Fractal Basins in a Magnetic Pendulum System

Galen Craven, Chris Martin, Fredrik Ekström, and Jung Byun
“Designed to illustrate the random forces that effect us all. Can you make order out of this chaos? If so, you could probably write a bestseller”

-ThinkGeek.com
Background

• Toy system that has complex dynamics and fractal formation
• Basins of attraction / fractal boundaries
• Interesting Patterns
Fig. 1. Schematic diagram of the physical pendulum and the forces acting on it.

- Gravitational force
- Magnetic force
- Damping
Final State Sensitivity

• The magnets are stable fixed points
• Simulation of trajectories from almost same initial conditions
• Example videos of a few runs (from almost same initial conditions)
Goals

- Recreate fractal using experimental data
- Investigate the dynamics of the system
- Optimize parameters for simulations
Experimental Setup

• Features
  – Pendulum
  – Free 2D pivot
  – Magnet on Rod
  – Base magnets
  – Correct Polarity “base attracting pendulum”
Measurements to Take

• Initial Conditions
  – Measured with Picture of initial state

• Final Position
  – Final position of pendulum
  – (1,2,3) or (r,g,b)
Things to Control

- Need initial Velocity to be zero
  
  \( V_0 = 0 \)

- Stabilize on aluminum ring

- Better than trying to hold steady
Special Considerations

- Magnet Strength
- Length of Pendulum
- Mass of Pendulum
- Separation of Magnets
- Orientation of magnets
- Distance of Base magnets from Pendulum
- Friction in pivot joint
Physical Experiment
Data Acquisition

- Static Pictures
- Whiteout on base to help
- Final Position tagged onto photo
Difficulties

- zero velocity initial condition
- Light intensity variations
Image processing

- Need to find position of white dot
- Tried using a threshold and compute centroid, but did not work
- Marked position manually instead
Model

- Gravitational force: \( mF_g = -C \begin{bmatrix} x \\ y \end{bmatrix} \)

- Damping force: \( mF_d = -R \begin{bmatrix} x' \\ y' \end{bmatrix} \)

- Force from magnet:

\[
mF_i = A \frac{1}{\left( (x_i - x)^2 + (y_i - y)^2 + d^2 \right)^{3/2}} \begin{bmatrix} x_i - x \\ y_i - y \end{bmatrix}
\]

Model

\[ x'' = -Cx + A \sum \frac{x_i - x}{\left((x_i - x)^2 + (y_i - y)^2 + d^2\right)^{3/2}} - Rx' \]

\[ y'' = -Cy + A \sum \frac{y_i - y}{\left((x_i - x)^2 + (y_i - y)^2 + d^2\right)^{3/2}} - Ry' \]
Parameter estimation
Parameter estimation

- Equation: \( x'' + Rx' + Cx \)
- Solution: \( x \propto \exp(-\alpha t) \sin(\omega t) \)
- Gives \( R \) and \( C \)
- \( d \) and \( A \) trickier
Parameter estimation

• Minimize

\[ N(A, C, R, d) = \# \text{misses} \]

• Improved agreement a bit but not much
Basins of attraction, weak magnets

d ≈ 6 mm

Points ≈ 2300
Basins of attraction, strong magnets

$d \approx 4 \text{ mm}$

Points $\approx 1200$
Basins of attraction, weak magnets

d \approx 6 \text{ mm}
Basins of attraction, strong magnets

\[ d \approx 4 \text{ mm} \]
Simulations
Varying $d$

- $C = 25 \text{ N/m}$
- $R = 0.41 \text{ N/m} \cdot \text{s}$
- $A = 0.0003 \text{ N/m}^2$
Simulations

Varying $A$

- $C = 25 \, \text{N/m}$
- $R = 0.41 \, \text{N/m} \cdot \text{s}$
- $d = 0.004 \, \text{m}$
Time Dependence of Final State  
(How long it takes to reach final state)

- $C = 25 \text{ N/m}$
- $R = 0.41 \text{ N/m}\cdot\text{s}$
- $d = 0.005 \text{ m}$
- $A = 0.0001 \text{ N/m}^2$
\[ \sqrt{(Vx1)^2 + (Vy1)^2} \text{ [m/s]} \]

\[ \sqrt{(Vx2)^2 + (Vy2)^2} \text{ [m/s]} \]

\[ \sqrt{\left( \sqrt{(Vx2[r])^2 + (Vy2[r])^2} - \sqrt{(Vx[r])^2 + (Vy[r])^2} \right)^2} \text{ [m/s]} \]
Conclusions

• Varying Opinions
  – Opinion 1: Experiment is so flawed that the measured points must be rejected. Simulation accepted
  – Opinion 2: Data is good. Reduction of dimensionality in simulation model is a poor approximation

• Agreement
  – Chaotic systems are difficult to probe experimentally
Improvements

• Automate: More points and less variance in initial velocities

• Working parameter optimization algorithm

• Automatic centroid tracking
Contributions

• Fredrick
  – Experimental Design
  – Data Acquisition
  – Data Analysis
  – Simulation Code

• Chris
  – Experimental Design
  – Data Acquisition
  – Data Analysis

• Jung
  – Experimental Design
  – Data Acquisition
  – Data Analysis

• Galen
  – Experimental Design
  – Data Acquisition
  – Data Analysis
  – Simulation Code
Acknowledgments

• Andrei
• Nick
• Dr. Goldman
Thank You