Proposal: Synchronization of Three Dimensional Oscillators

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I. INTRODUCTION

In 1665, the Dutch scientist Christiaan Huygens discovered that two pendulum clocks mounted on the same wall synchronize with one another—the bobs swing with the same frequency but exactly out of phase[1]. The origin of this effect is weak coupling of the clocks mediated through the walls vibrations. Ever since, the seemingly old topic of synchronization has developed into one of the most actively studied phenomena in the fields of applied mathematics, nonlinear dynamics, statistical physics and material science.

Modelling a system of interest using a system of coupled pendulums, is a very general approach that one can observe in a number of different applications. It can be used to take on very practical problems like studying the Millennium Bridge problem and designing dampening systems for buildings to compensate for wind or seismic disturbances[2]. However it is also employed in the study of far more complex and interesting systems where it use has ranged from modelling the signaling patterns of insects (either visual or auditory), studying the interactions between large ensembles of neurons in the brain during complex activities[3], and designing sensing elements for gravitational waves[4].

Crowd synchrony on the Millennium Bridge, as a specific example of synchronization, has been investigated[2]. In that study, pedestrians, while moving forward along the bridge, fell spontaneously into step with the bridges vibration. This was due to the fact that in addition to their forward progress, a small component of their step was directed laterally to compensate for the small lateral sway of the bridge. After a transient state where the net lateral motion was small, the people on the bridge began to synchronize their steps, at least the lateral component of their steps, with the sway of the bridge due to the tendency to be thrown off balance in stride if your lateral motion was out of phase with the ever increasing sway of the bridge. The net result was a significant lateral deviation which grew with time and could, if left unchecked, potentially cause catastrophic failure of the bridge.

Understanding synchronization of pendulums has been a rather hot topic with a considerable amount of research tackling the problem of $N = 2^5$. Furthermore, the majority of research completed considers simple, driven pendulums[6]. Previous results have shown that various modes may occur during synchronization of pendula lying on the same plane[7]. The potential configurations of the system are (i) complete synchronization, (ii) synchronization of clusters for three or five pendulums, and (iii) total antiphase synchronization.

The goal of this work is to expand upon previous efforts which has analyzed synchronization of Huygens' clock to three dimensional pendulums[5,7]. Particularly, we intend to extend previous results by allowing for spherical pendulums which do not necessarily lie in a straight line. Rather, the pendula will rest on a base which is held up by Meissner bodies[8]. By conducting physical, analytical, and numerical experiments we hope to quantify what modes may occur and to determine how various parameters impact the synchronization of multiple pendula.

To this end, we believe that a more generous model of such synchrony on one base-plane will be of great interest and benefit the study of all similar phenomena in future.

A. Basic Theory

A system of differential equations for N coupled simple harmonic oscillators takes the form

$$\dot{\theta}^n(t) = \omega + \frac{k}{N}\sum_{i=1}^{N} \sin\left[\theta^i(t) - \theta^n(t)\right]. \quad (1)$$

In Figure 1, we present the synchronization phenomena of 11 coupled harmonic oscillators.

The complexity increases when we consider the case of a single spherical pendulum where the bob is attached to a solid rod, which is expressed as

$$\begin{cases} mr^2\ddot{\theta} - mr^2 \sin\theta\cos\theta\,\dot{\phi}^2 + mgr\sin\theta = 0, \\ 2mr^2\dot{\theta}\sin\theta\cos\theta\,\dot{\phi} + mr^2\sin^2\theta\,\ddot{\phi} = 0. \end{cases} \quad (2)$$

II. METHODS

A. Experimental set-up

Before studying the synchronization of coupled pendulum on same base plane, we will first study non-linear dynamics of one pendulum. A pendulum was positioned

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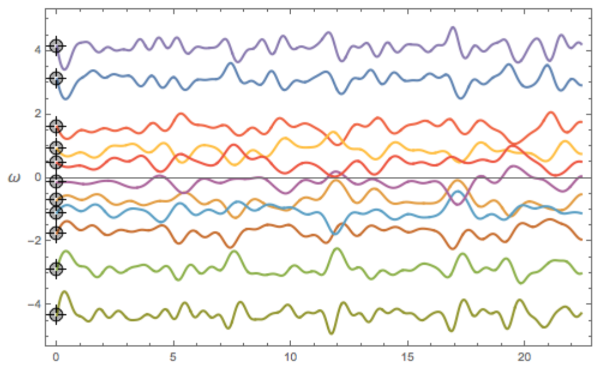

FIG. 1. Phases of 11 coupled harmonic oscillators

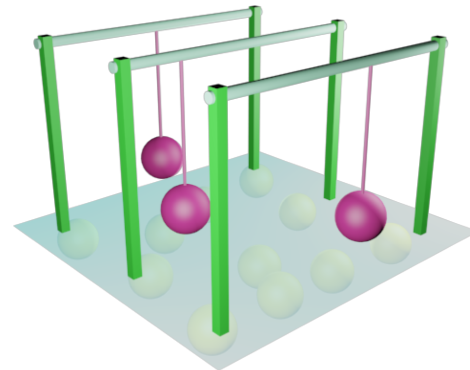

FIG. 2. Coupled pendulum on same base plane.

at the center of a base plane, with several glass spheres located randomly underneath the base plane. We found that the weight of the base plane strongly affected the dynamics of one pendulum, that is, when the base plane is relative light, it will move in large amplitude to accommodate with motion of pendulum. To this end, the synchronization of coupled pendulum could not be achieved. Thus we increase the weight of base plane in the final design of coupled pendulum on same base plane (shown in Figure 2). The spheres we used for pendulum is 25±0.2mm in diameter and 21.2 ±0.2g in weight, which are coated with IR reflective tape for 3D position recording. The spheres were hanged to the hard frame by soft strings; the weight of strings was neglectable. The dimension of the base plane was 18 inches by 25 inches, and we attached two wood stick on its two sides to increase the weight.

We studied the synchronization of coupled pendulum by placing three pendulums on the base plane in triangular positions. We did not try more than three pendulums in our study due to the limited space of the base plane. However, we found that even with three pendulums, the synchronization behavior was complicated, partially because that the dynamics of pendulum was nonlinear regime at the beginning stage (i.e., we manually released the pendulums at very large angle away from the center line). Notably, the motion of the base plane is subtle due to the large mass, thereby leading to a slow synchronization of the three pendulums.

B. Results

By recording the position of the pendula we will were able to generate phase plots in addition to a host of other qualitative and quantitative results of the relative positions of the pendula.

Through analyzing the configuration space of the initial positions of the pendula, we discovered quasiperiodic synchronization, beat formation, and chaos in the system.

The phase space of the respective pendula showed tori that decayed toward the origin over time. Thus showing the effect of friction on the system.

We also found drifting of the system over time when plotting every 15th point of the trajectory in phase space. This is due to the small effect of rotation of the Earth. During the course of our 7 minute experiment we observe an angular drift of

$$\delta\phi = \frac{7 * 2 * \pi}{24 * 60}. \tag{3}$$

We thus expect approximately a 2^o drift overall.

We observed that starting the system from essentially the same configuration with a slight perturbation led to radically different transient behavior, but similar synchronization and beat formation over the course of the experiment.

The tentacle structures in the phase space come from synchronization shooting pendula into other modes while it drifts.

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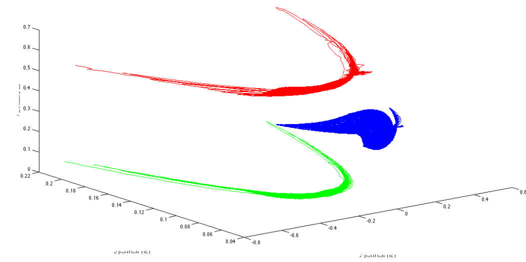

FIG. 3. Triangle Configuration Orbit with In Phase Pendula

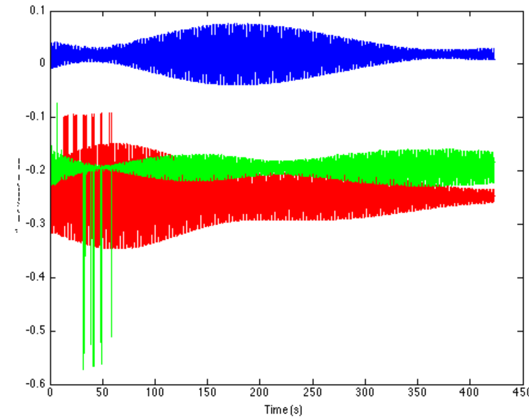

FIG. 4. Triangle Configuration showing beats and synchronization

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III. APPENDIX

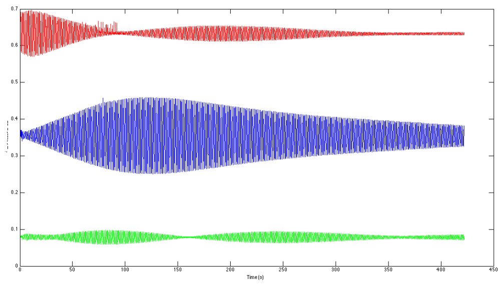

FIG. 5. Triangle Configuration Z Position

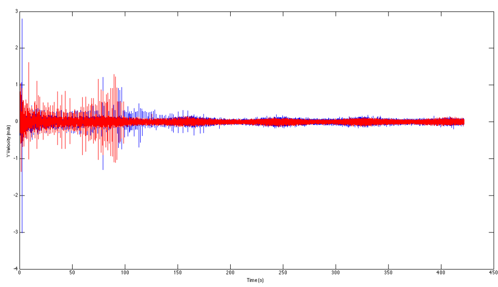

FIG. 6. Triangle Configuration Y Velocity

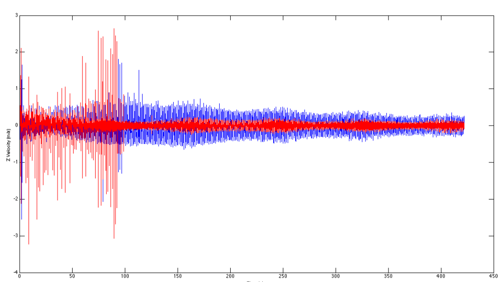

FIG. 7. Triangle Configuration Z Velocity

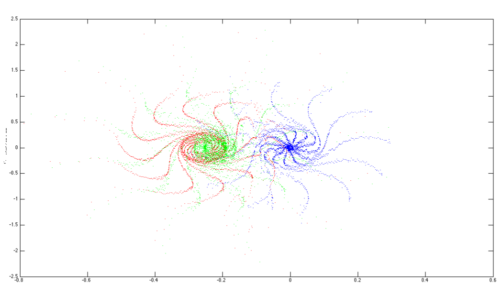

FIG. 8. Triangle Configuration X Phase Space

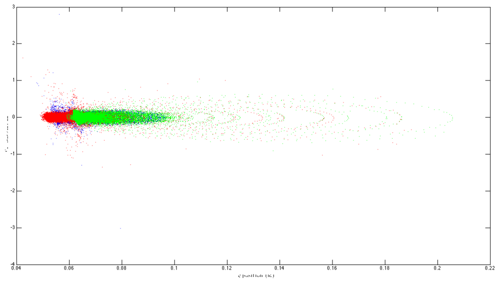

FIG. 9. Triangle Configuration Y Phase Space