation (in terms of tracking localized peak heights) as the amplitude and frequency are modulated up starting from 4.8V Amplitude, 10.0Hz Frequency, and 0.0V offset; the bifurcation is from a 2-period to a 4-period cycle.

Referring to Fig.2-4, varying the voltage offset (at 3.4V Amplitude and 7.0Hz Frequency) gave way to a bifurcation to a 2-period cycle between 0.0V – 0.1V. After that, the cycles seem to return to a 1-cycle state. There is the noticeable feature of a contraction of the frequency after 7 successive peak heights. This may be due to the instrument, though since it is not seen in the case of the 0.0V and 0.1V offset, it is questionable; this will be discussed later under the (Theory) section.

## IV. Theory

**Accounting for the Magnetization**

First, we consider an assembly of paramagnets, each acting as a magnetic moment corresponding to a spin state. The fundamental entropy (σ = ln(g(s)), where g(s) is the multiplicity of states in terms of some parameter s), is given as

$$\sigma(s) \approx -\left(\frac{1}{2}N+s\right)\ln\left(\frac{1}{2}+\frac{s}{N}\right)-\left(\frac{1}{2}N-s\right)\ln\left(\frac{1}{2}-\frac{s}{N}\right) \quad \text{[3]}, \tag{1}$$

where the spin excess (between number of spin up and spin down) is 2s. Minimization of the Helmholtz free energy gives

$$-2mB+k_B T\ln\frac{N+2s}{N-2s}=0 \quad , \tag{2}$$

where $m$ is the magnetic moment per spin, B is the external magnetic field, $k_B$ is the Boltzmann constant, and T is the temperature. From this, the expectation value of the spin states and the net magnetization becomes

$$\begin{gathered}\langle 2s\rangle = N\tanh\left(\frac{mB}{k_B T}\right)\\ M=\langle 2s\rangle\frac{m}{V}=nm\tanh\left(\frac{mB}{k_B T}\right)\end{gathered} \quad \text{[3]}, \tag{3}$$

where $n$ is the number of spin states per unit volume.

For ferromagnets, there is the additional aspect of the spin-spin coupling, which results from the exchange interaction between spins. The hamiltonian becomes

$$\hat{H}=\sum_{i,j} J_{i,j}\sigma_i\sigma_j-\sum_j H_j\sigma_j \quad , \tag{4}$$

in which the first term accounts for the spin-spin coupling ($J_{i,j}$ being exchange coefficients) and the second term accounts for the coupling of spins to the external magnetic field (Ising Model). Restating the spin-spin coupling as a coupling of the individual spins to the material's magnetization gives $B_E=\mu_0(H+\lambda M)$ [4], in which $B_E$ is the effective field experienced by the individual magnetic moments. The magnetization here takes on the form

$$M=nm\tanh\left(\frac{m\mu_0(H+\lambda M)}{k_B T}\right) \quad \text{[4]}, \tag{5}$$

where $n$ is the number of spin states per unit volume. For sufficiently high temperatures (those that effectively surpass the energy barrier involved in having the magnetization shift its axis of orientation), the magnetization takes on the form of the Langevin formula,

$$L(\xi)=coth\xi-\xi^{-1}, \xi=\frac{m\mu_0(H+\lambda M)}{k_B T} \text{ [2]}. \tag{6}$$

Here, the magnetization is explicitly a function of itself (this also marks the difference between the paramagnetic and ferromagnetic case). This feature allows for interpretation in terms of one-dimensional mappings and also explicitly shows a hysteresis effect.

**The Fluid Equation of Motion**

Modifying the Bernoulli fluid equation gives

$$\rho\frac{d\boldsymbol{v}}{dt}=-\nabla P-\rho\nabla\Omega+M\nabla H, \frac{d}{dt}=\frac{\partial}{\partial t}+(\boldsymbol{v}\cdot\nabla) \quad , \tag{7}$$

where ρ is the fluid mass density, P is a pressure field, Ω is the gravitational potential, and the last term is the magnetic coupling. The magnetic state equation M = M(H,T), is given by the Langevin formula

$$M=nmL(\xi), \xi=\frac{m\mu_0(H+\lambda M)}{k_B T}, L(\xi)=coth\xi-\xi^{-1} \text{ [2]}, \tag{8}$$

in which H here is $H = H_0 \sin(\omega t)$ with ω as the driving frequency.

The force expression for the magnetic coupling is $\boldsymbol{F}=M\nabla H=nk_B T\nabla\ln\left(\frac{\sinh(\xi)}{\xi}\right)$ [2].

Supposing that $\nabla\cdot\boldsymbol{v}=0, \nabla\times\boldsymbol{H}=0, \nabla\cdot B=0$ (the first term referring to an incompressible fluid), the equations may be reduced to

$$P+\rho g(h+z)+\frac{1}{2}\rho v^2-nk_B T\ln\left(\frac{\sinh(\xi)}{\xi}\right)=const \text{ [2]}. \tag{9}$$

where h is the surface level (height above bottom depth) and z is the oscillation height about the surface level. ξ is as mentioned in Eq.6. Additional terms of surface tension (dependent on the shape of the waveform as a function of time) and viscosity are neglected.

The given equation of motion is thus a first order ODE with nonlinear terms of kinetic energy density and magnetic coupling. When considering the z term as perturbative (experimentally, it is small compared to the contribution of the velocity term), it is evident that the velocity of the system has an oscillatory nature. Increasing $H_0$ and ω both show a theoretical period doubling route. Qualitatively, this attributes the period doubling behavior observed in Fig.2-7 to the magnetic coupling. Varying the amplitude and frequency also exhibits a frequency contraction and expansion similar to that shown in Fig. 4, which actually promotes not a return to the 1-period state, but an extension to larger period states consisting of contractions and expansions of the frequency (though, this does not decisively throw out the chance of the observation of this being an instrumental effect).

## V. Concluding Remarks

Investigations of the ferrofluid showed a period-doubling feature of the dynamics of the fluid, in which the bifurcations occurred with changes in the amplitude, frequency, and voltage offset. The model discussed accounts for the period-doubling and for an additional feature of frequency contraction and expansion (this particular feature is still questionable in terms of being a result of the instrument). There is some error attributable to the inhomogeneity of the magnetic field. The variation of the field occurred within 0.01mT and the the lateral forces present can be rounded off as being symmetric along the center of the peak, and so their effects on measurment are not expected to be large. The proposed equation of motion did not account for surface tension and viscosity. The surface tension term is dependent on the curvature of the waveform; this may not necessarily have been negligible considering the sharpness of the peak (the average diameter was a good deal less than ~3mm).

## VI. References

[1] Thomas Mahr and Ingo Rehberg, “Nonlinear Dynamics of a single ferrofluid-peak in an oscillating magnetic field,” Physica D (1998).

[2] Mark I. Shliomis, “Ferrohydrodynamics: Retrospective and Issues”

[3] Kittel, Charles and H. Kroemer, “Thermal Physics,” (W.H. Freeman and Co., 1980), 2nd Edition, Chpt 3, pg 69-70.

[4] Kittel, Charles and H. Kroemer, “Thermal Physics,” (W.H. Freeman and Co., 1980), 2nd Edition, Chpt 10, pg 295-298.