Jumping Robot

Julien Stalla-Bourdillon, Stephan Froehlich, Reuven Ballaban

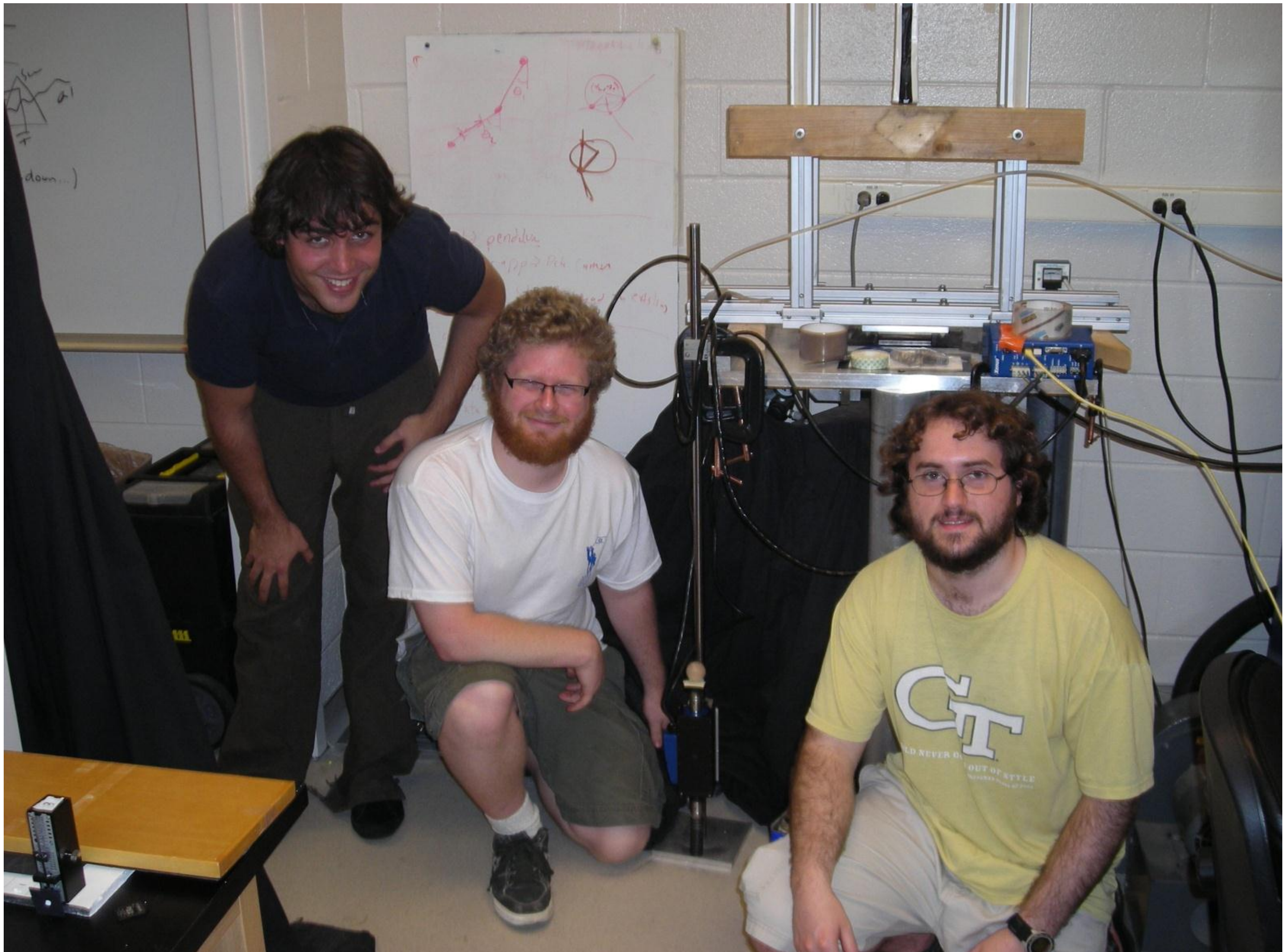
Introduction

Reuven Ballaban

My Contributions

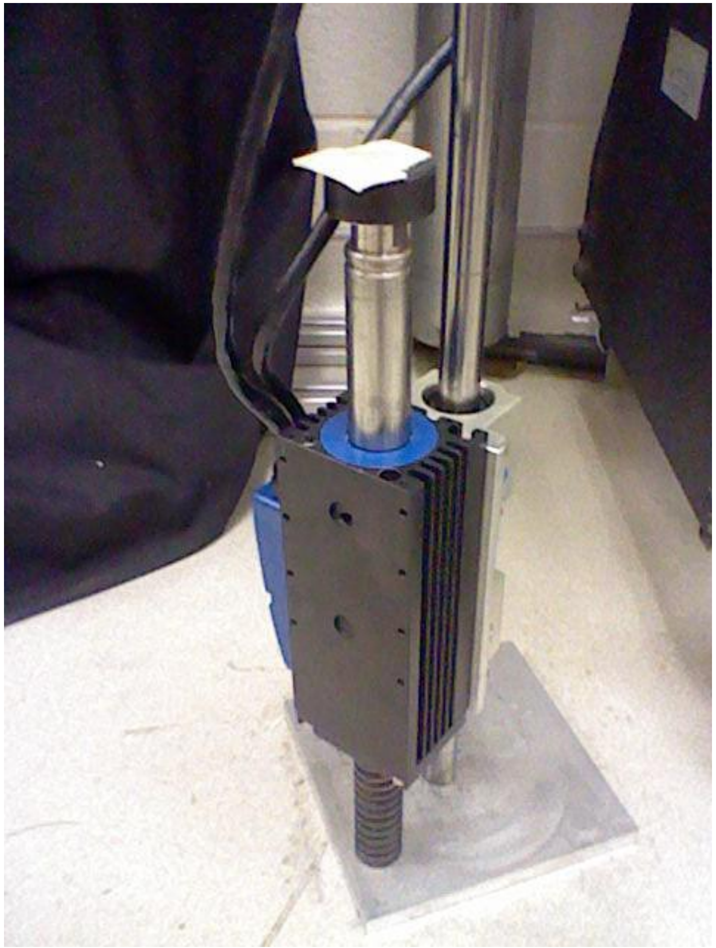
- Background Research
- Data acquisition & minor rig modifications
- Analysis

The Team



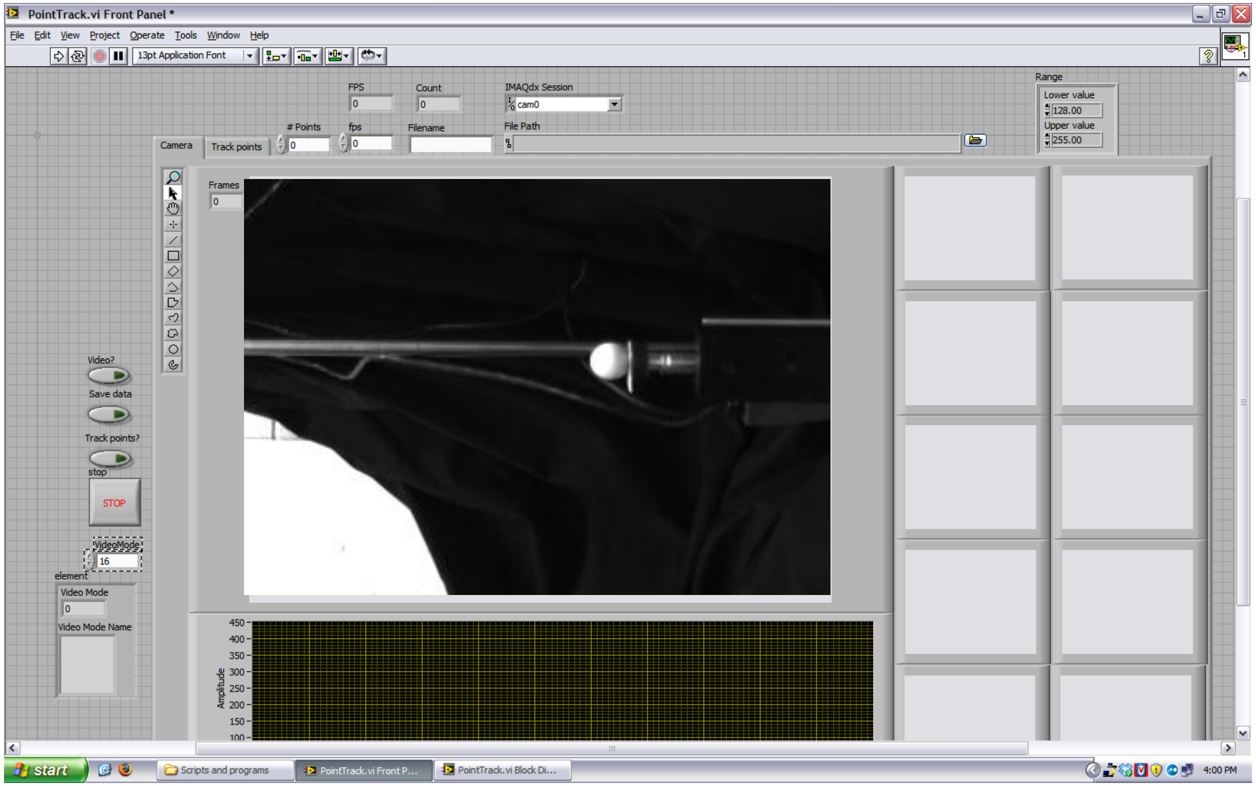
The Rig

- The Jumping Robot rig consists of a relatively heavy actuator attached to a pole.
- Spring attached to the bottom of the actuator allows robot to bounce.
- Floor, aluminum plate, power and control cables serve as damping.



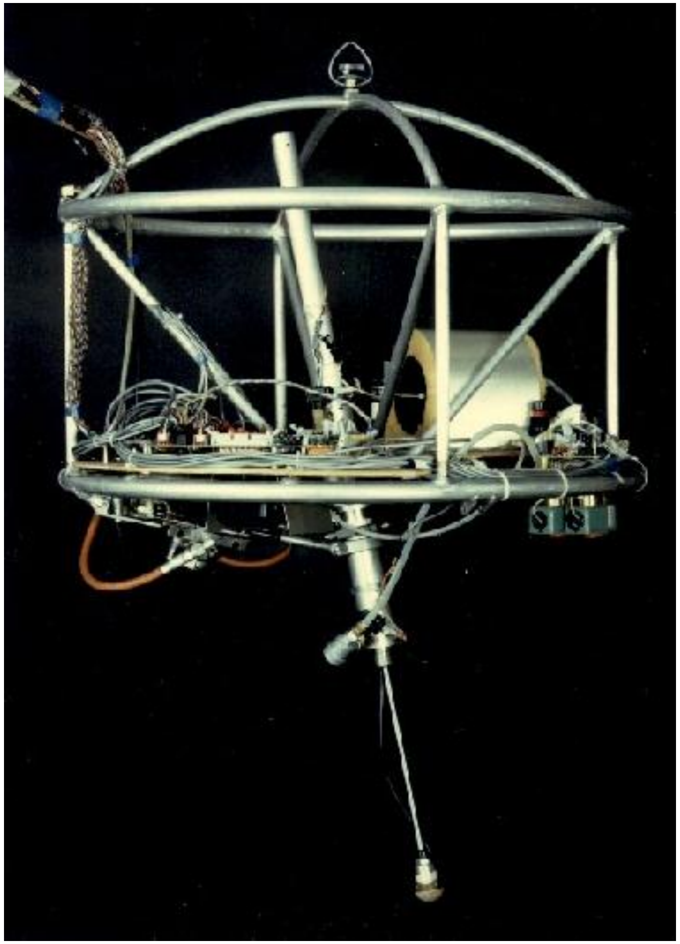
The Rig (pt.2)

- Motion tracking camera at 100fps captured the motion of the robot and output location.



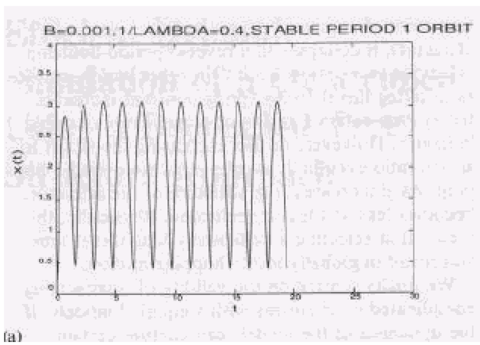
Background

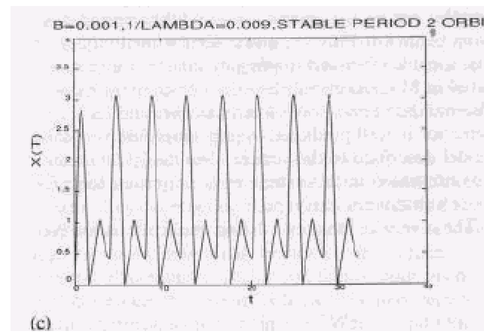
- Raibert's Hopper



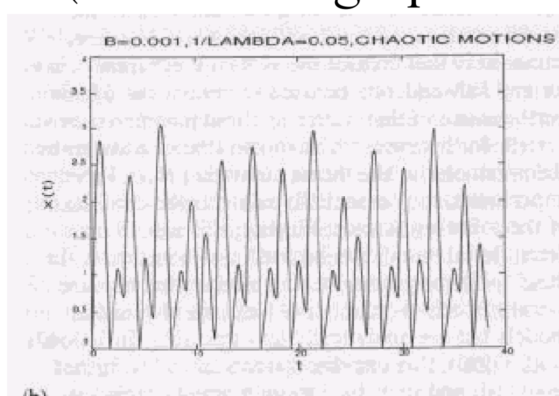
Background (pt.2)

- An Interesting Strange Attractor in the Dynamics of a Hopping Robot (A.F. Vakakis, J.W. Burdick, T.K. Caughey)
  - 1-cycle stable, "limping" gait (2-cycle)





  - Stable 3-cycle (no motion graph in article), chaos



Background (pt.3)

- Analysis of a Simplified Hopping Robot (Daniel E. Koditschek, Martin Bühler)

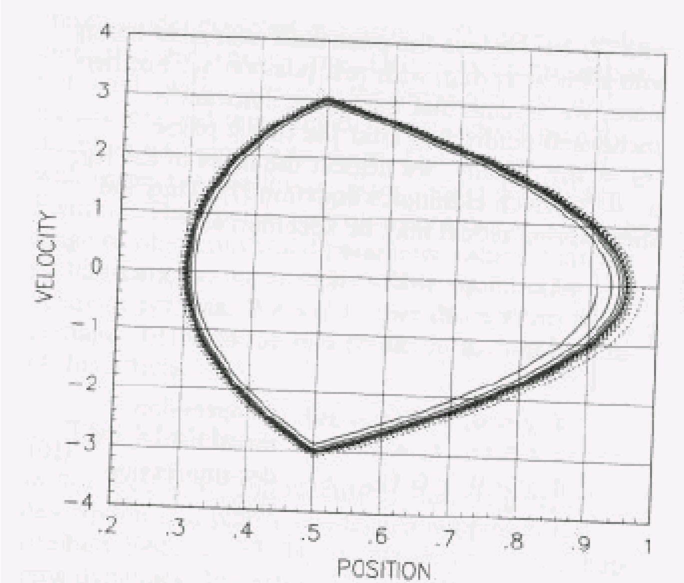
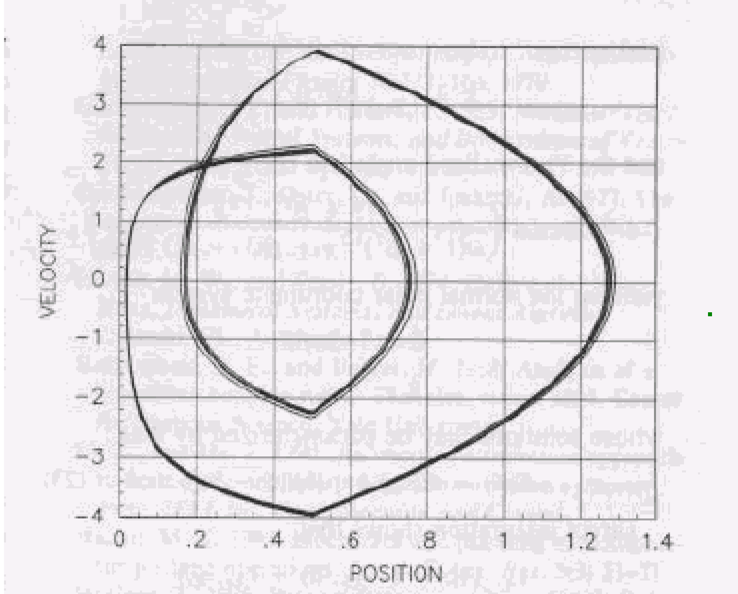
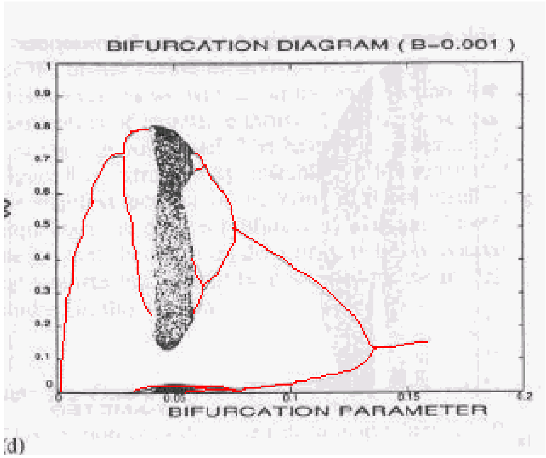


Fig. 8. Full linear spring model.



Procedure



Example from Vakakis, et al with highlights for clarity.

- Our original goal was to produce an orbital map.
- Accomplished this by using a set frequency and sweeping through different oscillation amplitudes.
- Results are noisy, but interesting.

Difficulties

- Camera sometimes lost lock on tracking point momentarily, causing (presumably) negligible noise.
- Rig put out enough force to gouge floor, solved by placing aluminum plate under spring
- Rig occasionally orbited pole, resulting in lost tracking.
- Original LabView program written to control robot did not work; had to use more basic tools resulting in more coarse frequency resolution.

Jumping Robot part 2: Results and Data

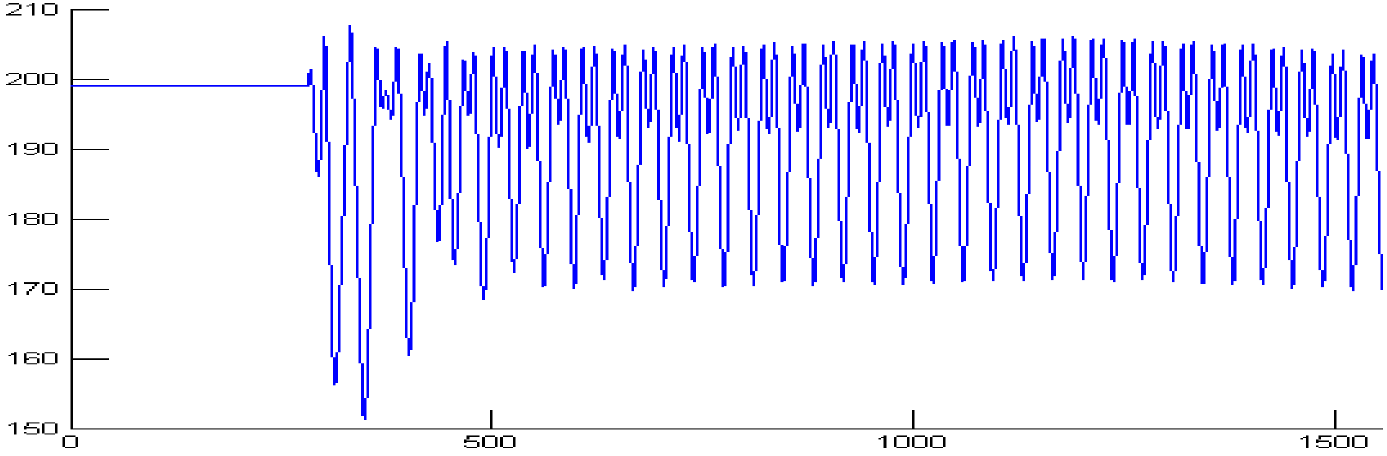
By: Stefan Froehlich

My contributions

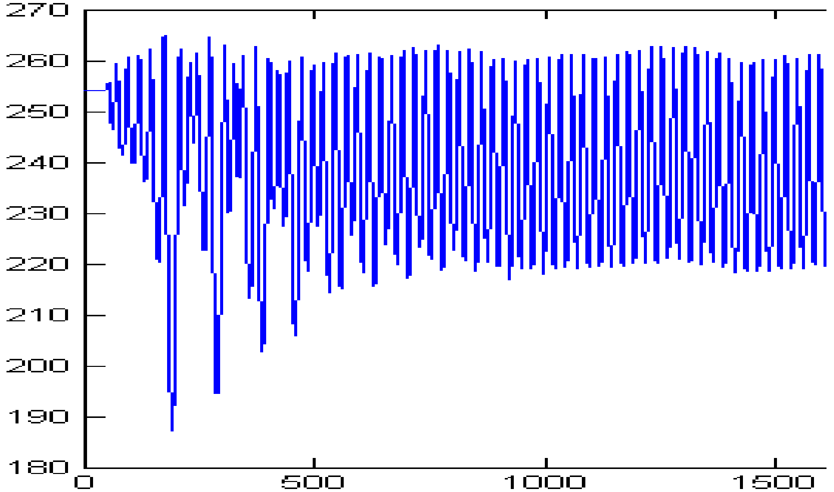
- Helped out with data taking
- Helped out with analysis of data

Example trajectories

2-Cycle at 6Hz with amplitude 1125

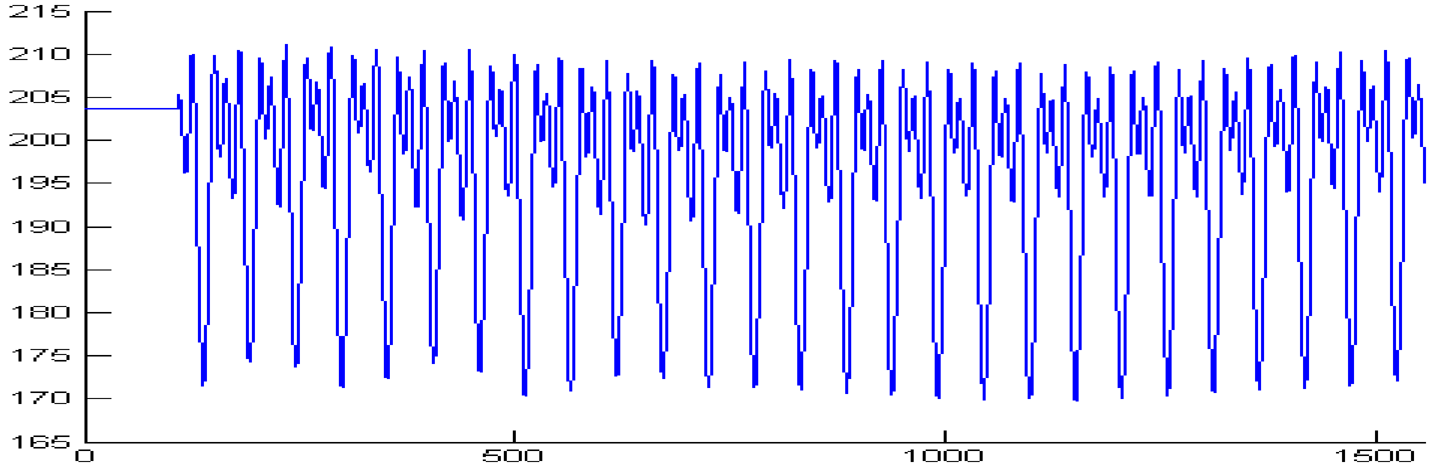


1-Cycle at 4Hz with amplitude 1250

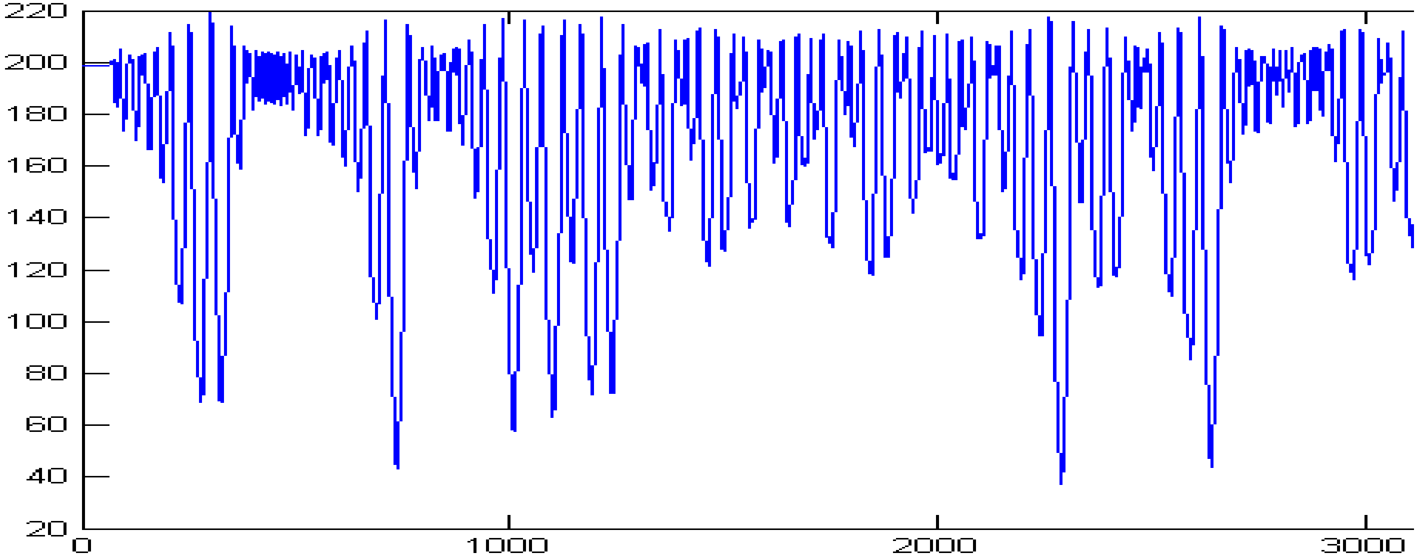


Example trajectories

3-Cycle at 6Hz with amplitude 825

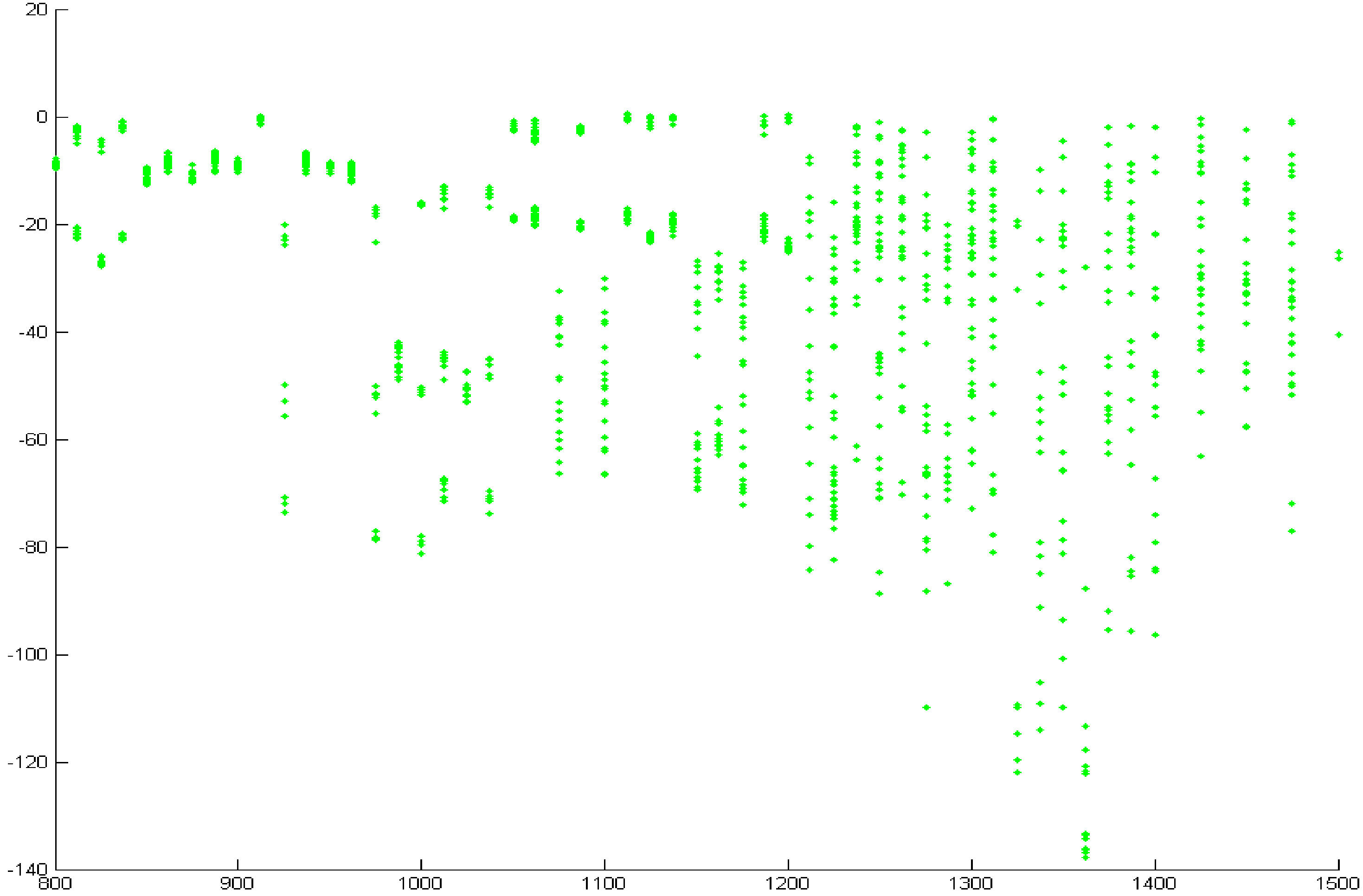


Chaos at 7Hz with amplitude 1200



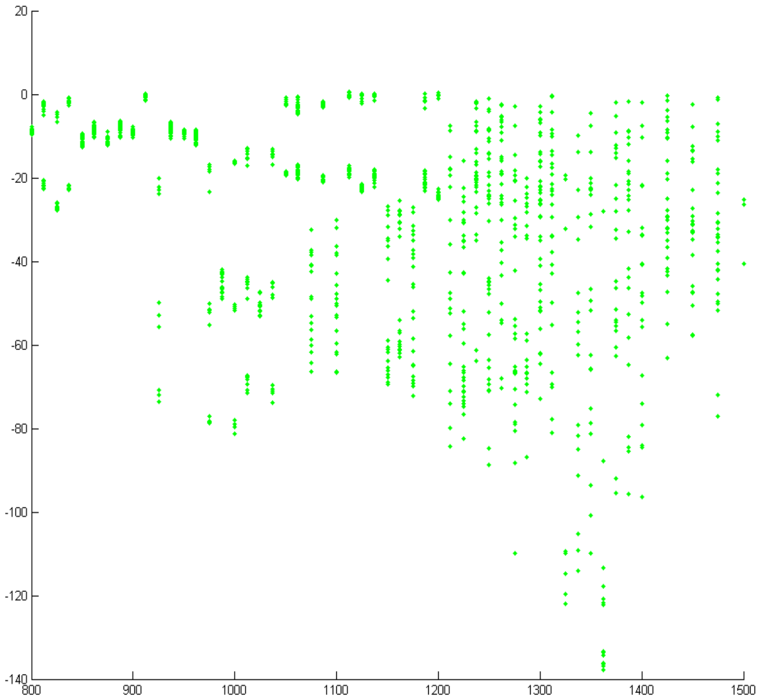
Orbit Diagrams

6 Hz



6 Hz

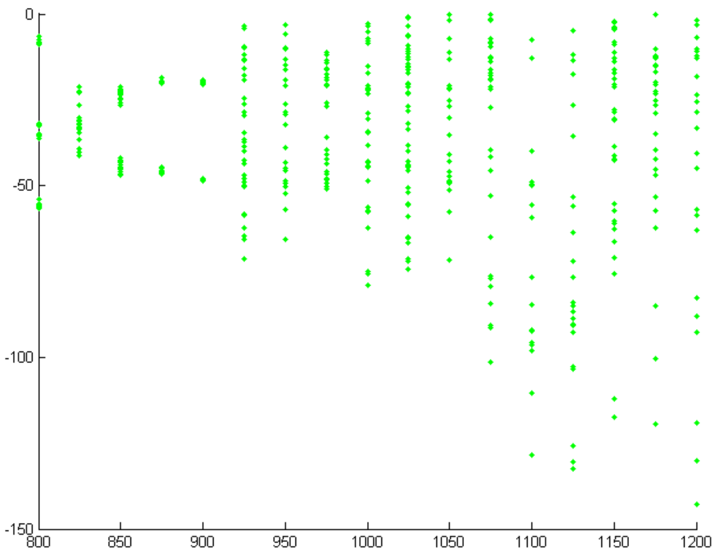
- As the amplitude increases there are:
  - 2-cycles initially
  - 1-cycles
  - 3-cycles with larger amplitude
    - 1-cycle in this region at 1025
  - 2-cycles
    - What appear to be chaotic trajectories with larger amplitudes interspersed
  - Chaos
    - Starts at 1212



7 Hz

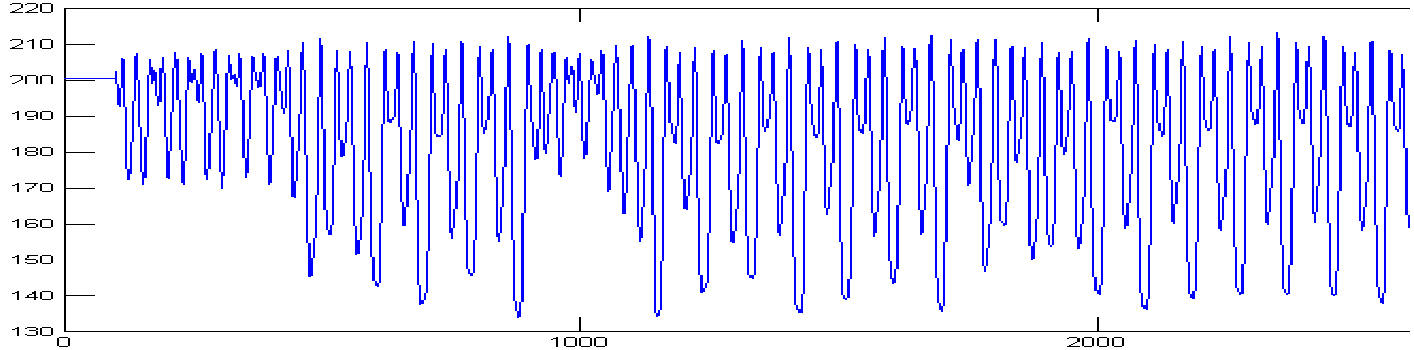
- 3-cycle initially
- 2-cycles
  - From 850 to 900
- Chaos
  - Starting at 950

- Notice the transition from 3-cycles to 2-cycles at 825

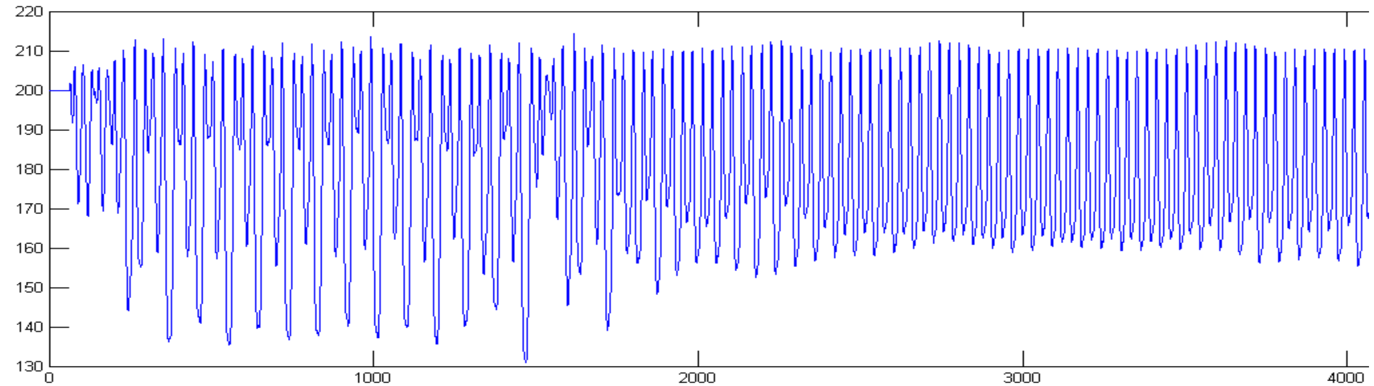


Transition

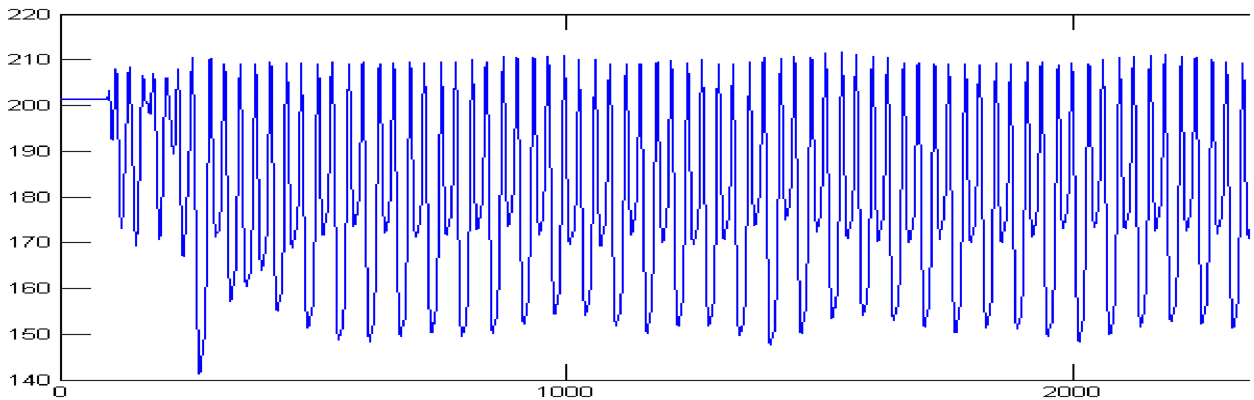
3-cycle at 800



Transition at 825

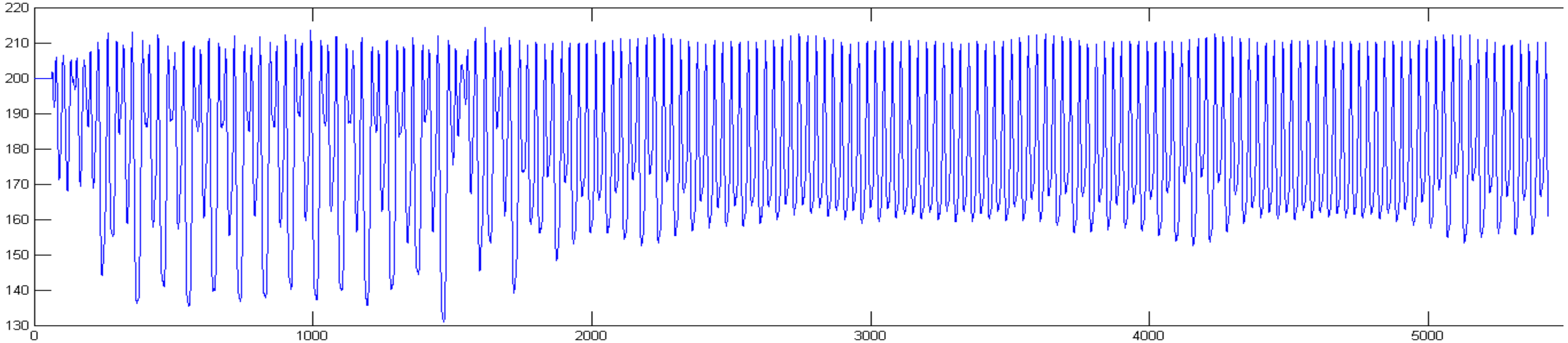


2-cycle at 850



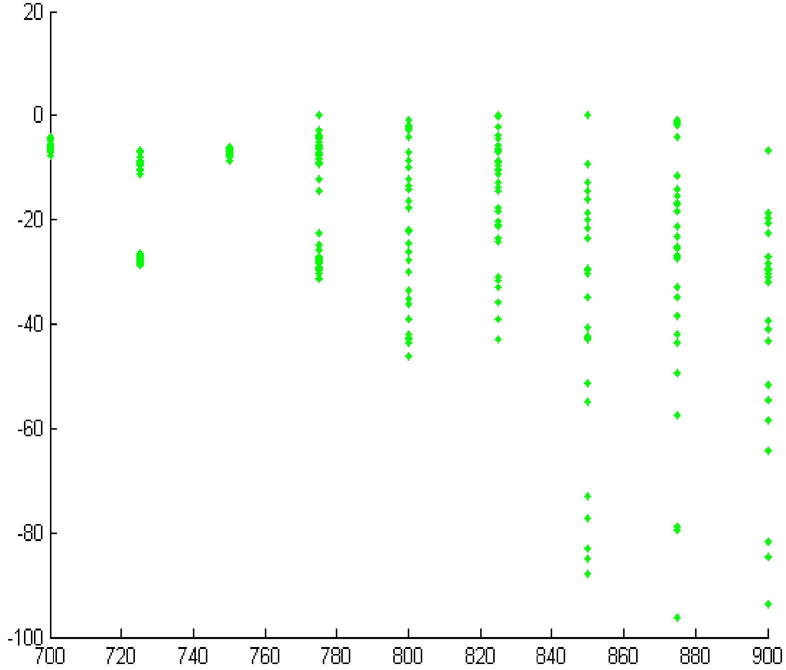
Transition

- Starts out with the 3-cycle
- Transitions into a state whose short term appears to be a 2-cycle, but has a different long term behavior
- These types of states, with short term behavior different from long term occur frequently where the cycle length bifurcates



8 Hz

- 1-cycles and 2-cycles initially
- Quickly diverges into chaos starting at 775
- Not as much of a cascade before it reaches chaos

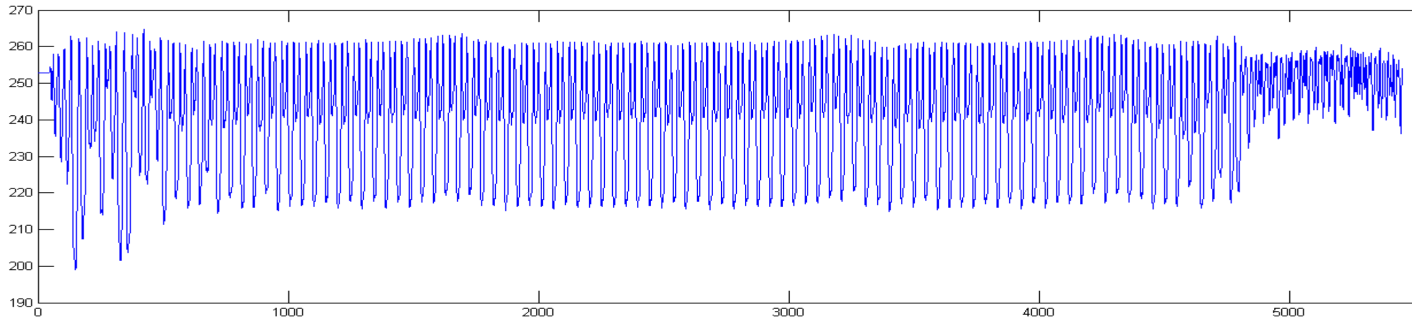


Chaos

Took data for 8Hz at 775 three times

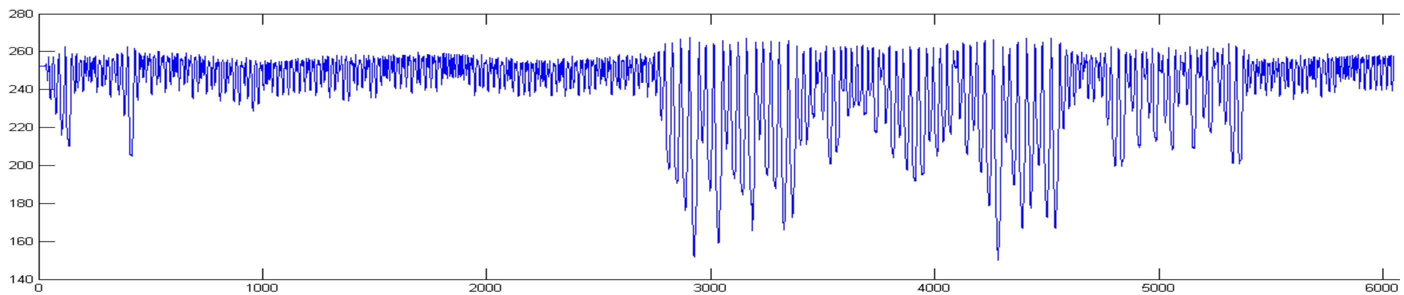
Trial 1

Follows 3-cycle for a long time before finally becoming chaotic



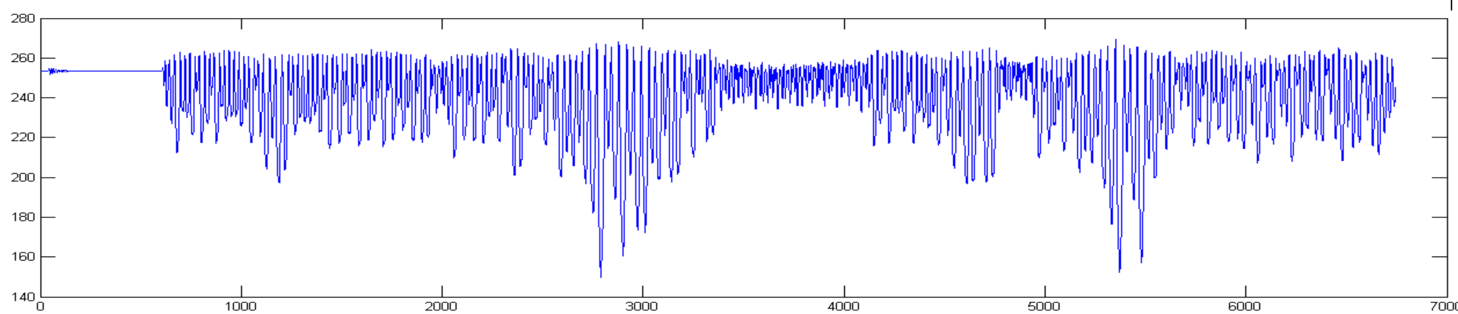
Trial 2

Begins chaotic and has large regions of small amplitude.



Trial 3

Begins chaotic and again has regions of small amplitude

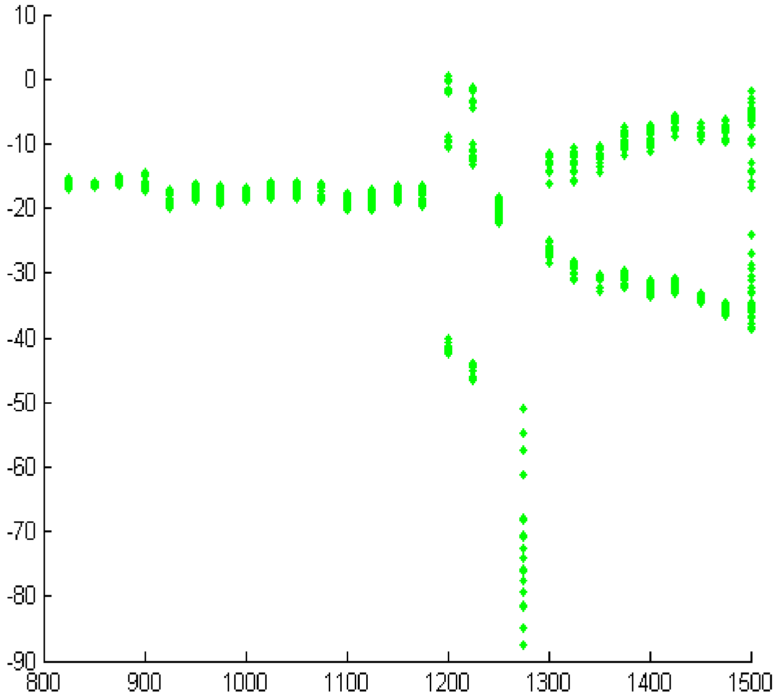


These regions of small amplitude chaotic motion were common for 8 Hz

5 Hz

- Large region of 1-cycles
- 3-cycles and chaos in transition region
- Followed by 2 cycles

- Interesting bi-stability occurs in transition region



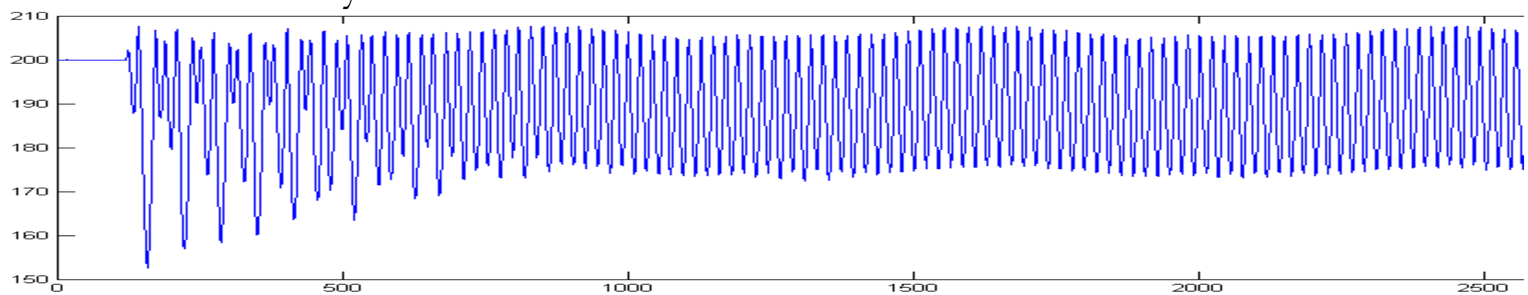
Bi-stability

Took data for 5Hz at 1212 in the transition region three times

There are stable 1- and 3-cycles at these conditions

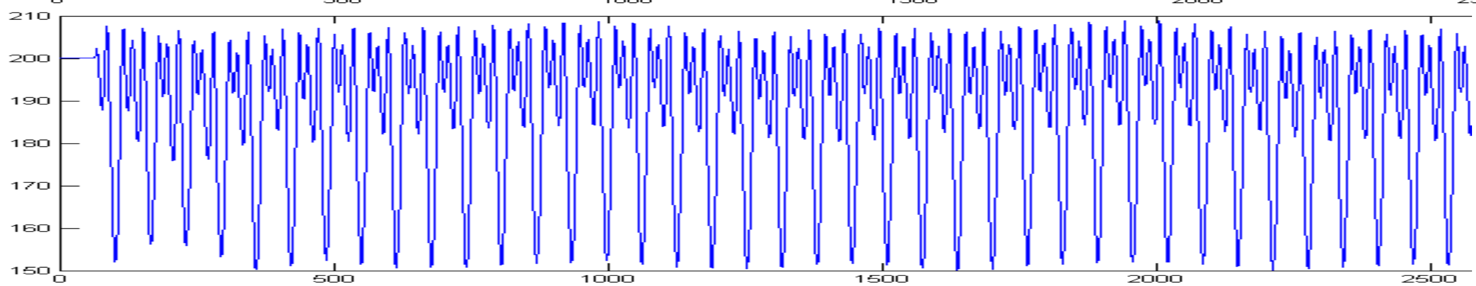
Trial 1

Transitions to 1-cycle quickly



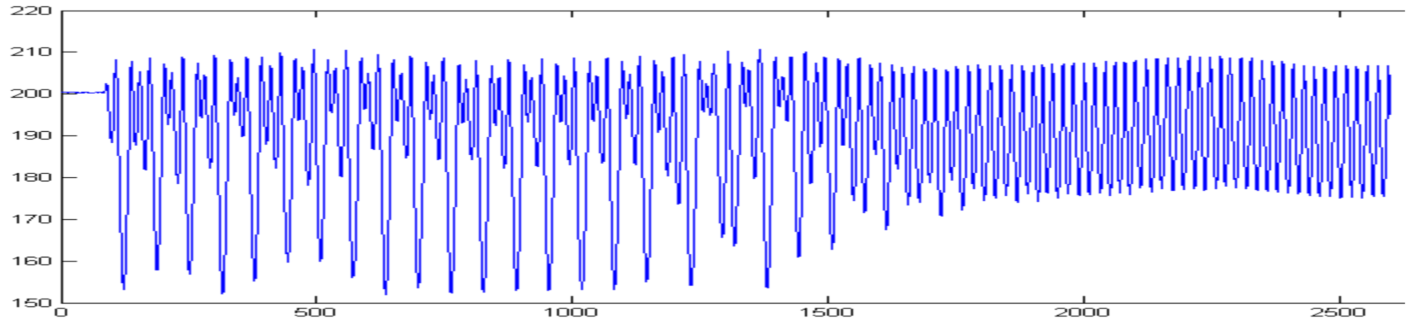
Trial 2

Stays at stable 3-cycle



Trial 3

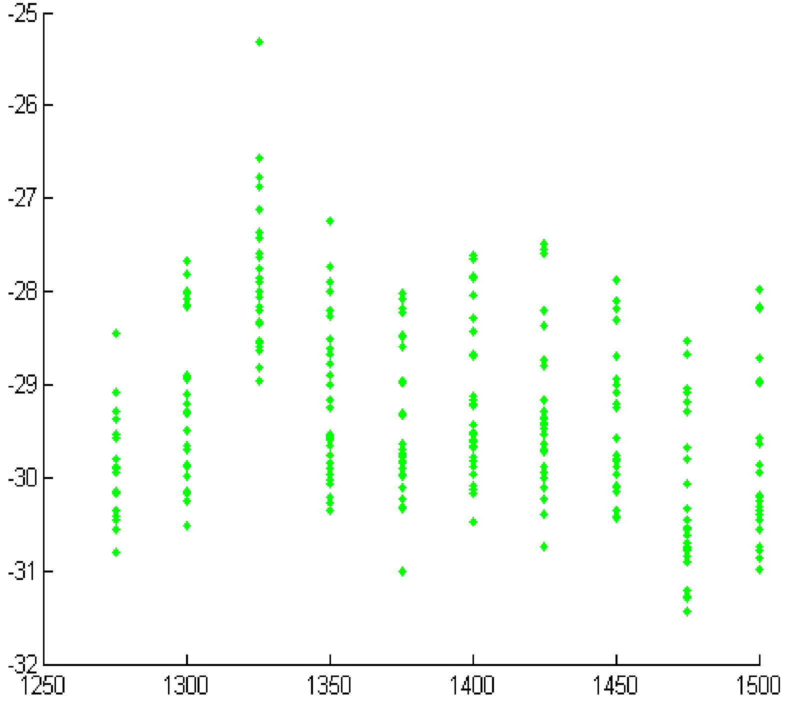
Stays at 3-cycle for long time before transition to 1-cycle



Bi-stability

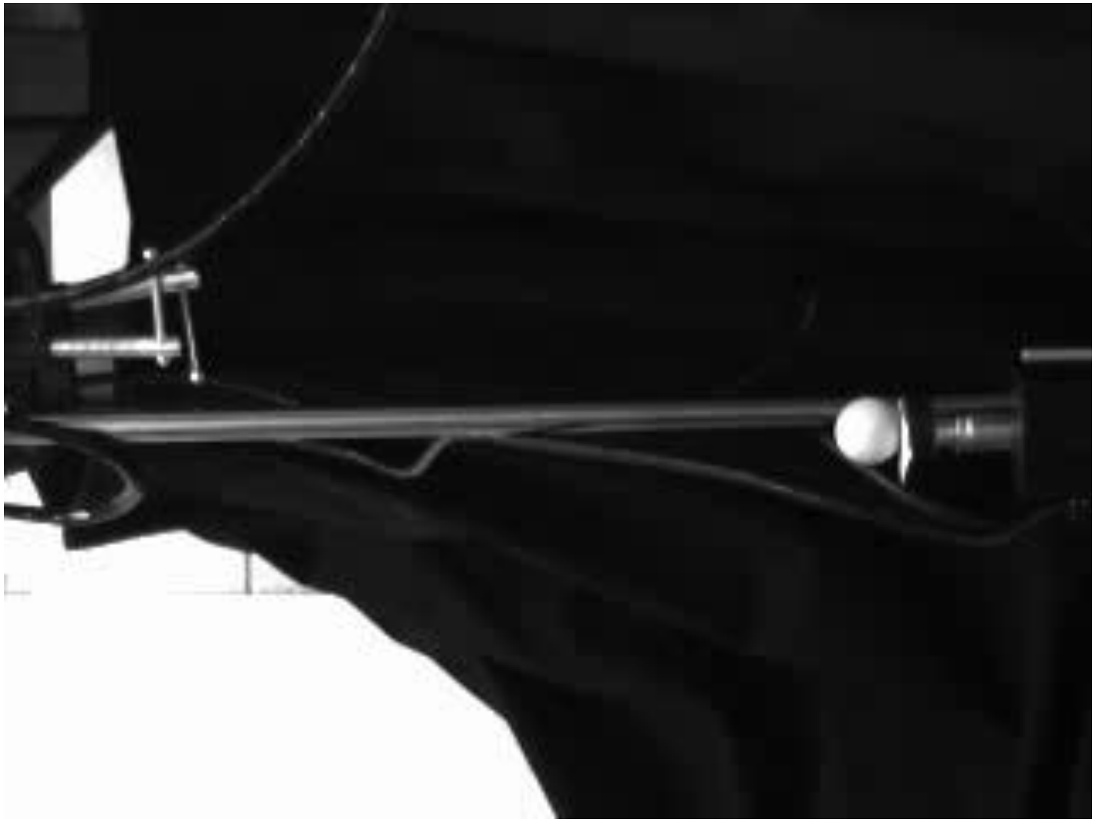
4 Hz

- There only 1-cycles
  - The 1-cycles are stable
  - We were unable to find anything else



Stable 1-cycle

Video was taken for frequency of 4 Hz and amplitude of 1700



Chaotic regions

- At 8 Hz the system quickly progresses into chaos with very minimal cascade
- For 6 and 7 Hz there was a cascade into chaos
- For 5 Hz there was the transition to 2-cycles but there was no chaotic region in the range tested
- For 4 Hz we only saw 1-cycles

- As frequency decreases, the chaotic region for the system disappears

Jumping Robot part 3: Analysis and Conclusion

Julien Stalla-Bourdillon

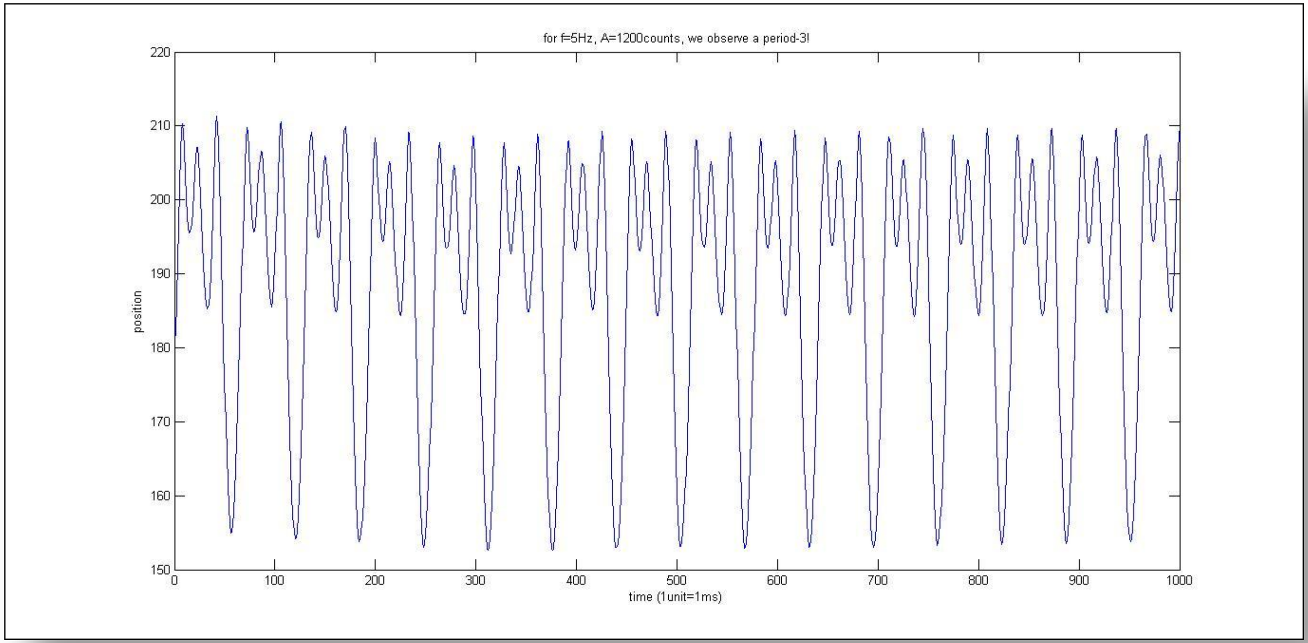
My Contributions

- Data Acquisition
- Data Analysis
- Research

Data Analysis

- Observation of a ‘period-3’, so thanks to Sarkovskii’s relation order and the famous article ‘period-3 implies chaos’, we know that our system will exhibit some chaotic behavior.

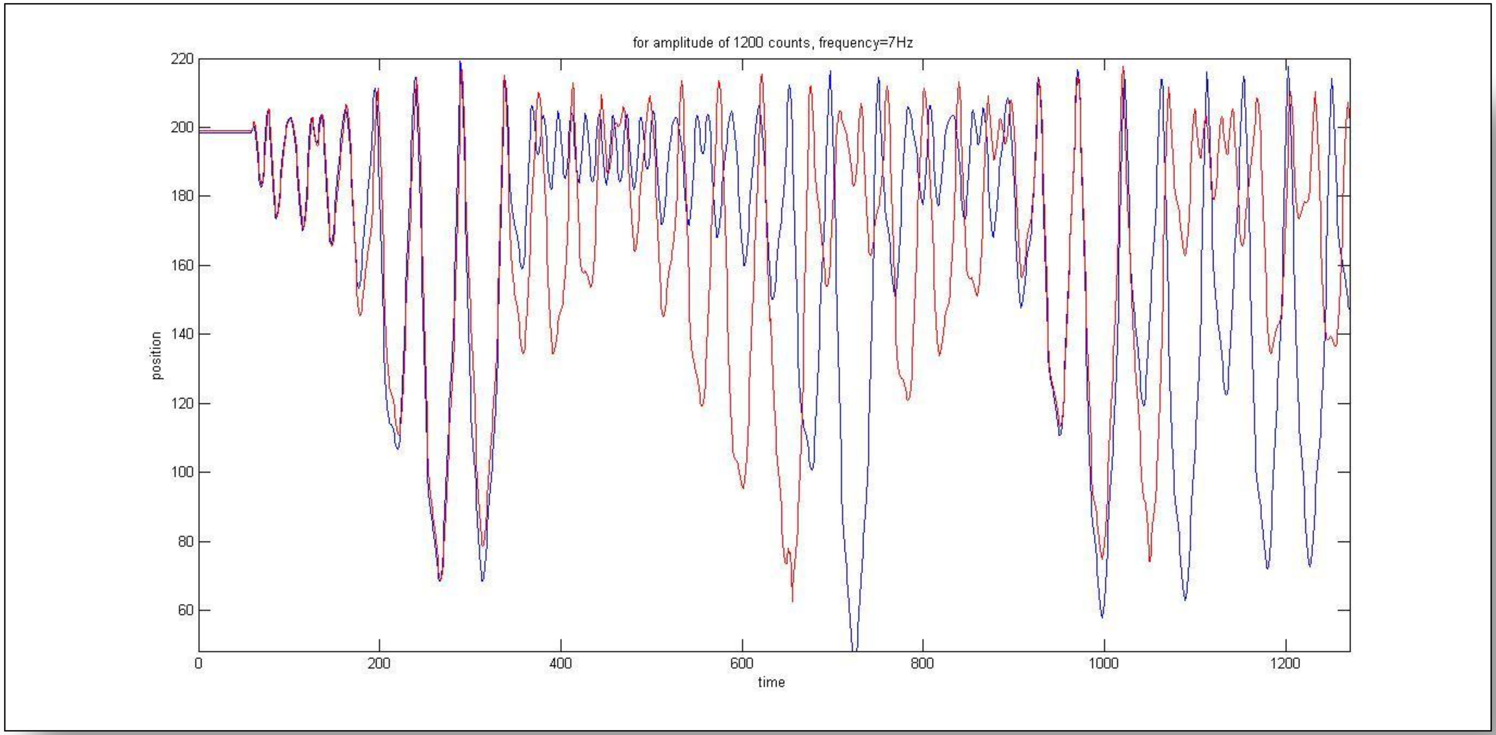
Example with f=5Hz, amplitude=1200counts:



Data Analysis

- High sensibility to initial conditions, for a fixed set (frequency, amplitude, initial position and speed), we observe two really different trajectories.

Example with f=7Hz, amplitude=1200counts:



Data Analysis

- How to determine if our series is chaotic or not? Different possibilities:

➢ Graphic analysis

➢ Power Spectrum Density and the Autocorellation of the signal
(comparison with white noise)

➢ Use of the Bifurcation Diagram

➢ Use the '0-1 test for chaos'*

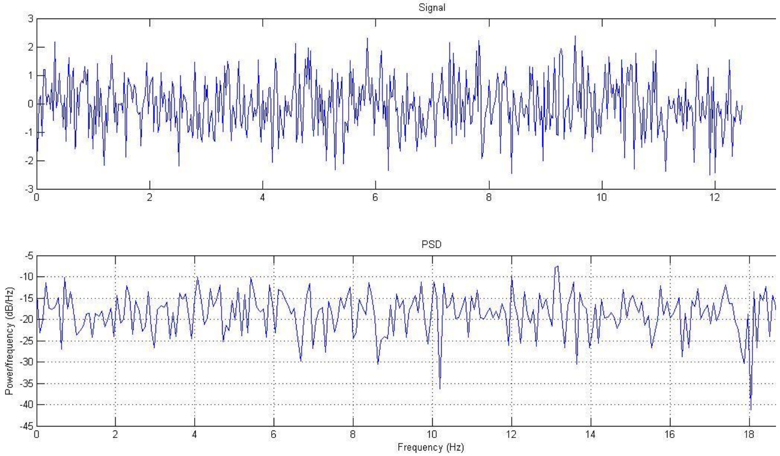
➢ Determine the Largest Lyapunov Exponent
Recall: the sum of the Lyapunov exponents must be negative and at least one of them is positive

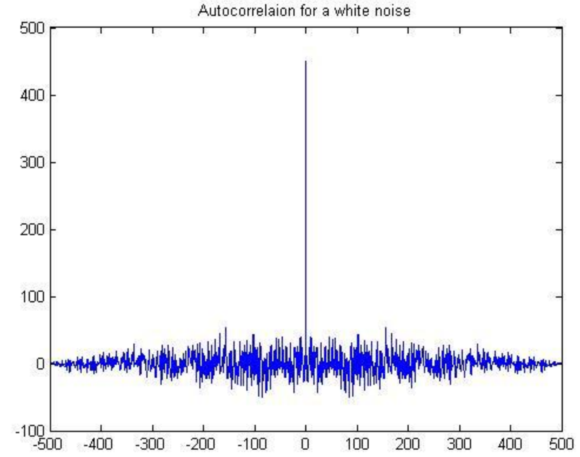
* See 'On the Implementation of the 0–1 Test for Chaos' from Georg A. Gottwald & Ian Melbourne

Data Analysis

- Graphic Analysis is not sufficient
- PSD and Autorocorellation of our data exhibts only ‘weak chaos’:

<u>PSD and Autocorellation of a white noise:</u>

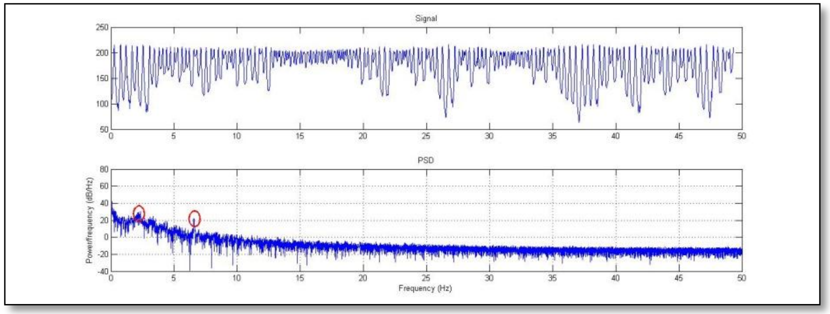


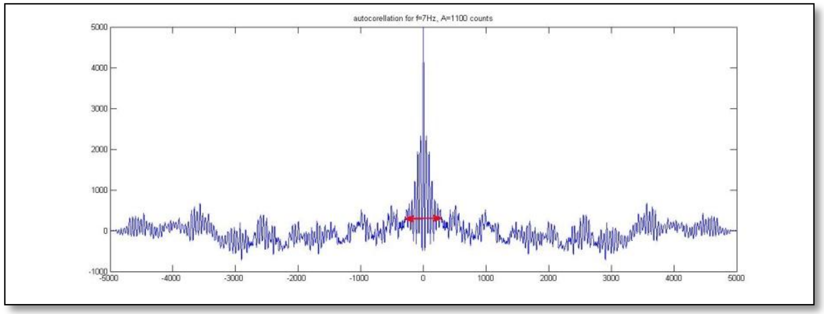


Data Analysis

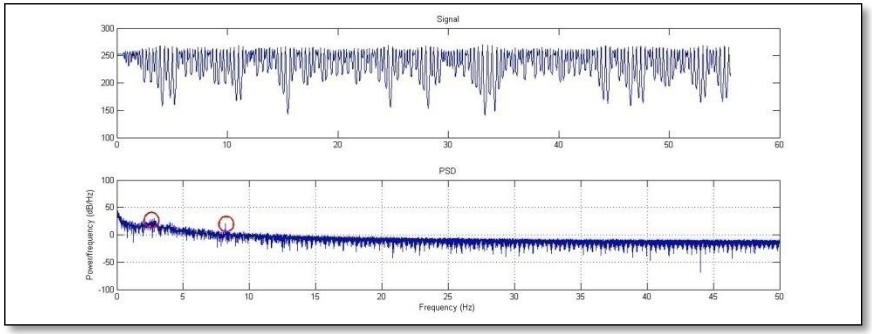
- For our data, the time-series which look like white noise we have this type of PSD and autocorrelation:

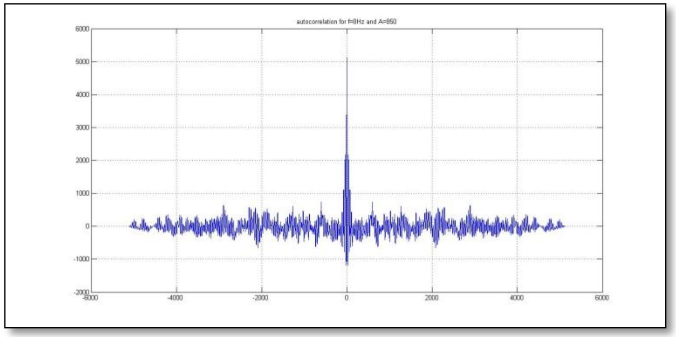
<u>For f=7Hz, A=1100:</u>





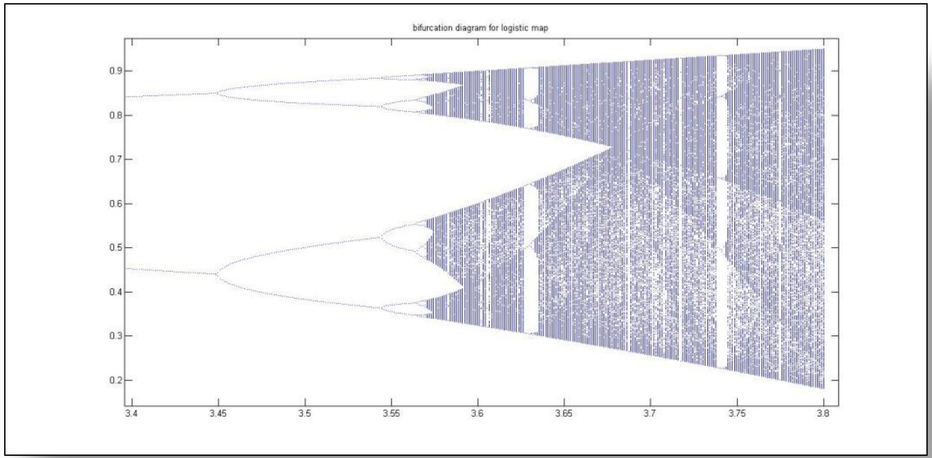
<u>For f=8Hz, A=8500:</u>

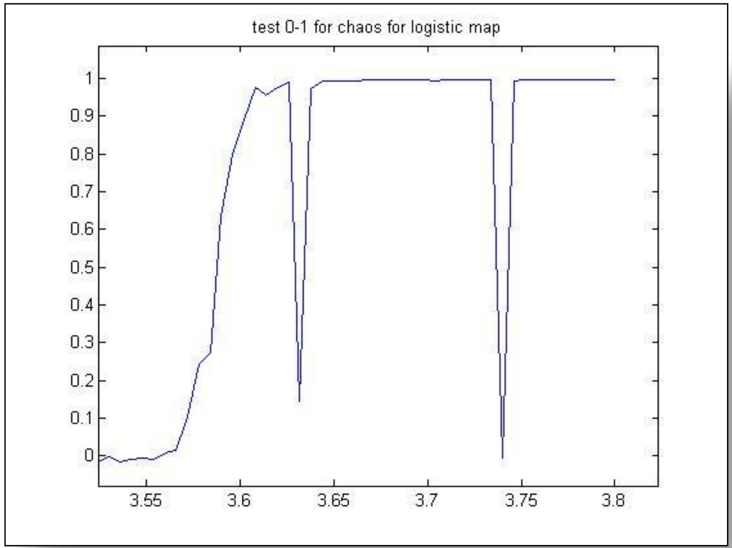




Data Analysis

- The '0-1 test for chaos' has been test on a lot of well-known systems but is not perfect* but gives a good idea of the range we need to work on.
- Example with the logistic map:

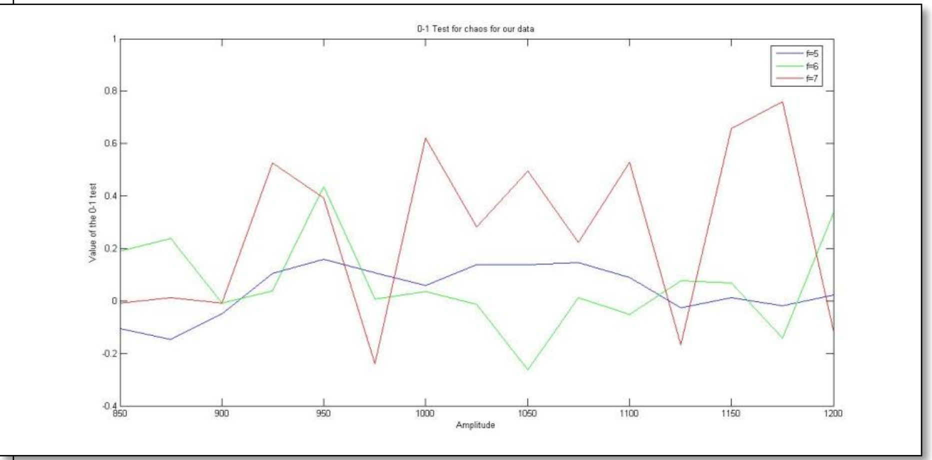


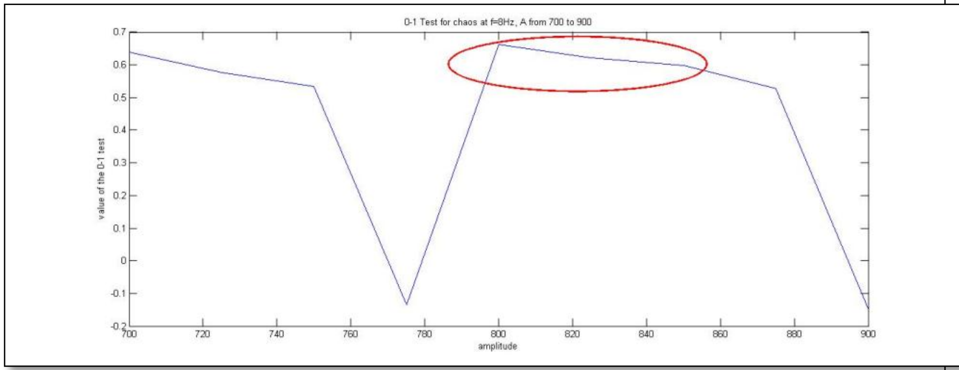


* Cf 'Reliability of the 0-1 test for chaos' from Hu, Tung, Gao and Cao and the answer 'Comment on "Reliability of the 0-1 test for chaos"' from Gottwald and Melborune.

Data Analysis

- On our data, with f=5,6,7Hz and A from 850 to 1200, we get:





➢ Best results for f=7Hz and amplitude around 1100 counts and for f=8Hz between 800 and 875 counts.

Data Analysis

- Lyapunov exponents are difficult to extract from a time-series, several methods exist. The method we used comes from the article 'A practical method for calculating largest Lyapunov exponents from small data sets' from Rosenstein, Collins and de Luca.

- On the time-seris with f=8Hz and A=800, the estimation of the Largest Lyapunov Exponent was 0.1904 for m=3 and 0.2102 for m=2 with τ=10ms.

>> we already knew that this system is chaotic, this gives us more proof. What could be interesting is to evaluate the whole set of Lyapunov exponents and then apply the conjectures which link Lyapunov exponents and the dimension of the strange attractor. (Mori or Kaplan and Yorke)

Data Analysis

- To rebuild the phase-space, we need an estimation of 2 variables: tau and m.

Several methods have been submitted so far, the time delay (tau) such as the minimum of the mutual information[1] or the first minimum of the autocorrelation[2].

For the embedding dimension (m, with the relation m≥ 2.D+1 and D the dimension of the attractor), one common method is to use the nearest neighbor algorithm (but it can be biased). We evaluate C(r):

$$C(r) = \lim_{m \to \infty} \frac{1}{m^2} . \sum_{i,j=1}^{m} H(r - |x_i - x_j|)$$

And with $C(r) \propto r^{v}$, we get the correlation dimension (when m is too big, we observe a saturation)

1- ‘Independent coordinates for strange attractors from mutual information’ from Fraser and Swinney

2-’Proper choice of the time delay for the analysis of chaotic time series’

Data Analysis

- Other methods suggest evaluation of m and tau in the same time

(see 'A Differential Entropy Based Method for Determining the Optimal Embedding Parameters of a Signal' from Gautama, Mandic and Van Hulle)

When we apply these two methods on our most chaotic time series (f=8Hz, A=800 counts) we find:

$$(\tau, m) = (15\text{ms}, 3)_1 \text{ and } (10\text{ms}, 2)_2$$

Conclusion

- This jumping robot exhibits chaotic behavior (period-3). We observe several specific aspects such as bistability and a return to stability (for frequency of 5Hz and 6 Hz, as we go up in amplitude).

- The system suffers from a lot of uncertainty and noise but we managed to piece together a nice bifurcation diagram. Some improvement could be made here (and try to use not only integer frequencies).

- Our time-series look like 'weak chaos' and a study for $f \geq 8$Hz and bigger amplitude could be interesting. Moreover, our algorithms present some defaults and an improvement could be made to get a better analysis of the attractor.

- In one article, scientists were able to 'destroy' the strange attractor and then control chaos by changing a parameter.

Thank you for your attention!