Experimental Characterization of Nonlinear Dynamics from Chua's Circuit

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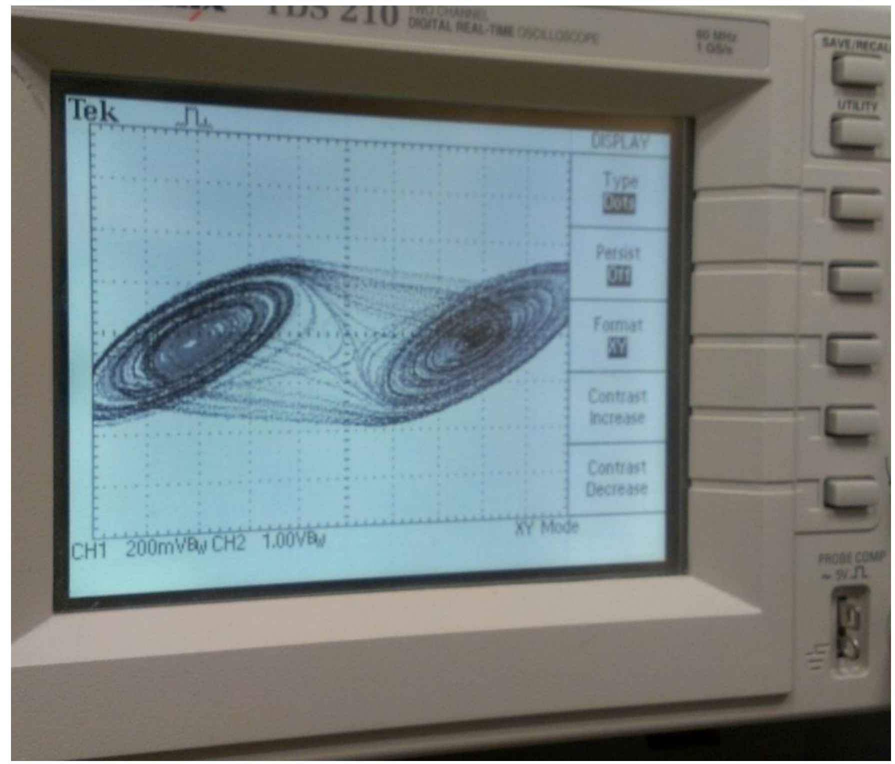
NLD class final presentation

12/04/2012

Outline

- Introduction
- Experiment setup
- Collected data
  - Route to chaos
  - Lyapunov exponent
- Computer simulation
- Post processing
  - Attractor reconstruction
  - Correlation dimension
- Outlook



Chaotic system

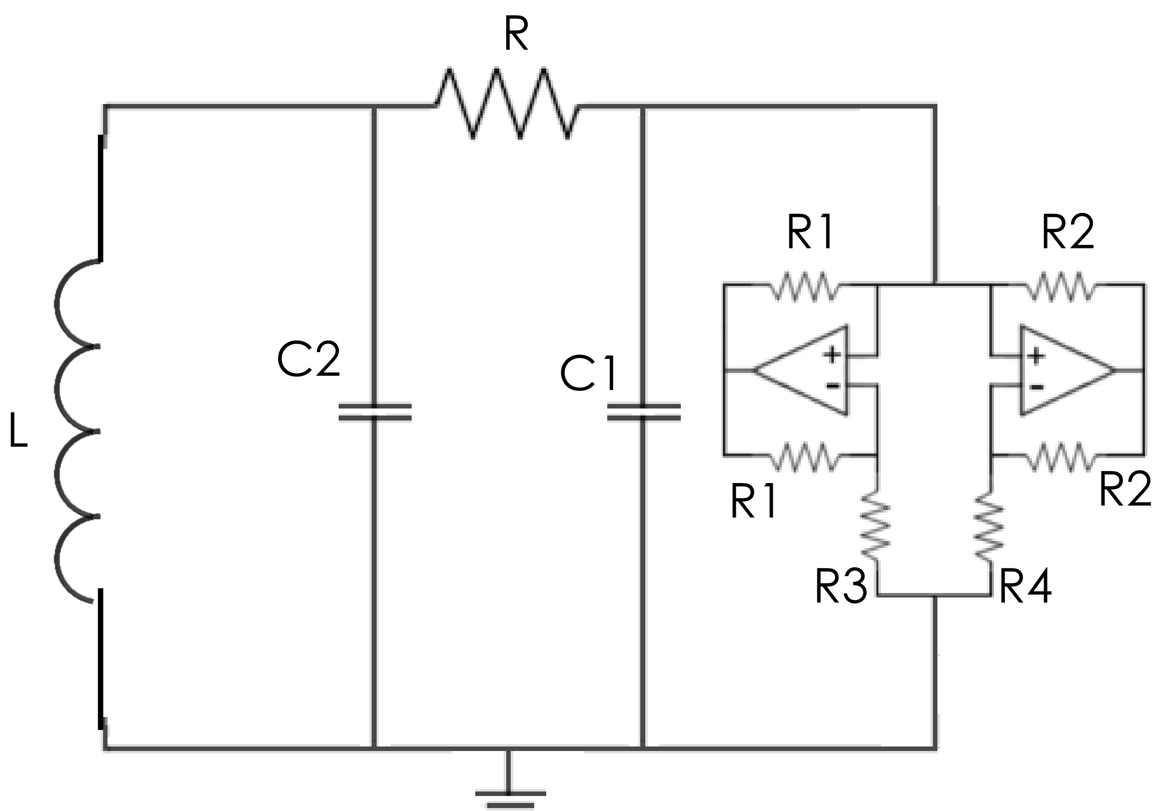
- Many application for chaotic system
  - atmosphere
  - human body
  - communication
- An **autonomous circuit** made from standard components (resistors, capacitors, inductors) must satisfy three criteria before it can display chaotic behavior. It must contain:
  1. one or more nonlinear elements
  2. one or more locally active resistors
  3. three or more energy-storage elements.

http://en.wikipedia.org/wiki/Chua's_circuit

Pecora1990

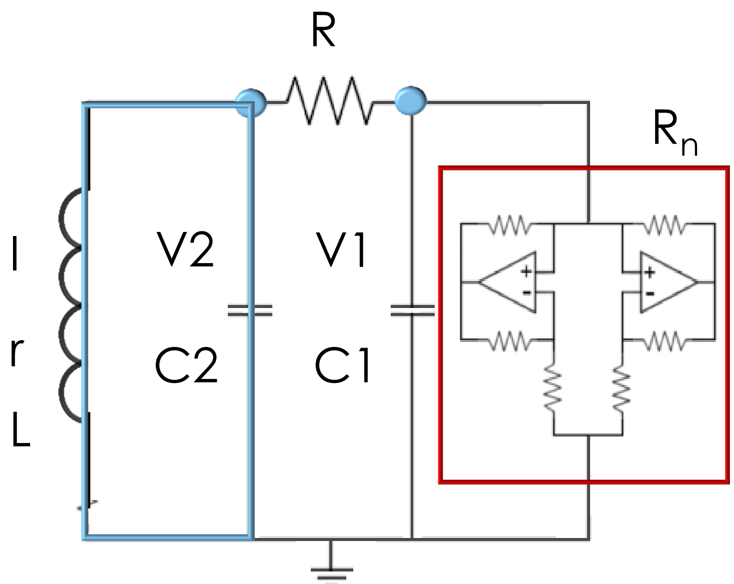
Simple circuit produces nonlinear

Chua's Circuit

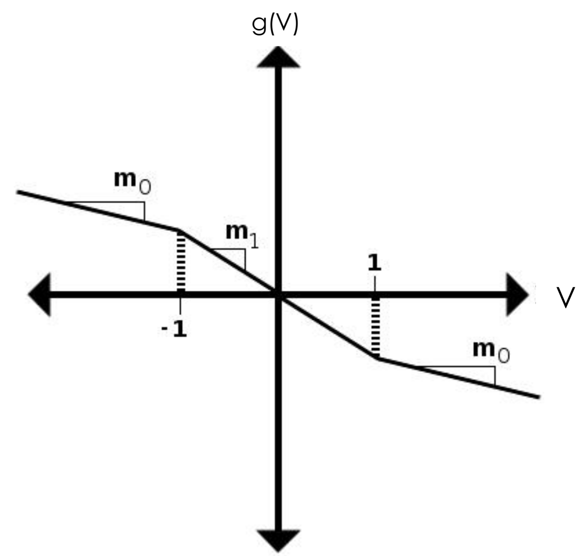


- L = 15mH
- C1 = 5nF
- C2 = 100nF
- R1 = 220Ω
- R2 = 22kΩ
- R3 = 2.2kΩ
- R4 = 3.3kΩ

Chua's circuit



g(V) = m0*V+m0-m1, V≤1
m1*V, -1≤V≤1
m0*V+m1-m0, 1≤V

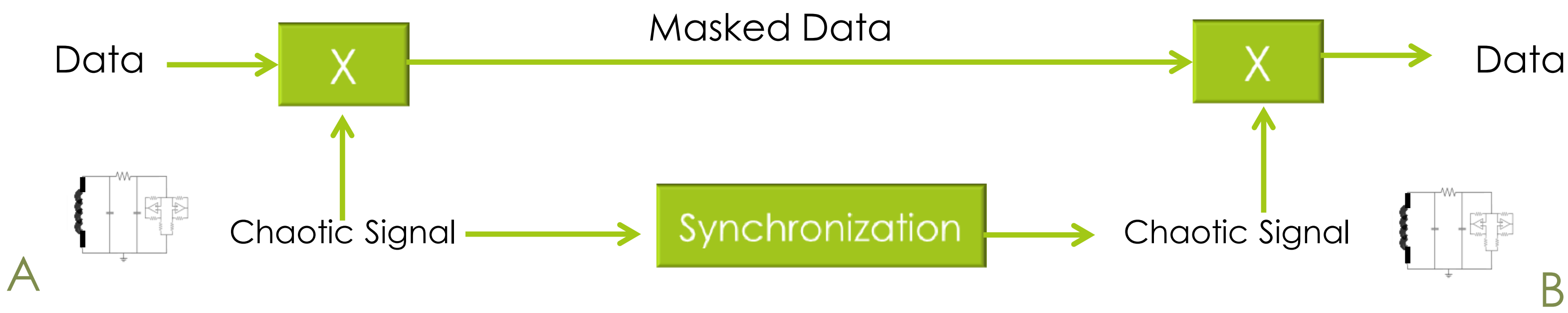


C1(dV1/dt)+g(V1) = (V2-V1)/R

C2(dV2/dt) +(V2-V1)/R = I

L(dI/dt) +rI+V2=0

Chaotic signal to mask information



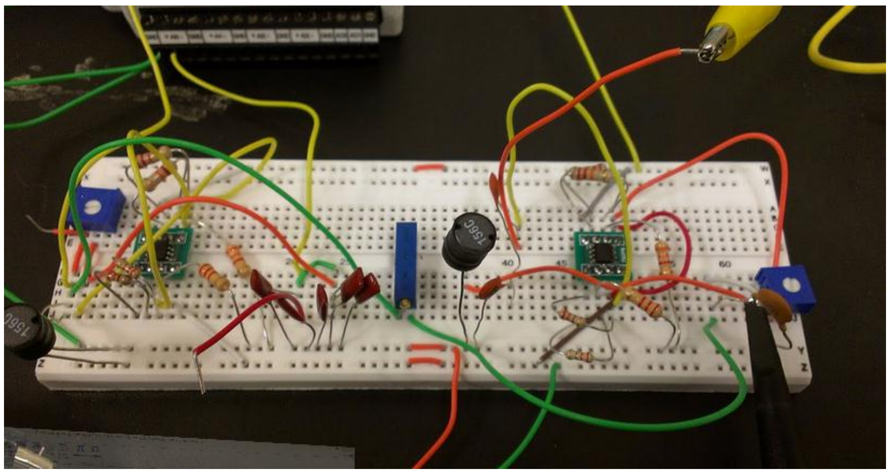
- Chaotic signal masks the information
- For demodulation we need an exact replica of the masking signal
- Subtract chaotic signal from masked data signal to reveal the data

Sadly, we couldn't finish it for this class.

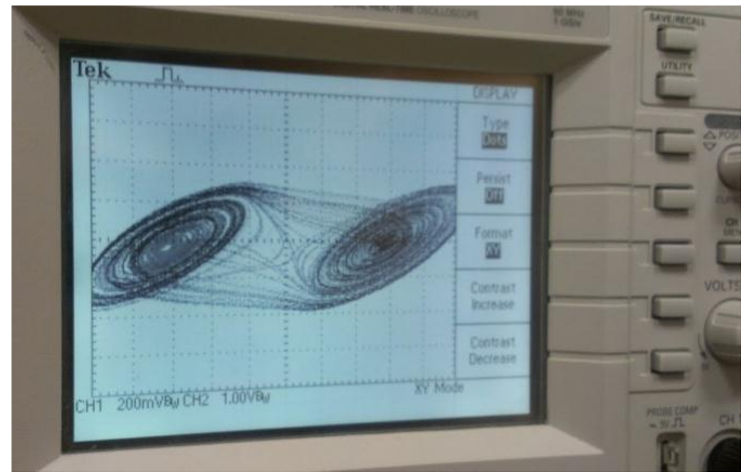
Dedieu1993

Experimental setup

Circuit board with two Chua's Circuits



Double scroll on oscilloscope

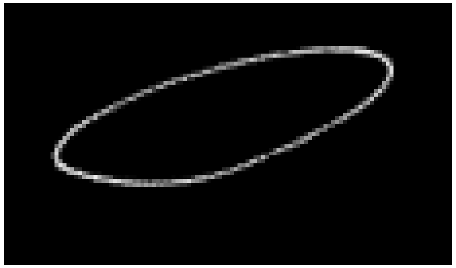


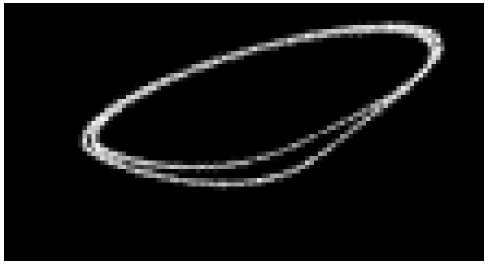
Components: two capacitors (the voltage across these are measured), an inductor, and resistors and op-amps (which make up the non-linear component)
Sample Rate: 48,000/s

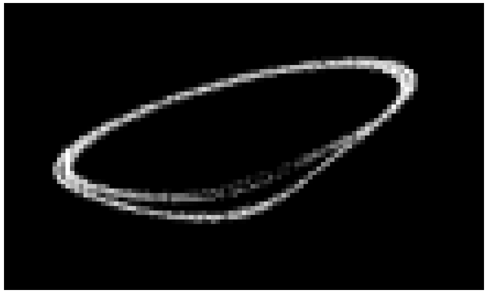
Route to chaos

Limit Cycle → Screw Attractor → Double Scroll (Harada 1996)

Period Doubling (Chua 1993)





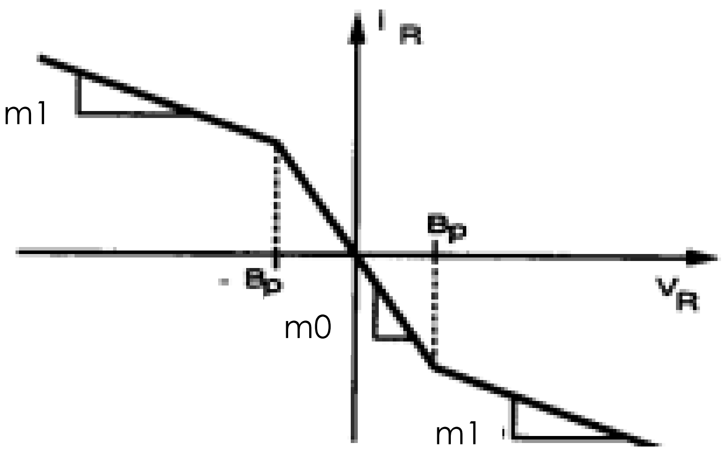


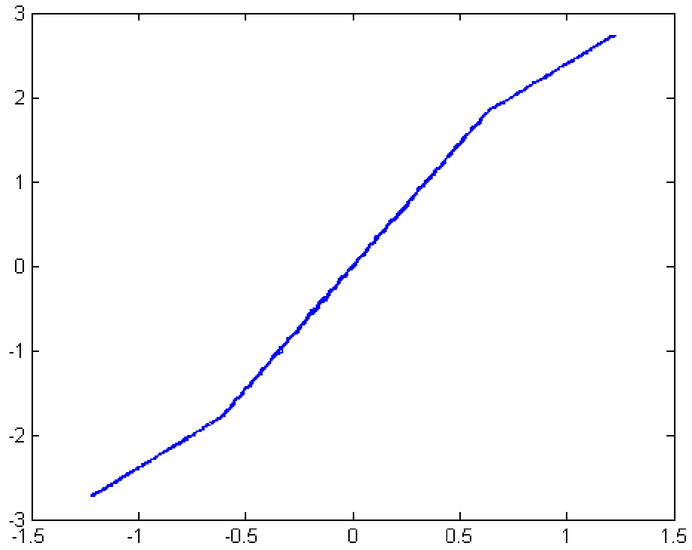
Route to Chaos

To obtain period doubling route to chaos, the following conditions must be satisfied:

$m_0 < -1$ and $-1 < m_1 < 0$

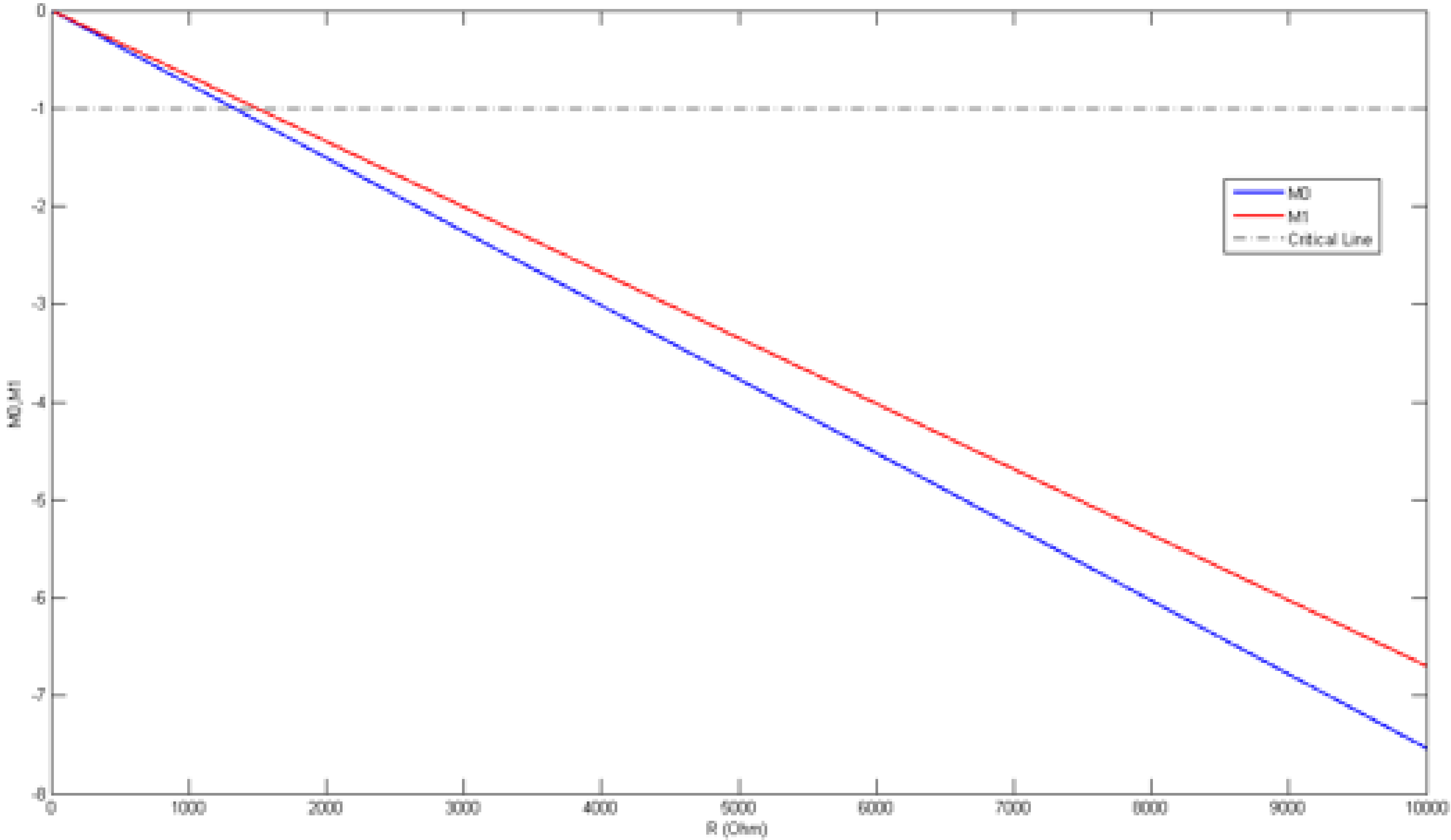
(Chua 1992)





(inverted)

Route to Chaos



Data collection

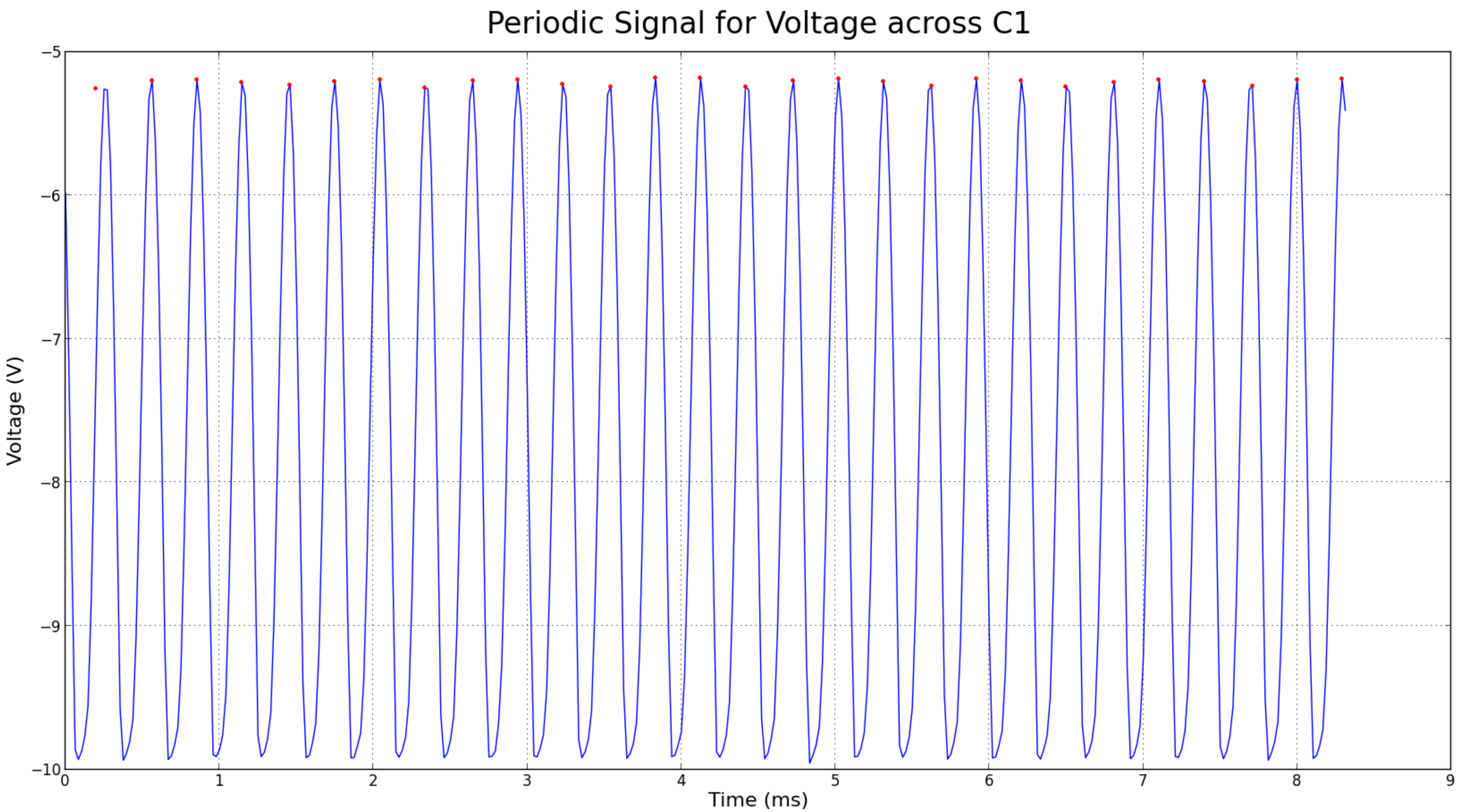
By increasing the resistance R, the behavior of the circuit changes:

1) Periodic Behavior

2) Semi-periodic Behavior

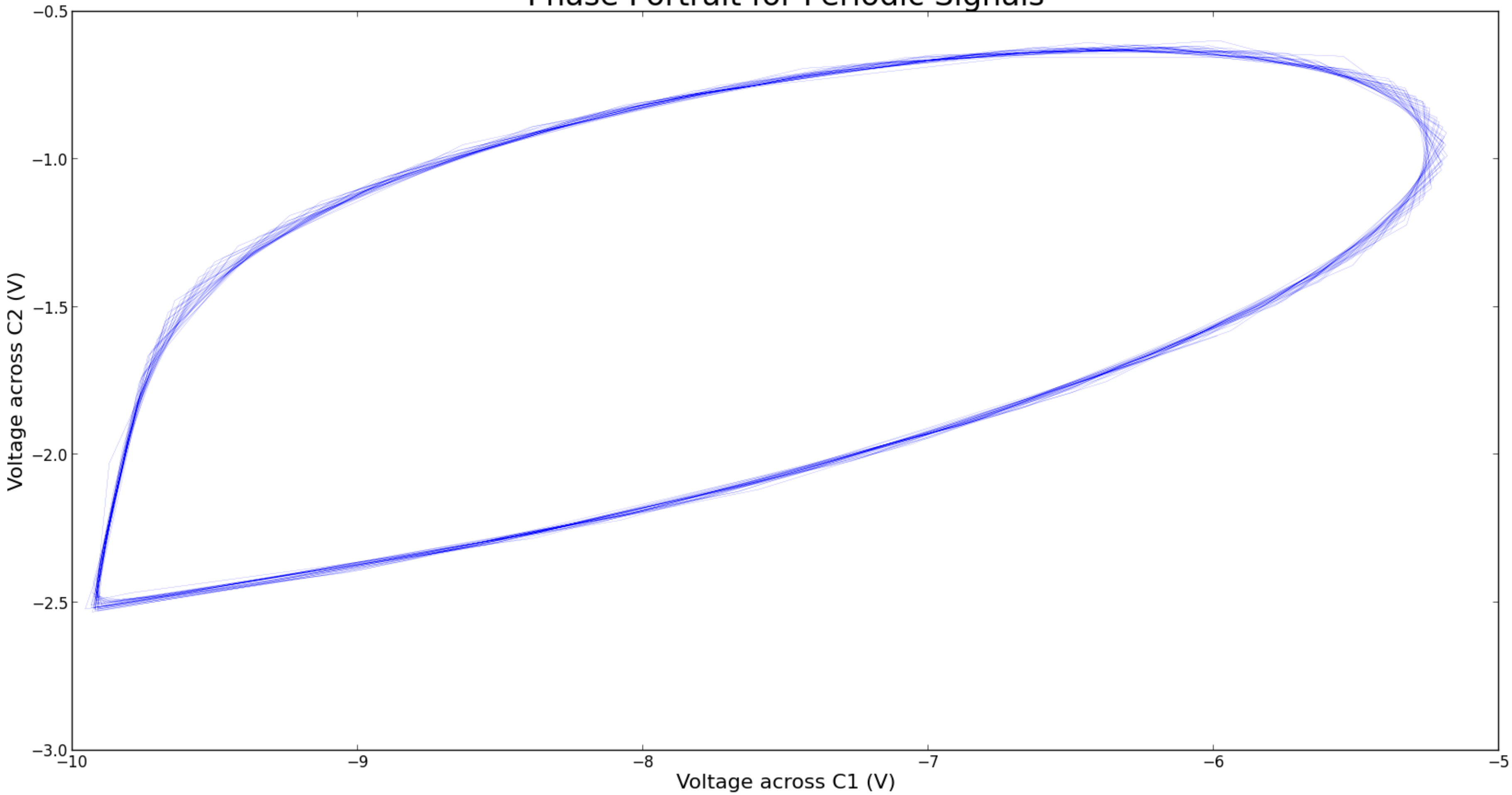
3) Chaotic Behavior

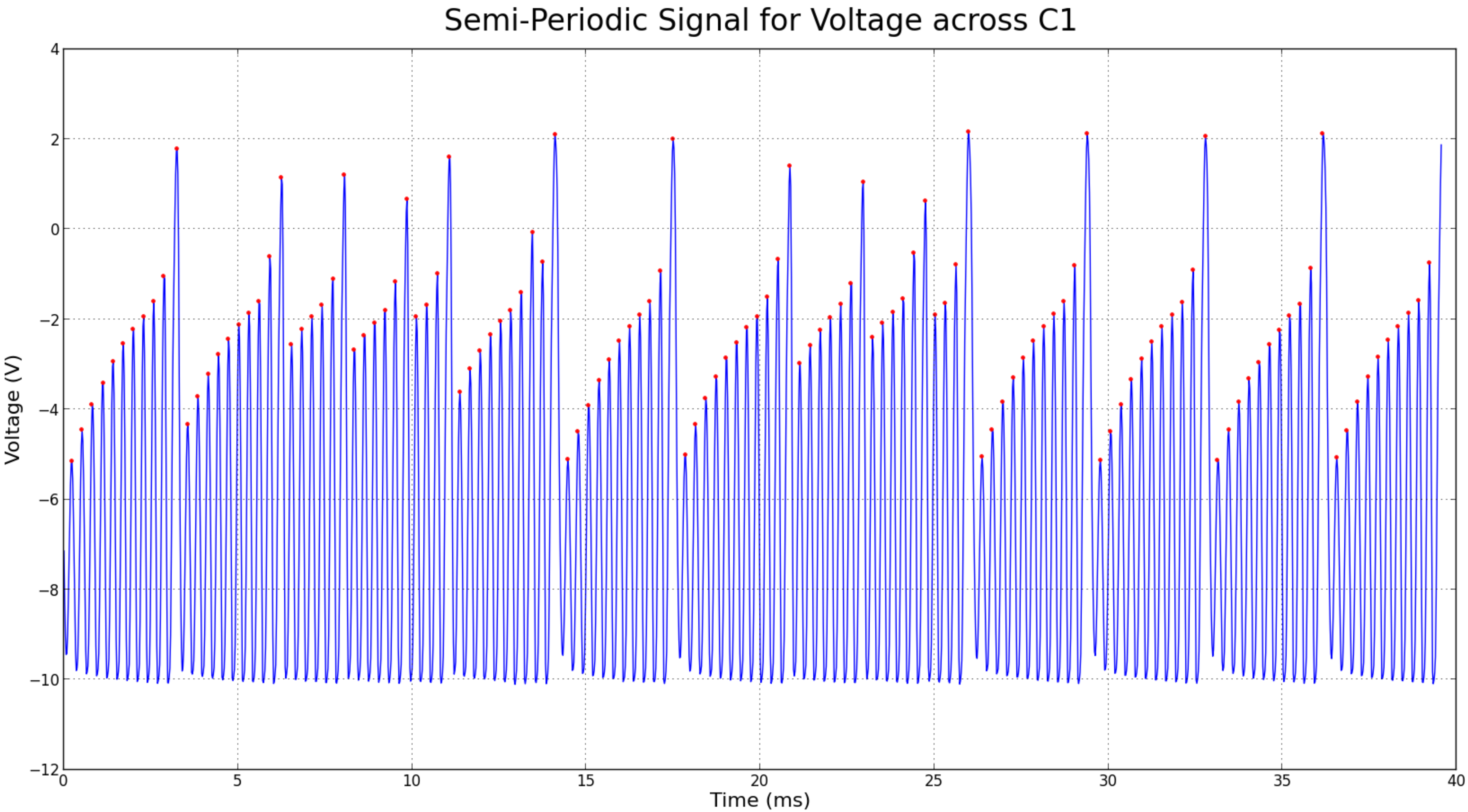
These stages are analyzed by looking at phase portraits and time-scale plots



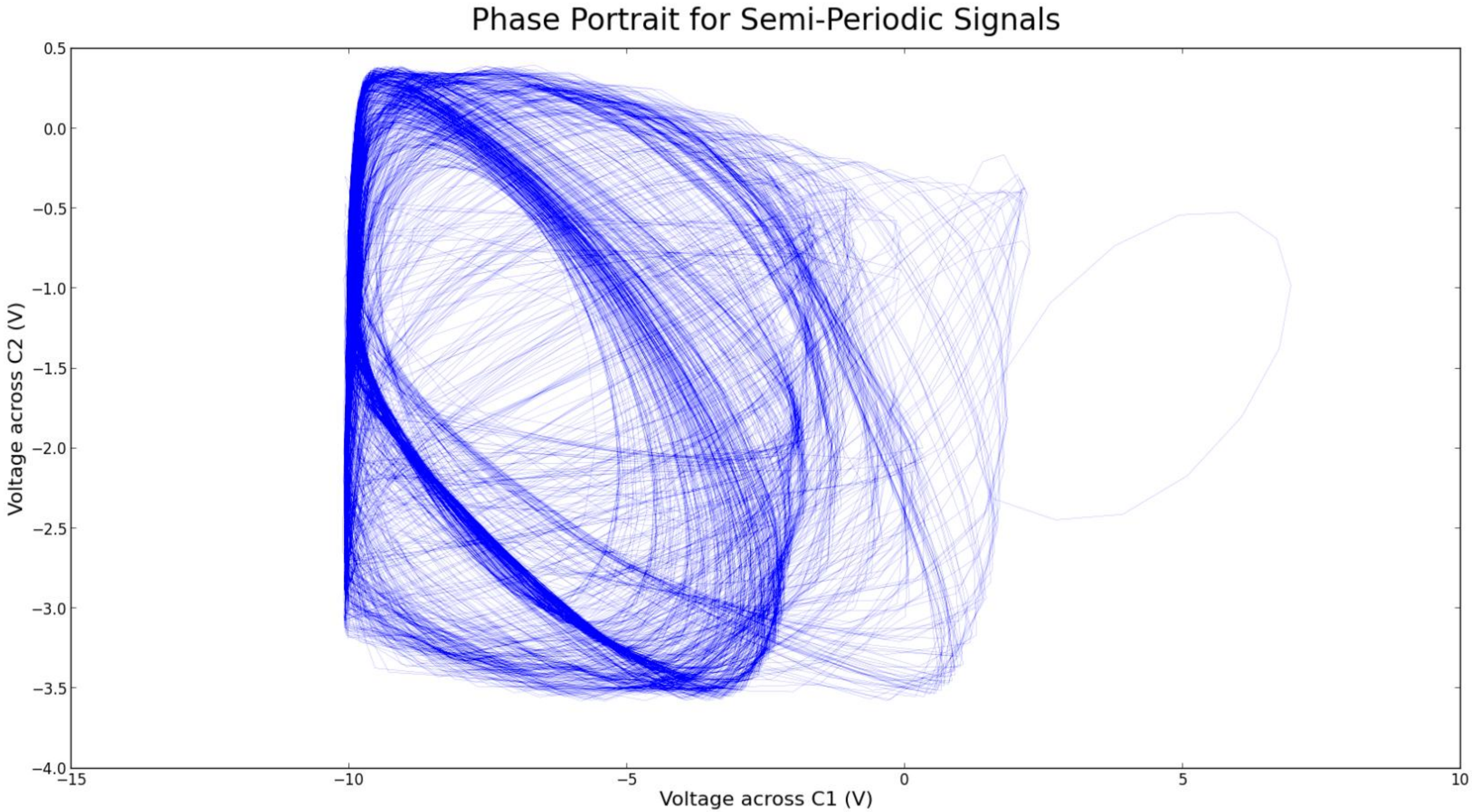
Periodic Signal for Voltage across C1
Voltage (V)
Time (ms)
−5
−6
−7
−8
−9
−10
0
1
2
3
4
5
6
7
8
9

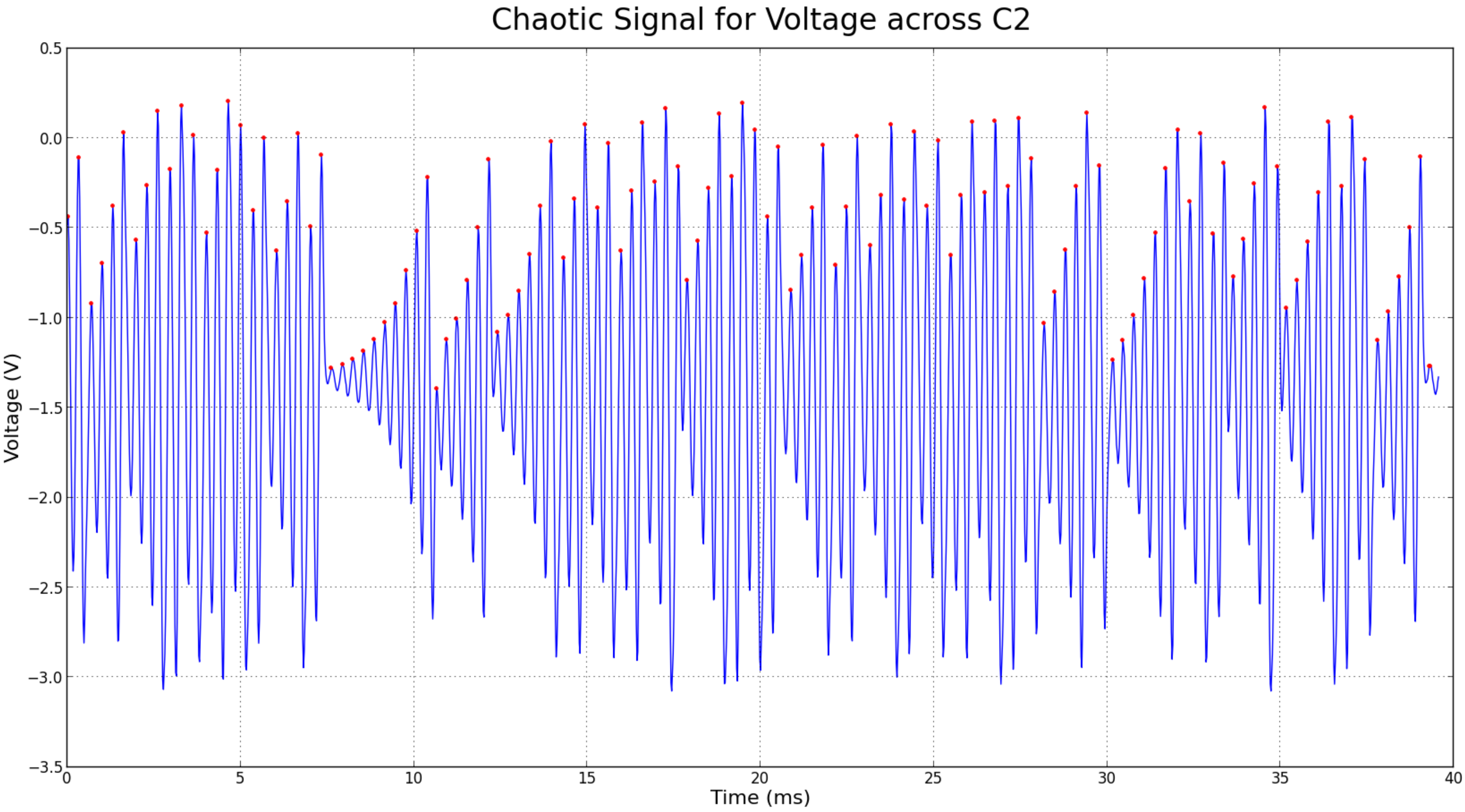
Phase Portrait for Periodic Signals



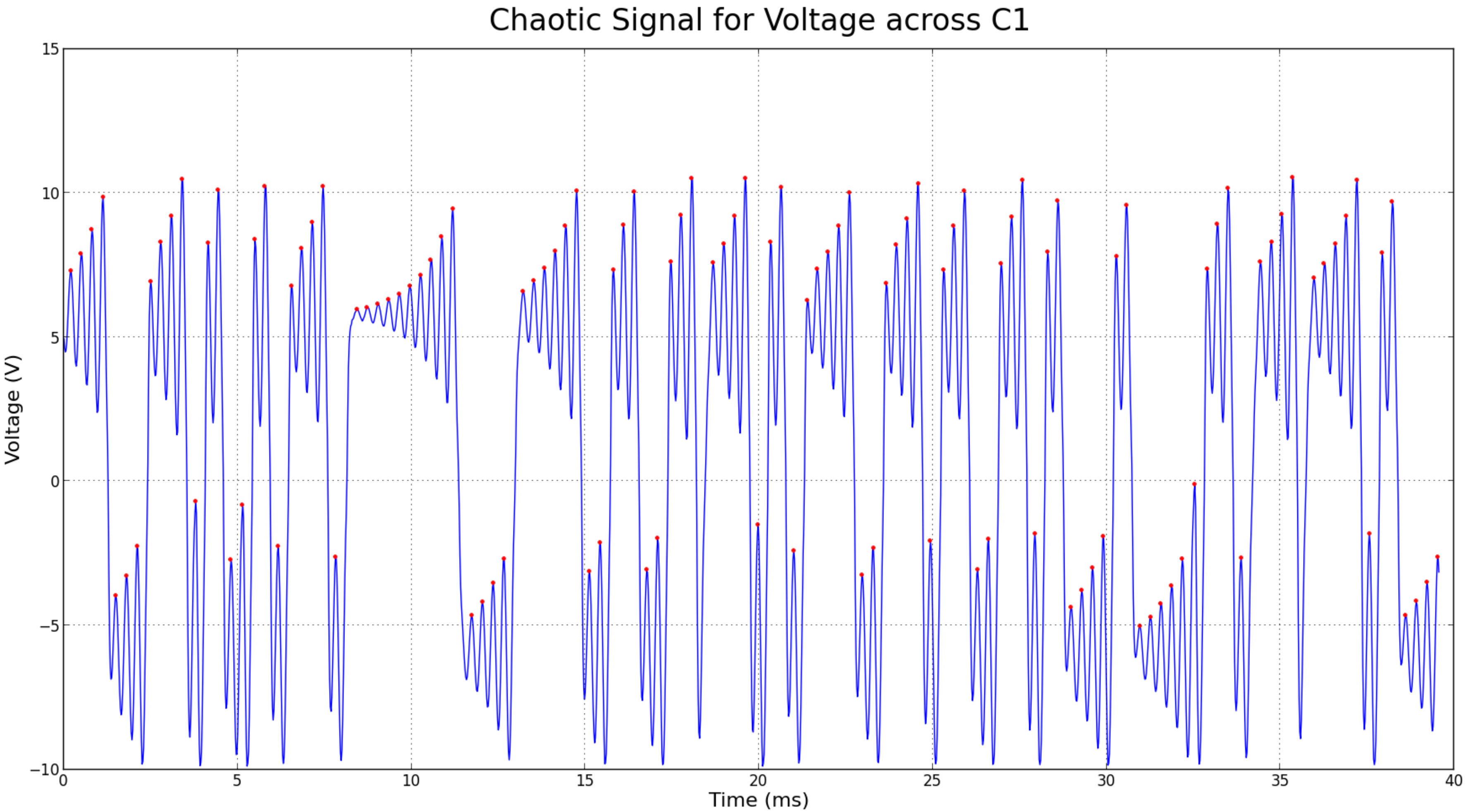


Semi-Periodic Signal for Voltage across C1
Voltage (V)
Time (ms)
4
2
0
−2
−4
−6
−8
−10
−12
0
5
10
15
20
25
30
35
40

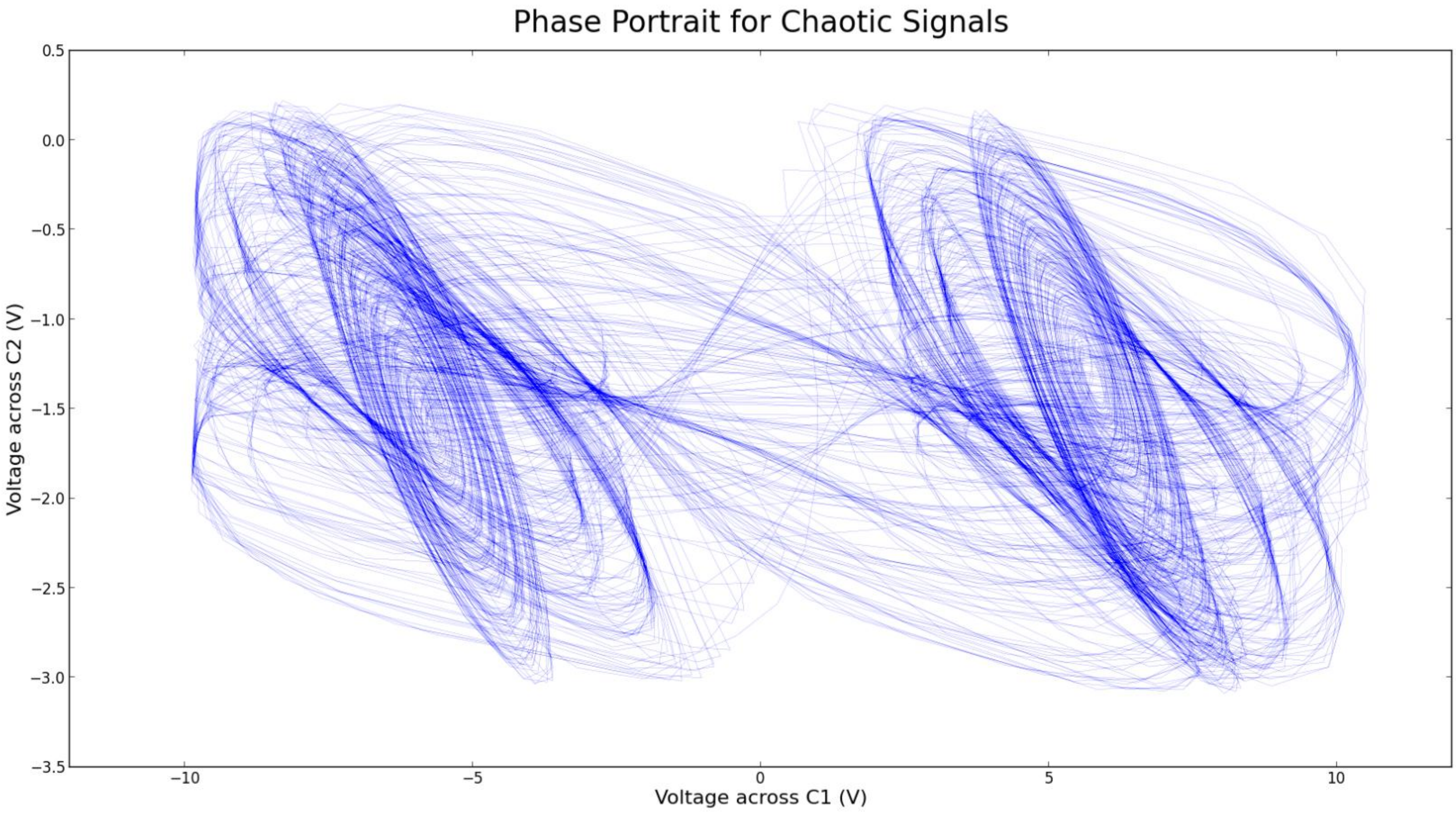

Phase Portrait for Semi-Periodic Signals
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
Voltage across C2 (V)
−15
−10
−5
0
5
10
Voltage across C1 (V)



Chaotic Signal for Voltage across C2
Voltage (V)
Time (ms)
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
0
5
10
15
20
25
30
35
40



Chaotic Signal for Voltage across C1
Voltage (V)
Time (ms)
15
10
5
0
−5
−10
0
5
10
15
20
25
30
35
40



Phase Portrait for Chaotic Signals
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
Voltage across C2 (V)
−10
−5
0
5
10
Voltage across C1 (V)

The Lyapunov Exponent

A measure of how two nearby trajectories diverge from each other:

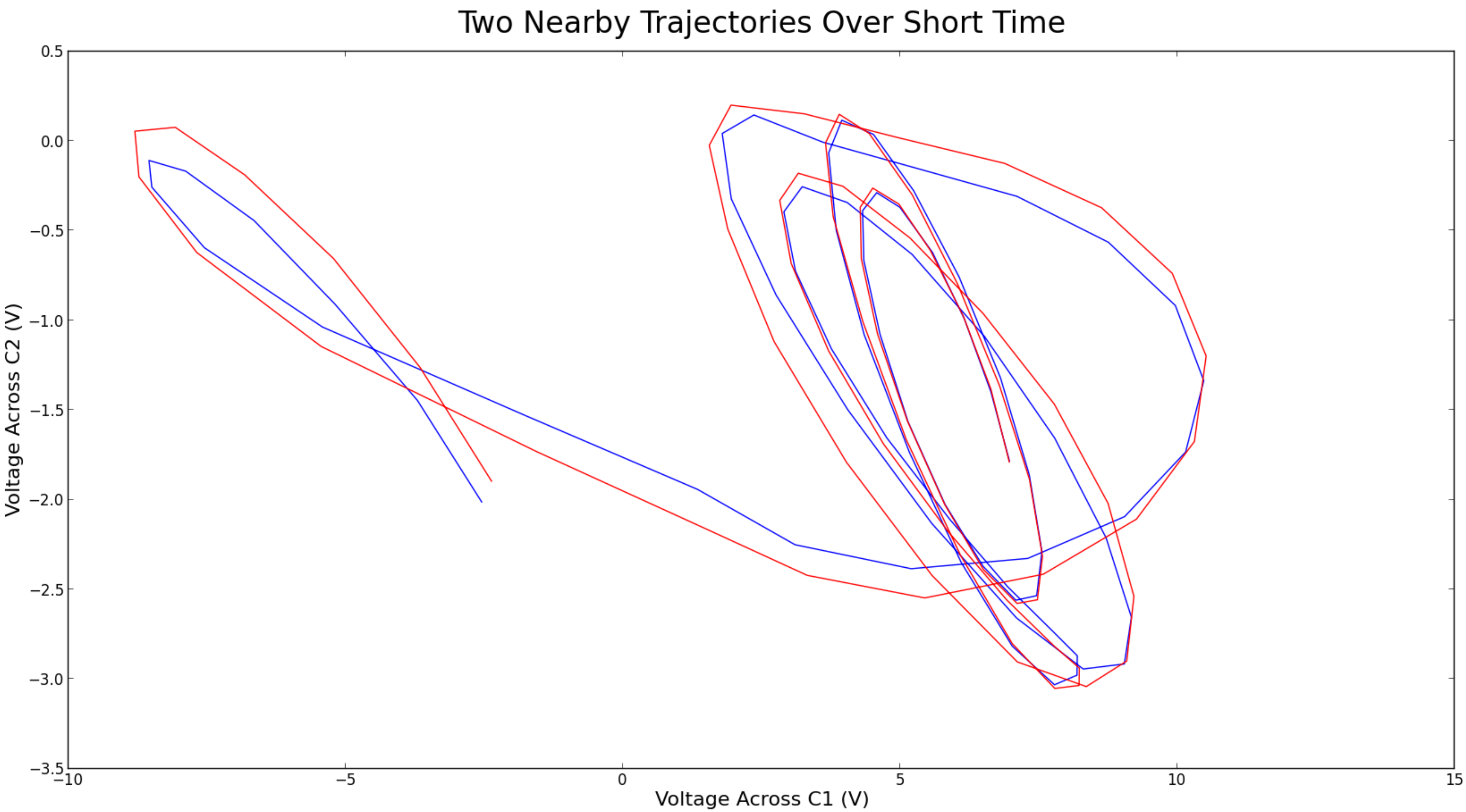
$$|\delta \mathbf{Z}(t)| \approx e^{\lambda t} |\delta \mathbf{Z}_0|$$

For an n-dimensional phase space, n Lyapunov exponents are needed to quantitatively describe the separation distance

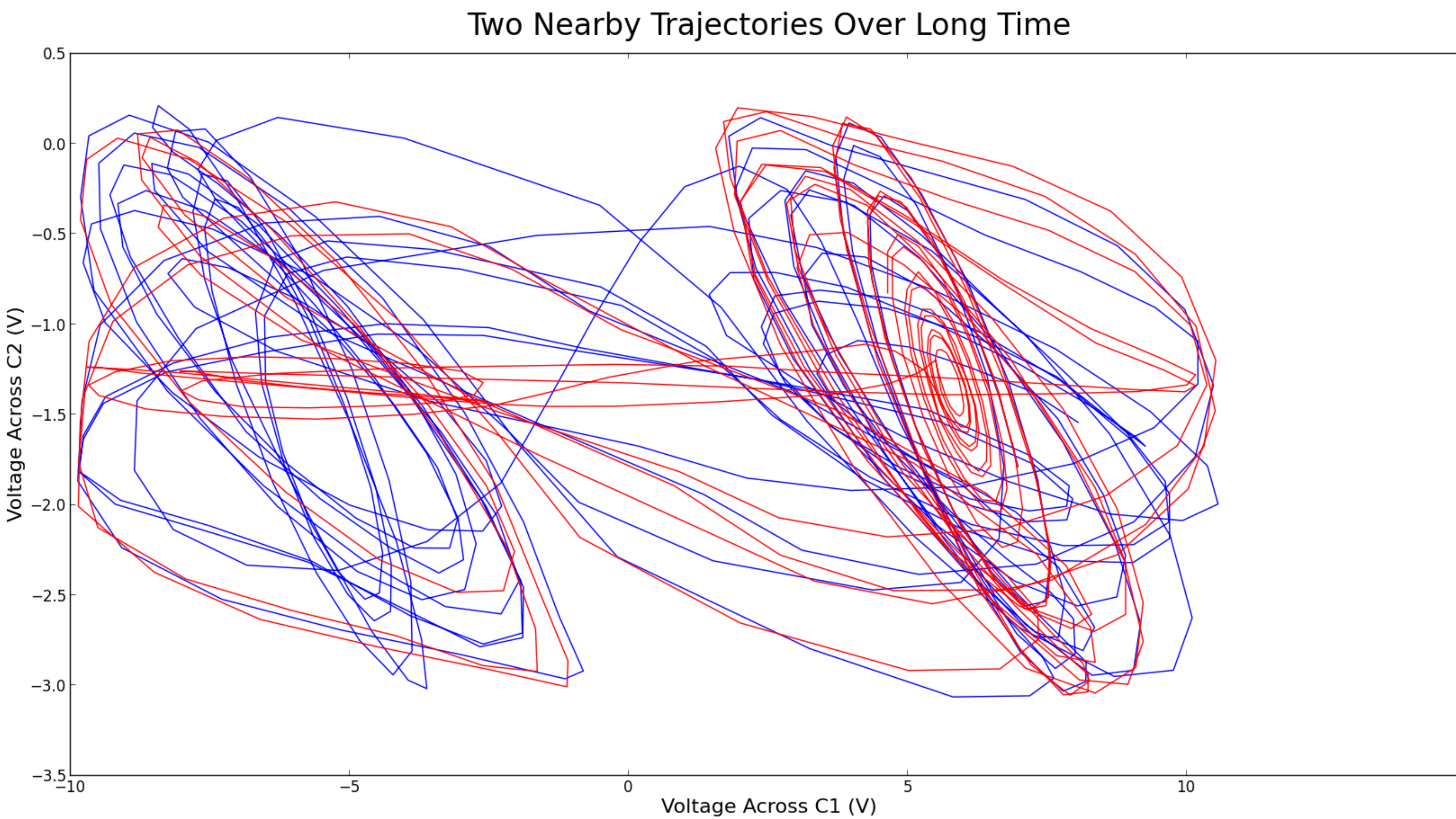
The Lyapunov Exponent

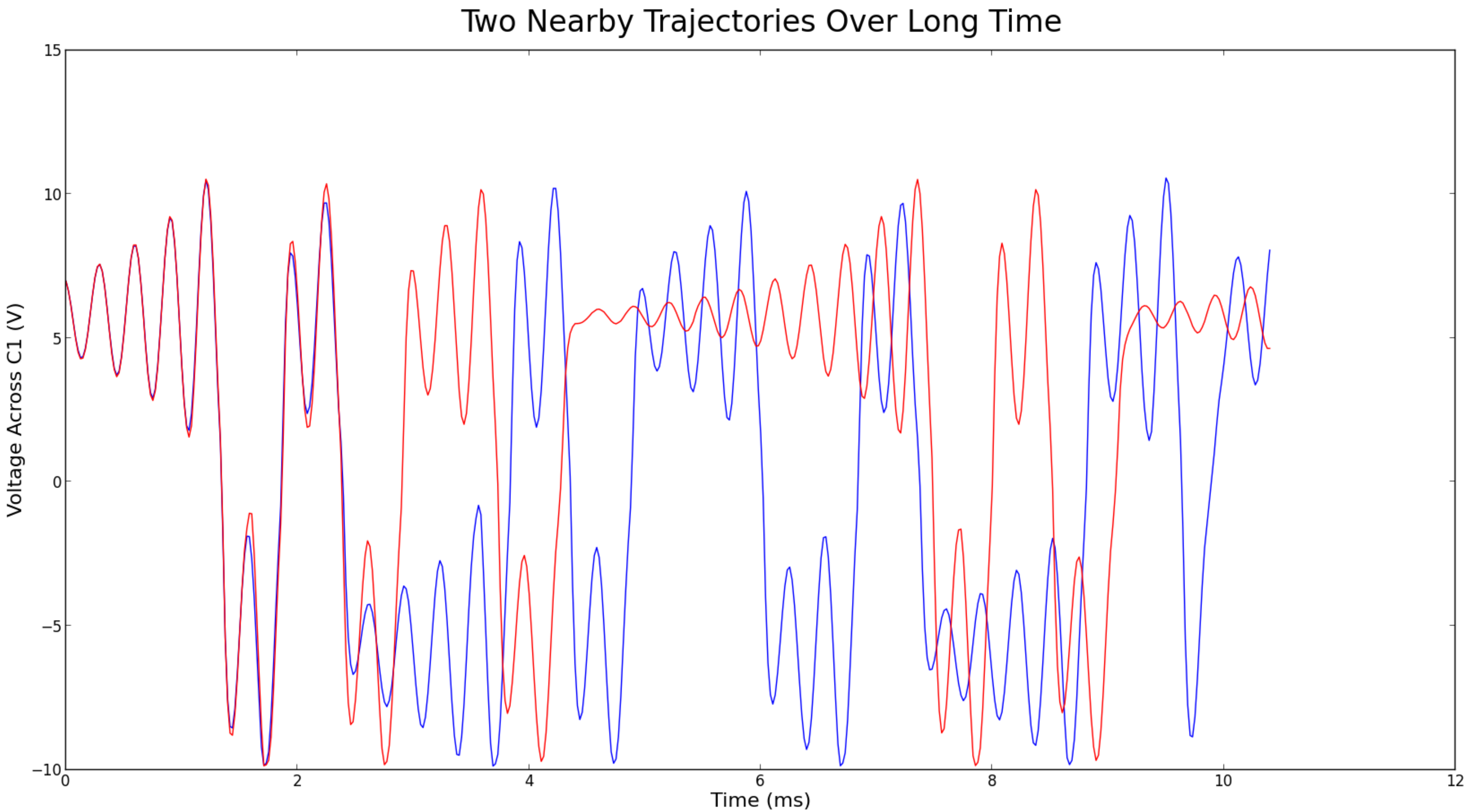
The maximal Lyapunov exponent can be calculated in any dimension by monitoring the separation distance in phase space

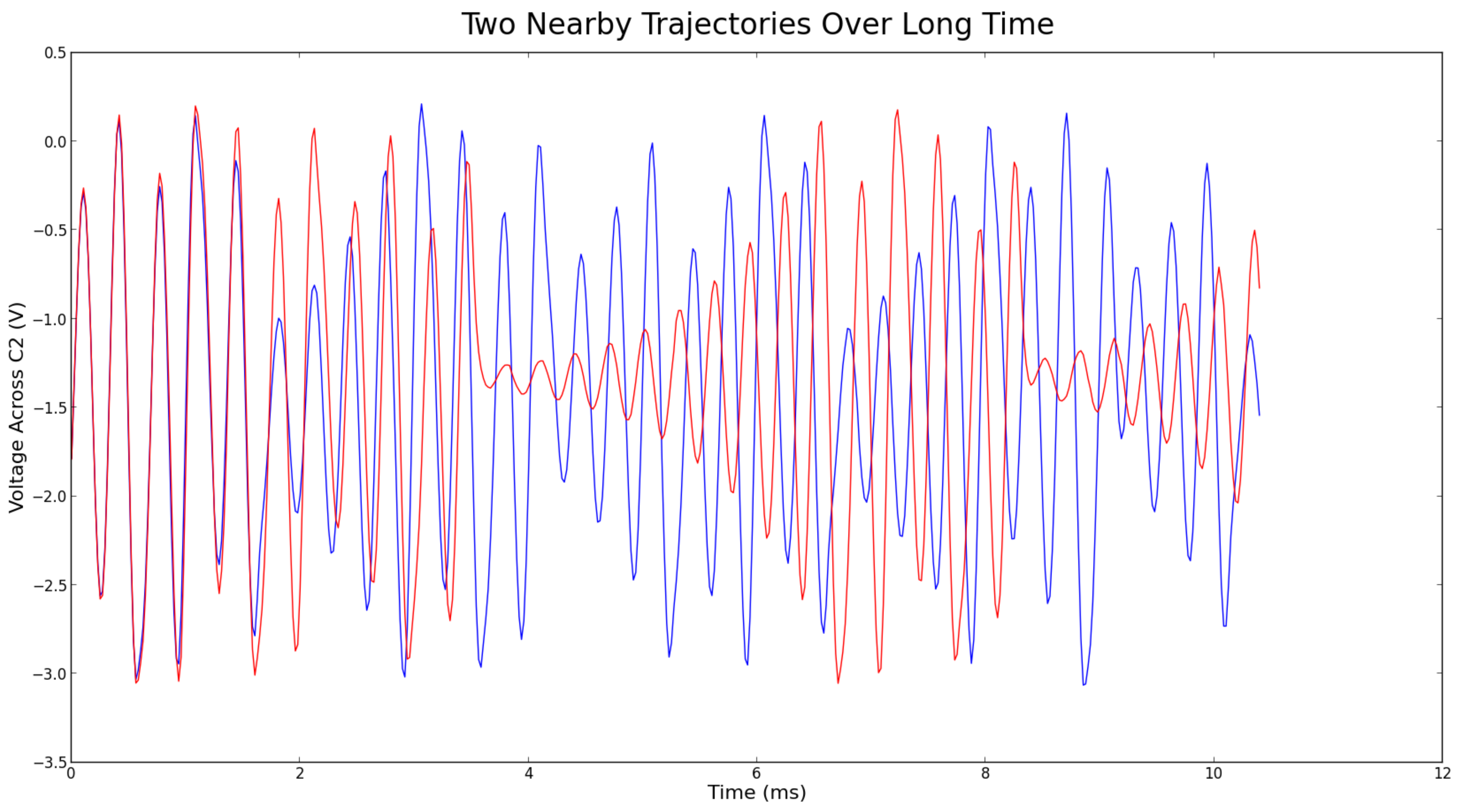
The behavior varies by attractor, so a Lyapunov exponent should be assigned to each attractor



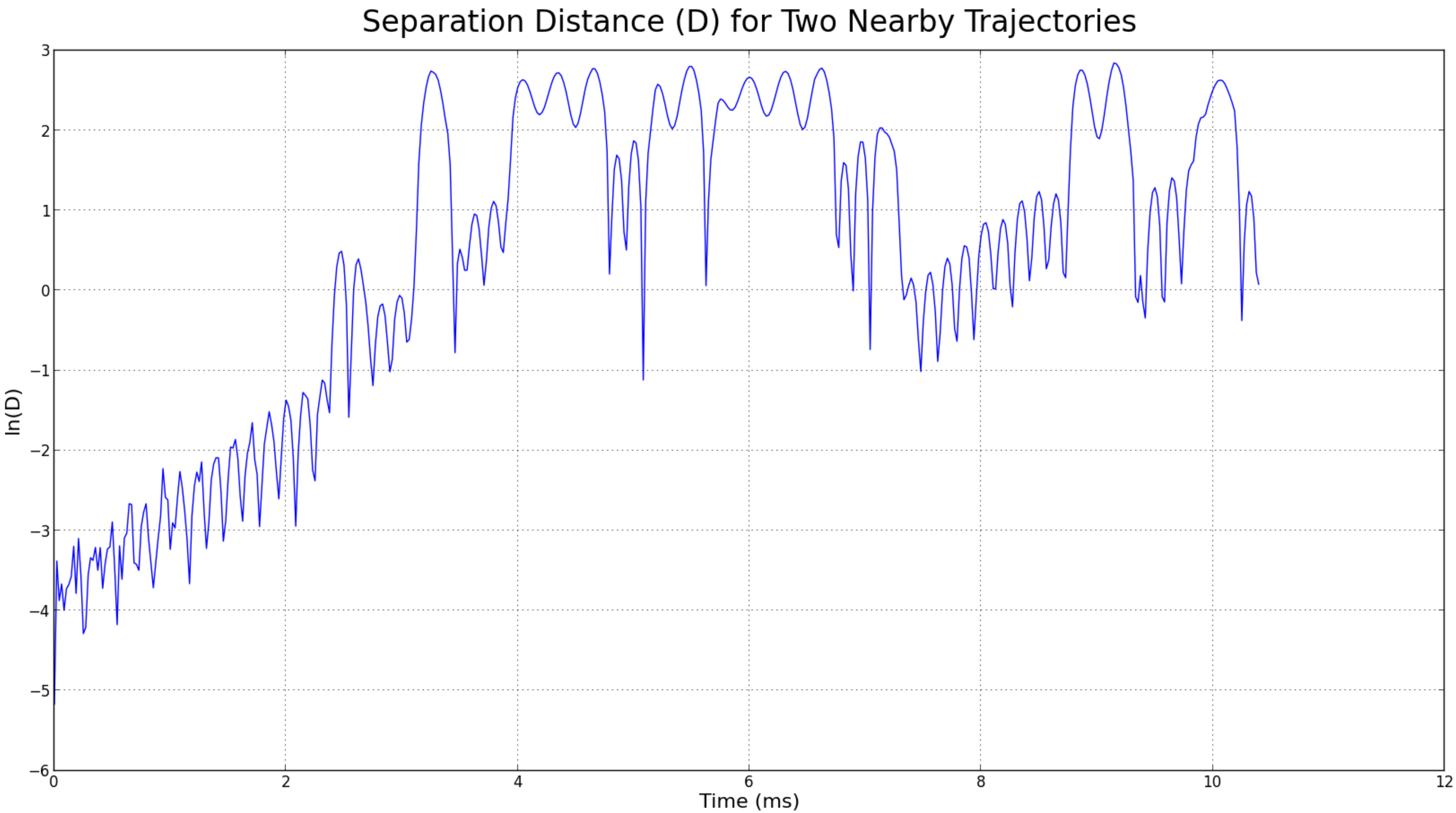
Two Nearby Trajectories Over Short Time
Voltage Across C2 (V)
Voltage Across C1 (V)
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−10
−5
0
5
10
15


Two Nearby Trajectories Over Long Time
Voltage Across C2 (V)
Voltage Across C1 (V)
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−10
−5
0
5
10
15

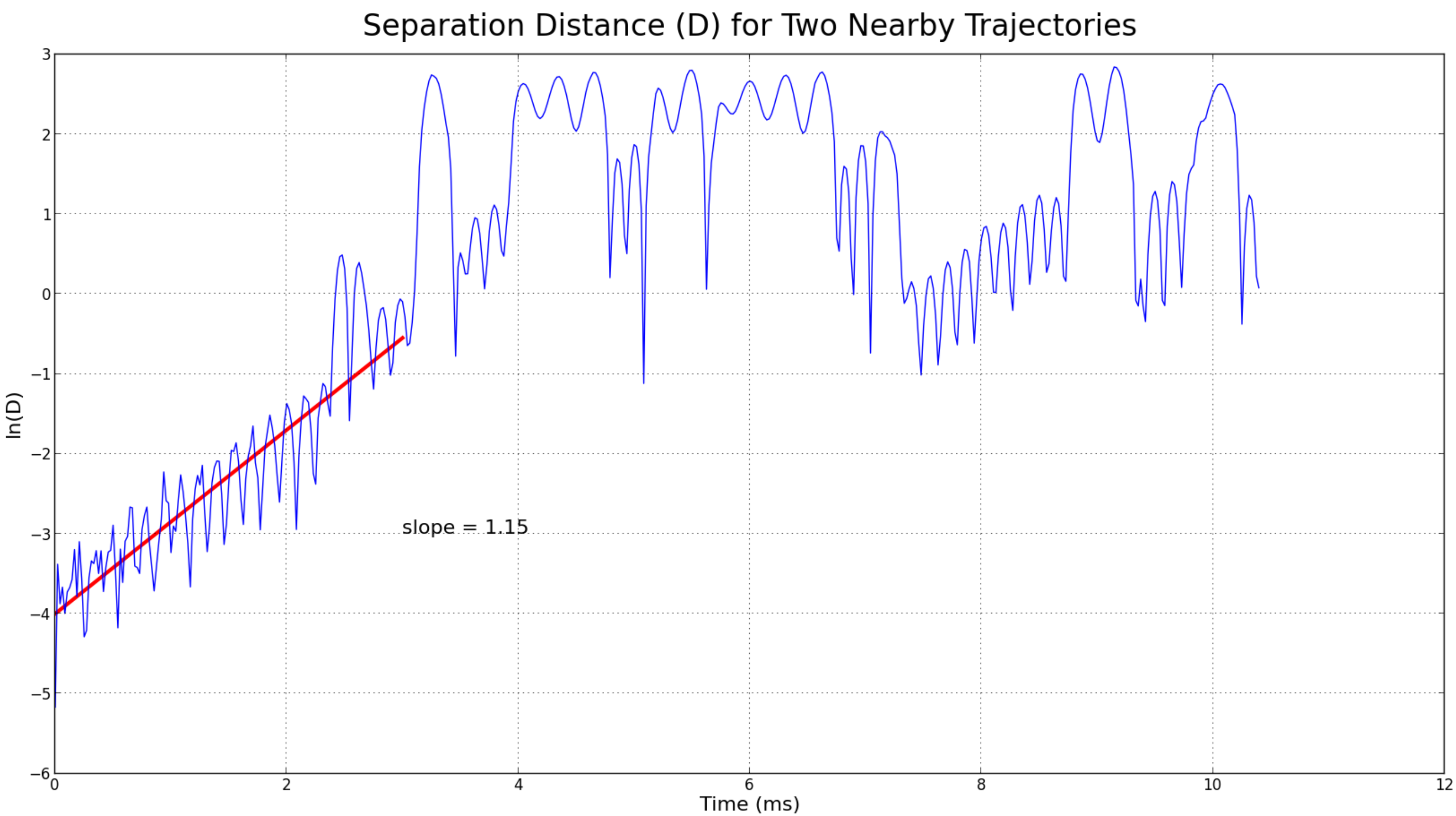

Two Nearby Trajectories Over Long Time
Voltage Across C1 (V)
Time (ms)
15
10
5
0
−5
−10
0
2
4
6
8
10
12



Two Nearby Trajectories Over Long Time
Voltage Across C2 (V)
Time (ms)
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
0
2
4
6
8
10
12

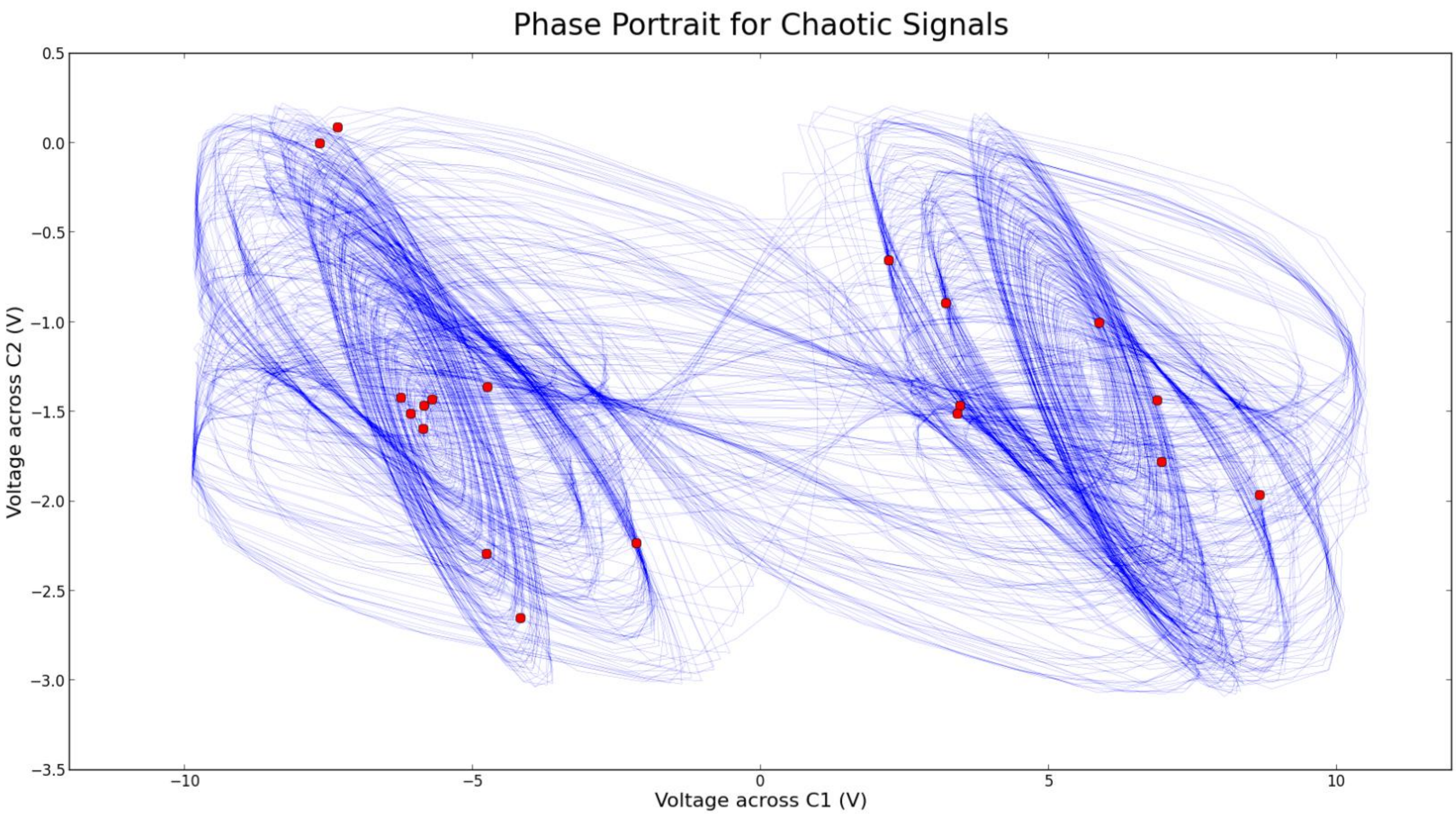


Separation Distance (D) for Two Nearby Trajectories
ln(D)
Time (ms)
3
2
1
0
−1
−2
−3
−4
−5
−6
0
2
4
6
8
10
12


Separation Distance (D) for Two Nearby Trajectories
ln(D)
Time (ms)
slope = 1.15
3
2
1
0
−1
−2
−3
−4
−5
−6
0
2
4
6
8
10
12

Calculating the Lyapunov Exponent

Since the slope is only roughly defined and may vary depending on the initial condition, a better value of the exponent is calculated by averaging over many different nearby trajectories


Phase Portrait for Chaotic Signals
Voltage across C2 (V)
Voltage across C1 (V)
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−10
−5
0
5
10

The Lyapunov Exponent

The average Lyapunov exponent for each attractor (time-scale = milliseconds):

$\lambda_{left} = 1.01$ $\lambda_{right} = 1.65$

Taking an average $\lambda = 1.33$, we define

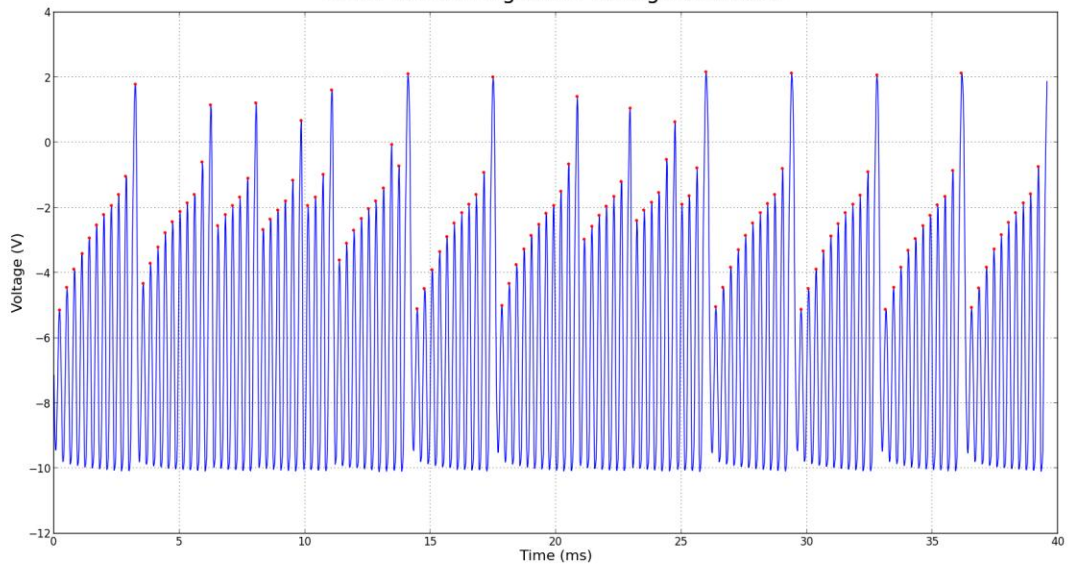
$t_{horizon} \sim 1/\lambda \ln(a/||\delta_0||) = 3.46\text{ms}$

An Iterative Map

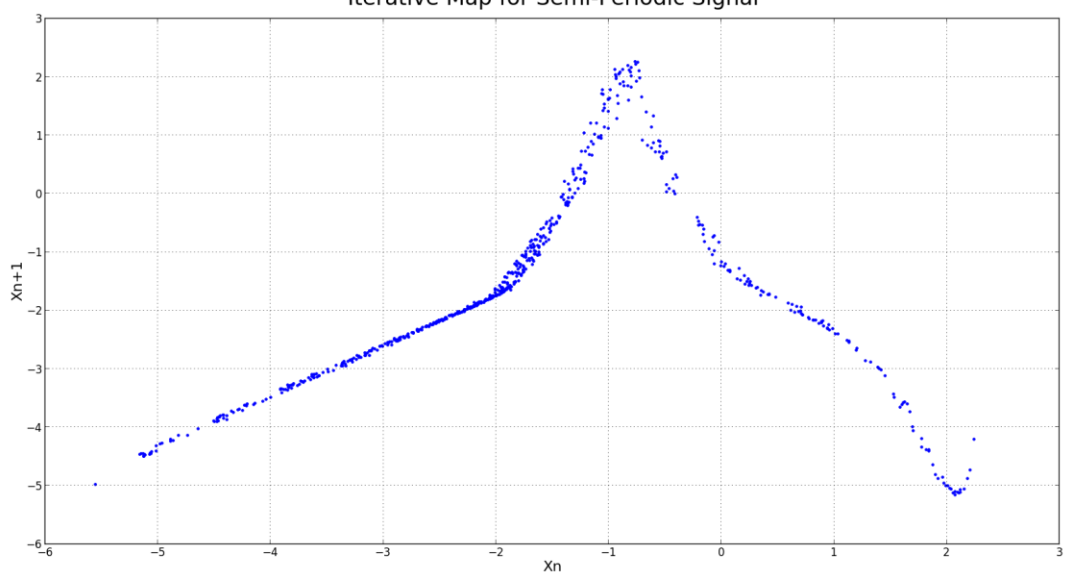
Motivation from Lorentz suggests that there may be an interesting relation between successive local maximums for the time-scale plots

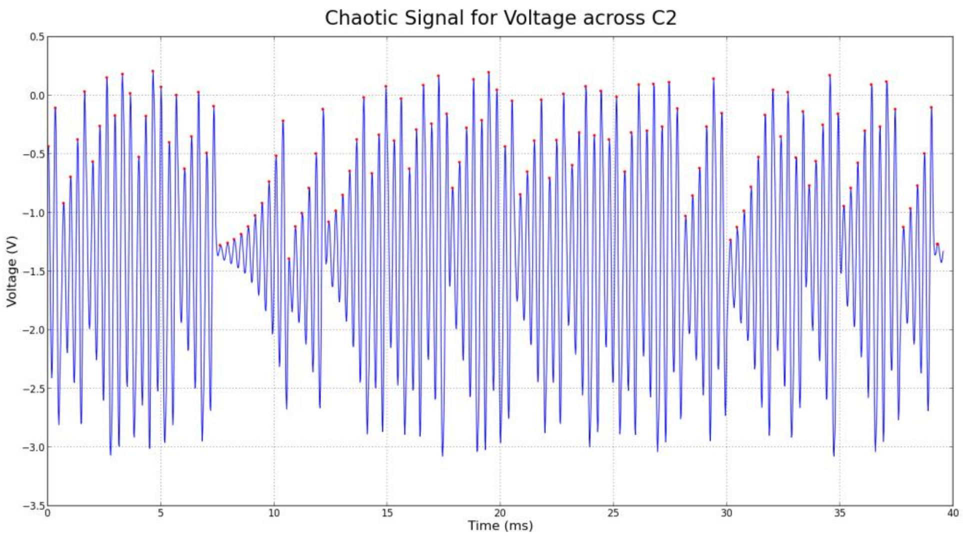
x_{n+1} and x_n are plotted against each other for the screw attractor and double scroll

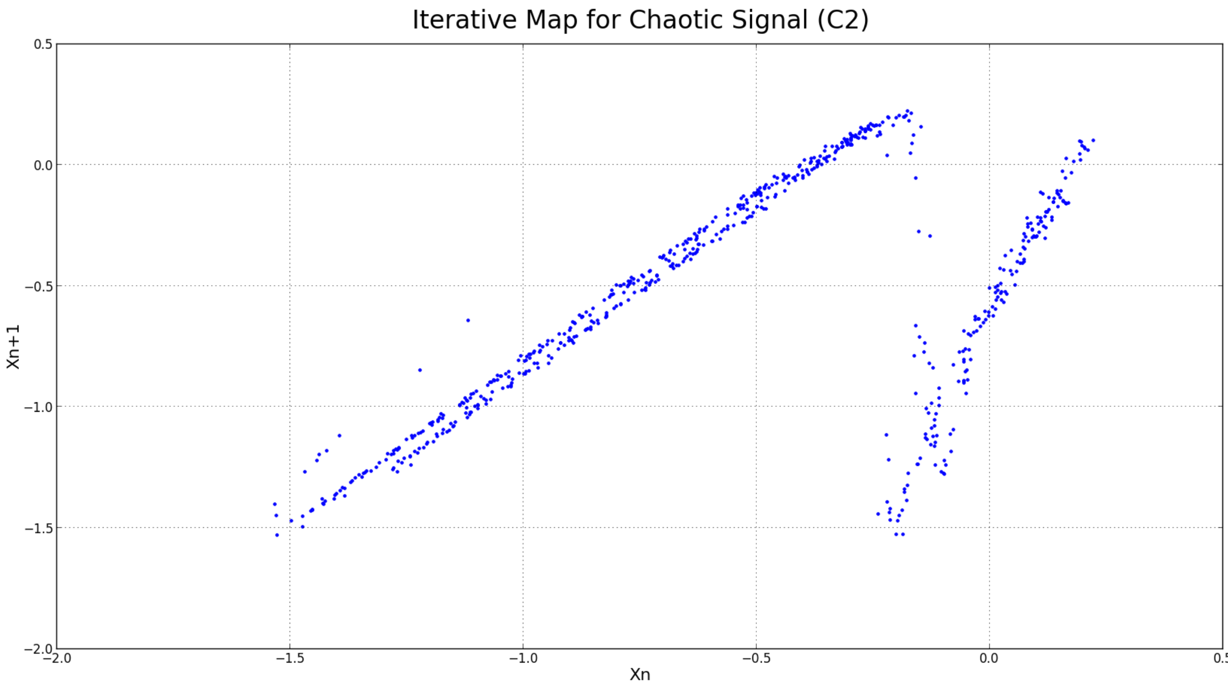
Semi-Periodic Signal for Voltage across C1

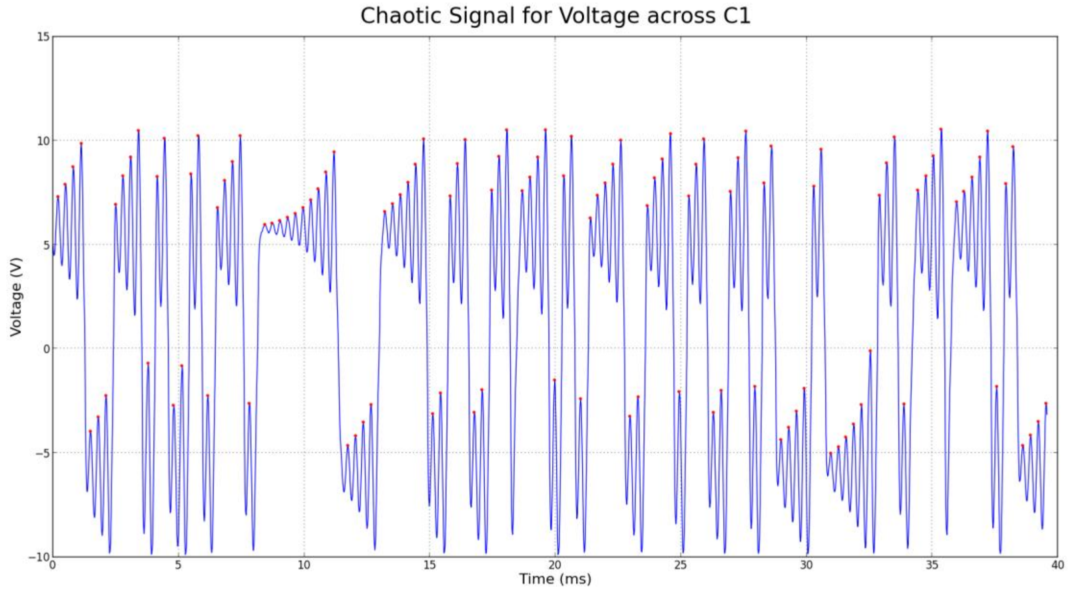


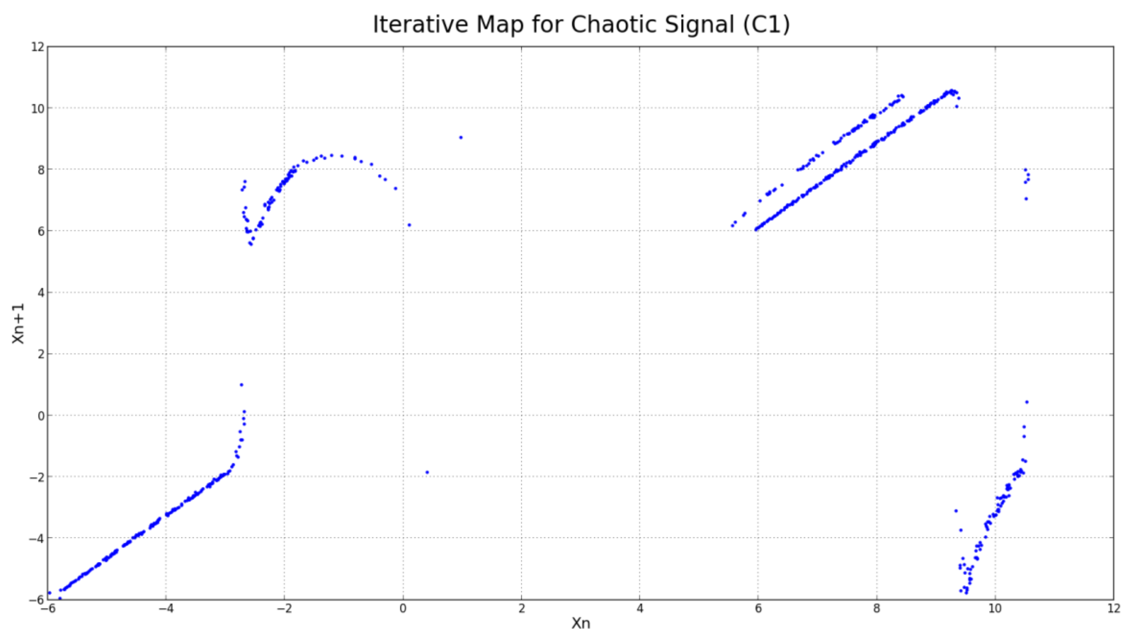
Iterative Map for Semi-Periodic Signal




Chaotic Signal for Voltage across C2
Voltage (V)
Time (ms)
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
0
5
10
15
20
25
30
35
40


Iterative Map for Chaotic Signal (C2)
Xn+1
Xn
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.0
−1.5
−1.0
−0.5
0.0
0.5


Chaotic Signal for Voltage across C1
Voltage (V)
Time (ms)
15
10
5
0
−5
−10
0
5
10
15
20
25
30
35
40


Iterative Map for Chaotic Signal (C1)
Xn+1
Xn
12
10
8
6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
8
10
12

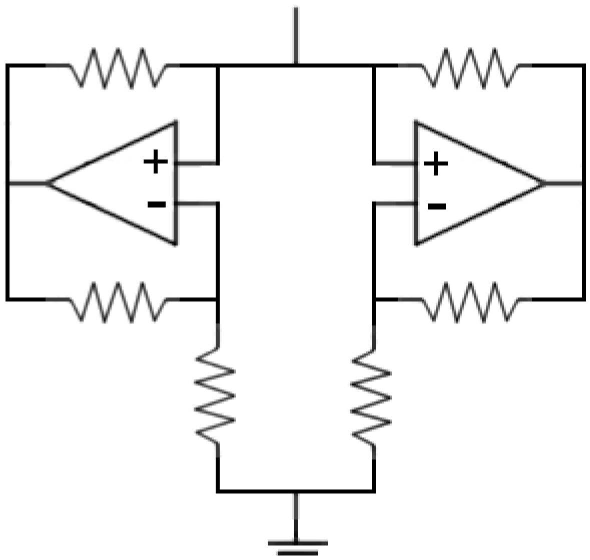
Simulation

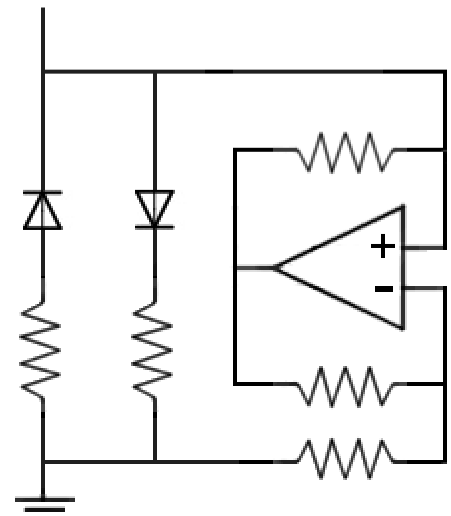
- Used Matlab's ode45 which uses a variable step Runge-Kutta.
- Runge-Kutta 4th order works by taking partial steps to calculate what the actual step is most likely to be.
- The variable time step allows for more accurate calculations during the difficult times while speeding calculations during the easier parts.

Equations and model used

- For the simulations done, the previous equations where used to write the Matlab code.
- Used the stated values for resistors to calculate the G function.
- For bifurcation diagram, and other attempted simulations, G was calculated using a different circuit layout.

Two different circuit designs for the active resistor





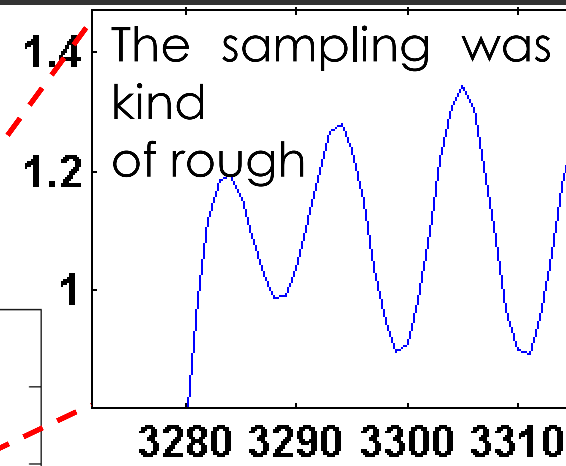
They are equivalent in their affect, but the calculations for G differ between the two.

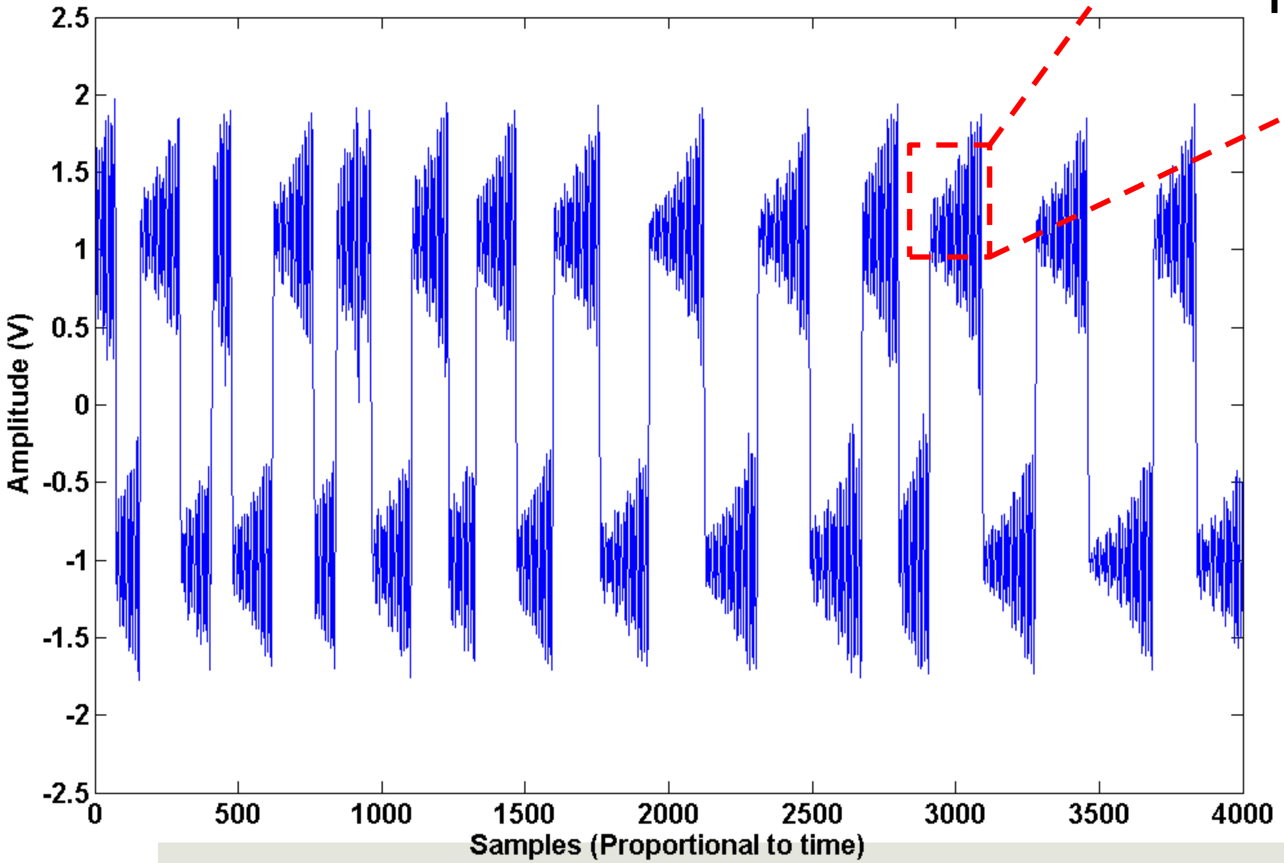
Expected Results

- Matlab simulations for circuits at their stated values for multiple values of the potentiometer.
- 1500 kΩ, 1700 kΩ, 1874.3 kΩ, 1900 kΩ, 1935 kΩ, 1982 kΩ, and 2100 kΩ were all simulated in 2D and 3D.

Time domain signal

Data were collected by direct sampling of voltage across the capacitor.
Fs=24K





The associated time can be calculated by considering the sampling rate at which the signal was sampled.

$$T_n = n\Delta t = \frac{n}{f}$$

Attractor reconstruction method

- Time Delay Method

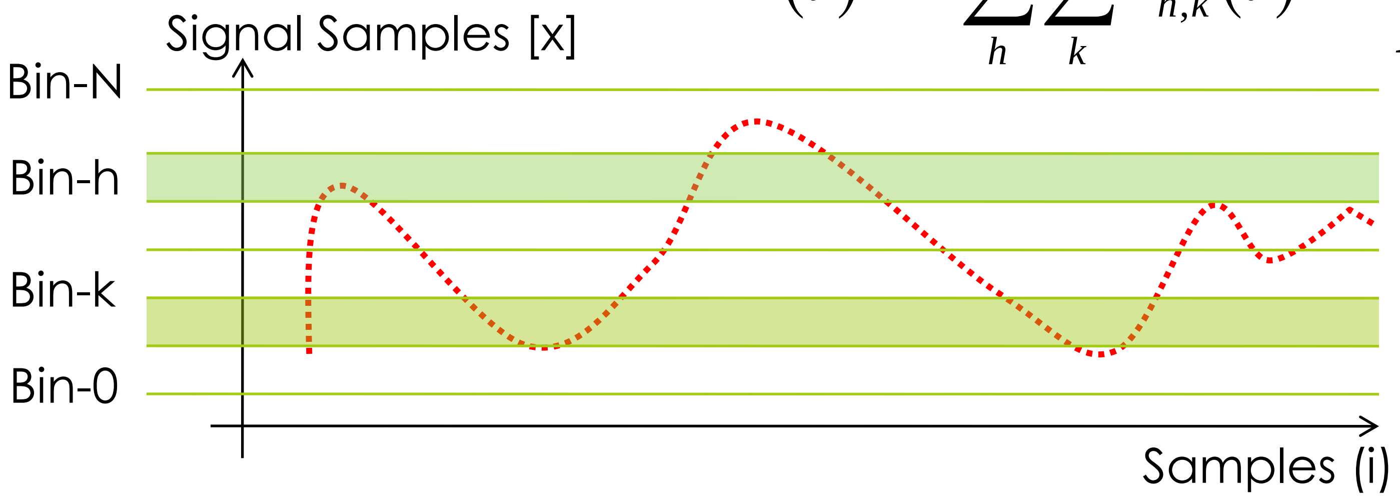
$$2D: \; X = X(n) \qquad Y = X(n+\tau)$$

$$3D: \; X = X(n) \qquad Y = X(n+\tau) \qquad Z = X(n+2\tau)$$

- How to choose τ ?
  - Can be guessed
  - Can be calculated using information theoretic concepts:
    - Minimize the mutual information between the signal and its time delayed version.

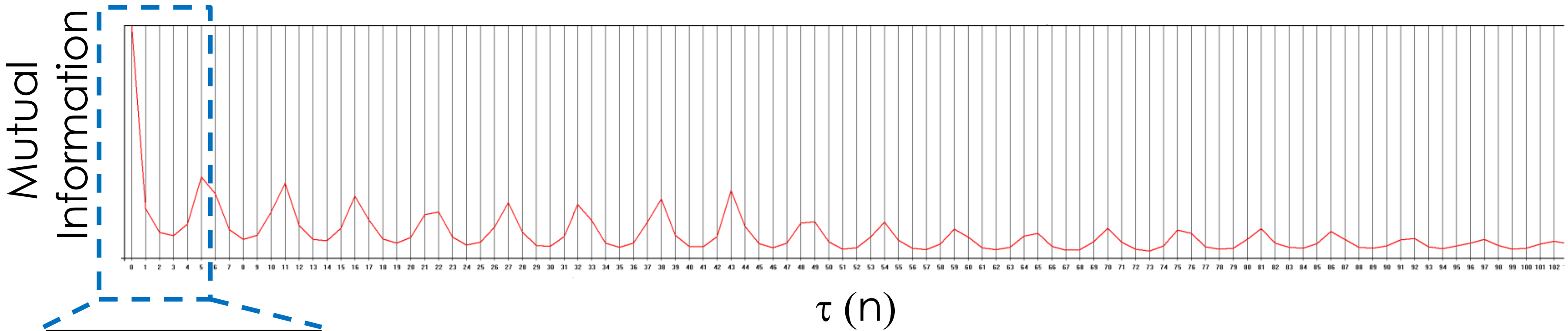
How to calculate mutual information?

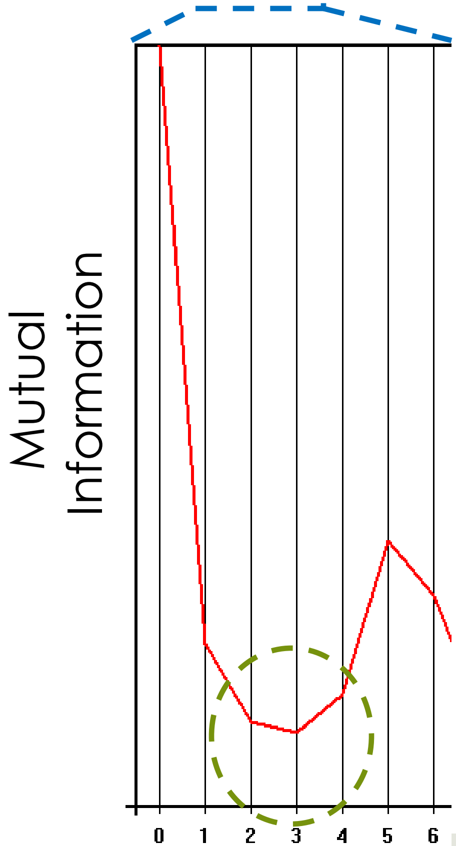
$$I(\tau) = -\sum_{h}^{j}\sum_{k}^{j} P_{h,k}(\tau)\ln\frac{P_{h,k}(\tau)}{P_k P_h}$$



- P_h and P_k denote the probabilities that the signal assumes a value inside the *h*th and *k*th bins.
- $P_{h,k}$ (τ) is the joint probability that x_i and $x_{i+\tau}$ is in bin h and k respectively.
- j is a large enough number
- How To estimate P_i?
  - By counting the points in each box and dividing by Total number of samples. $P_i = \frac{N_i}{N_i}$

Optimal value of τ ?





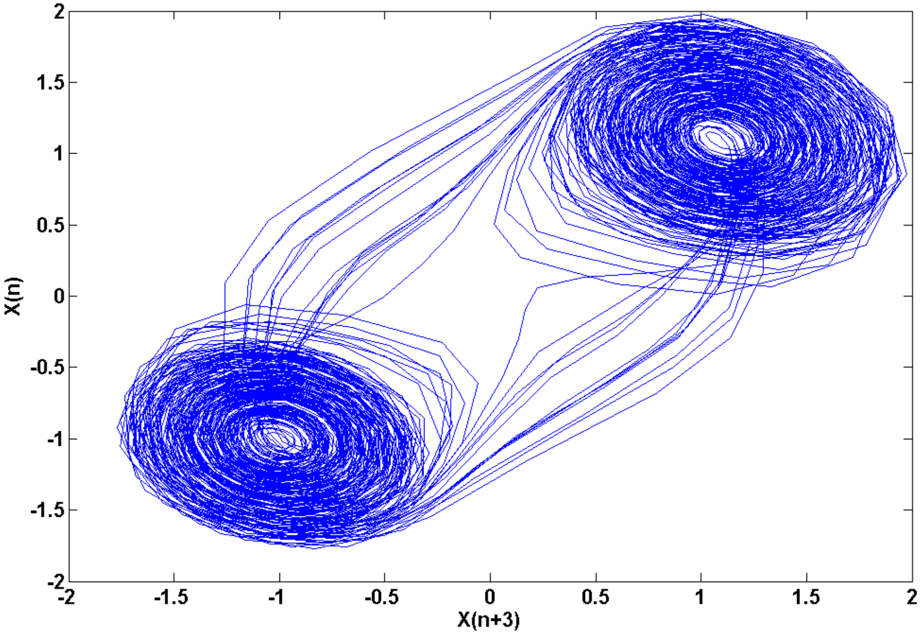
$$I(\tau) = -\sum_{h}^{j}\sum_{k}^{j} P_{h,k}(\tau)\ln\frac{P_{h,k}(\tau)}{P_k P_h}$$

τ for which the mutual information is minimal can be used as an optimum value of time delay.

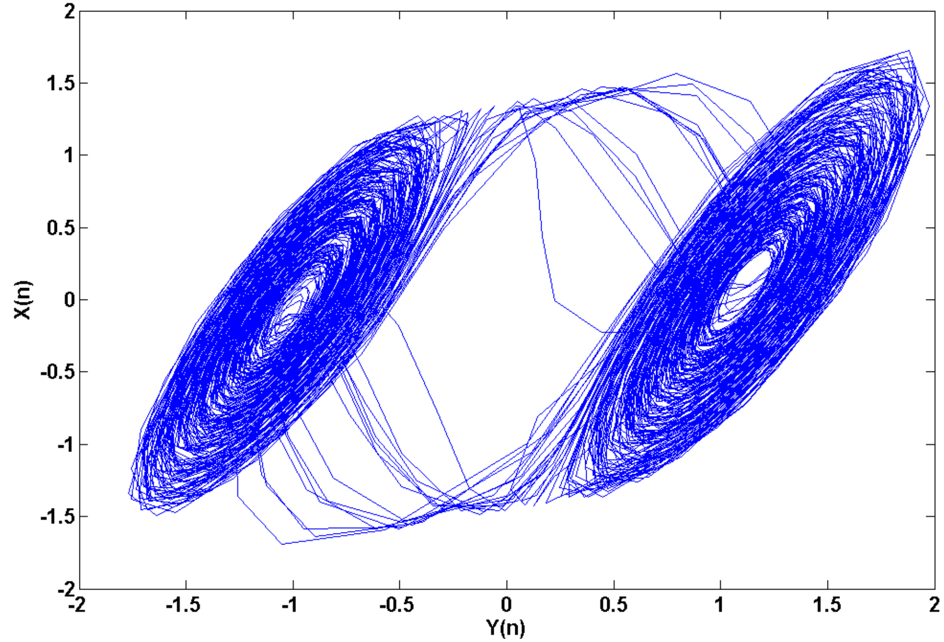
For this calculation N has been set to 100.

First minimum occurs at n=3

Reconstructed Attractor in 2D



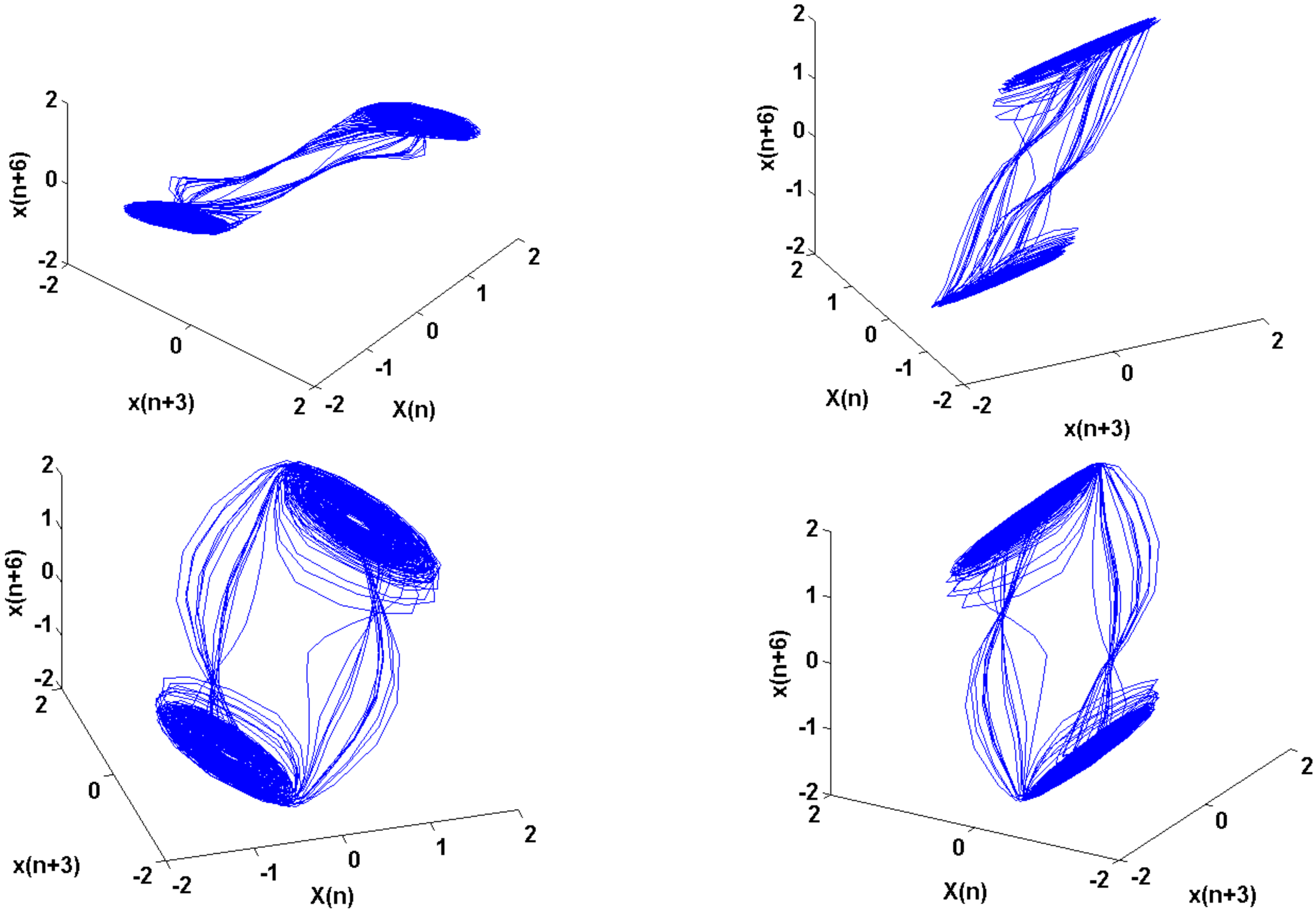
Reconstructed.



Measured

In good agreement with the measured experimental 2D attractor.

Reconstructed Attractor in 3D

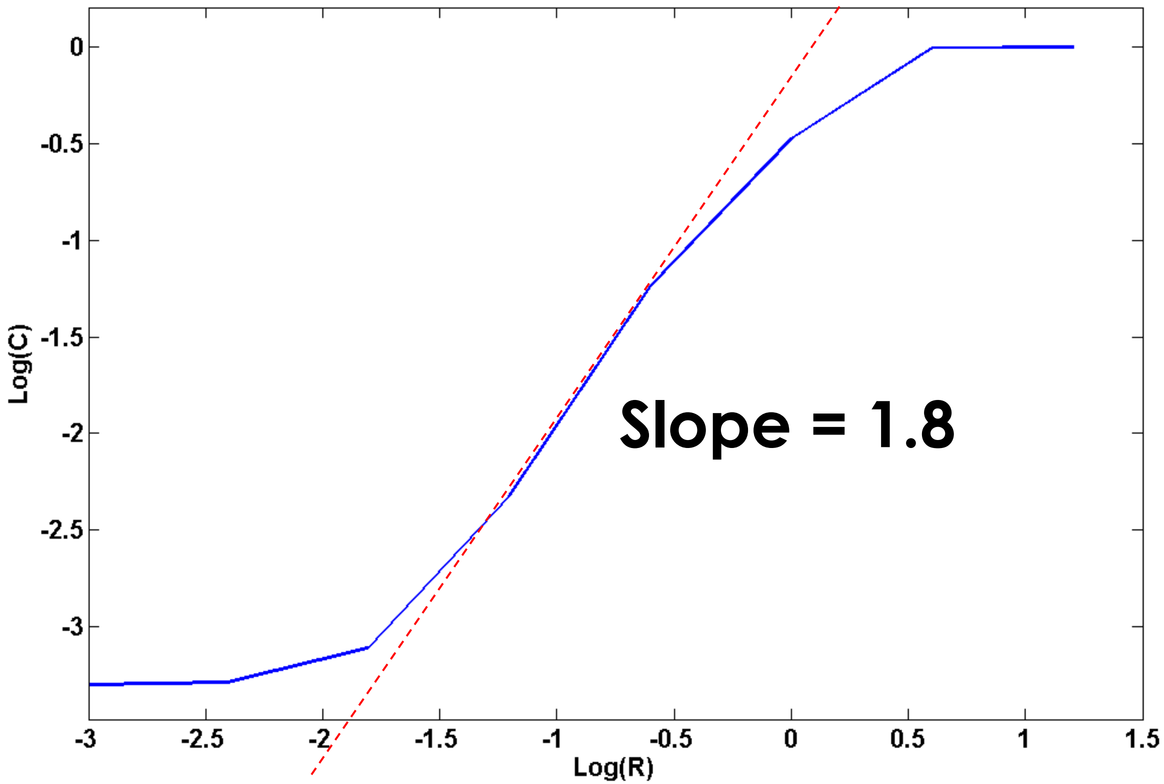


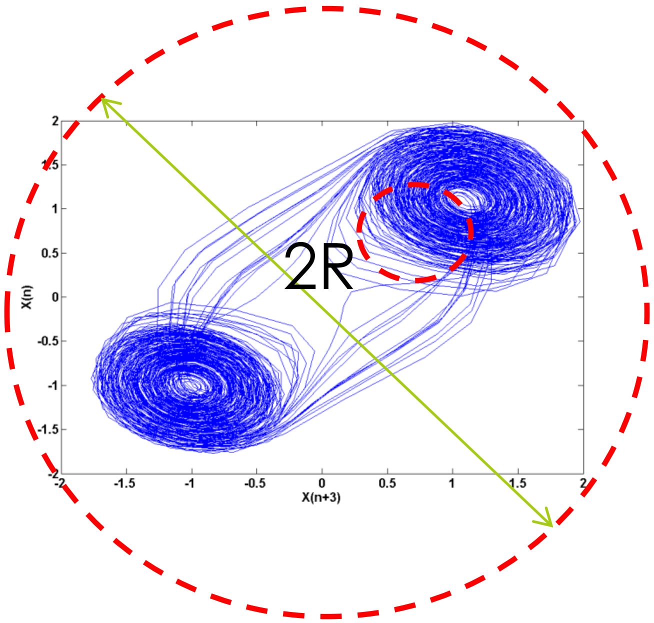
Although we didn’t get the chance to get the 3D attractor in experiment but the reconstructed attractor look s similar to that of simulation.

Correlation Dimension

$$C(R) = \lim \left[\frac{1}{N^2} \sum_{i,j=1}^{N} H\left(R - | X_i - X_j |\right) \right]$$

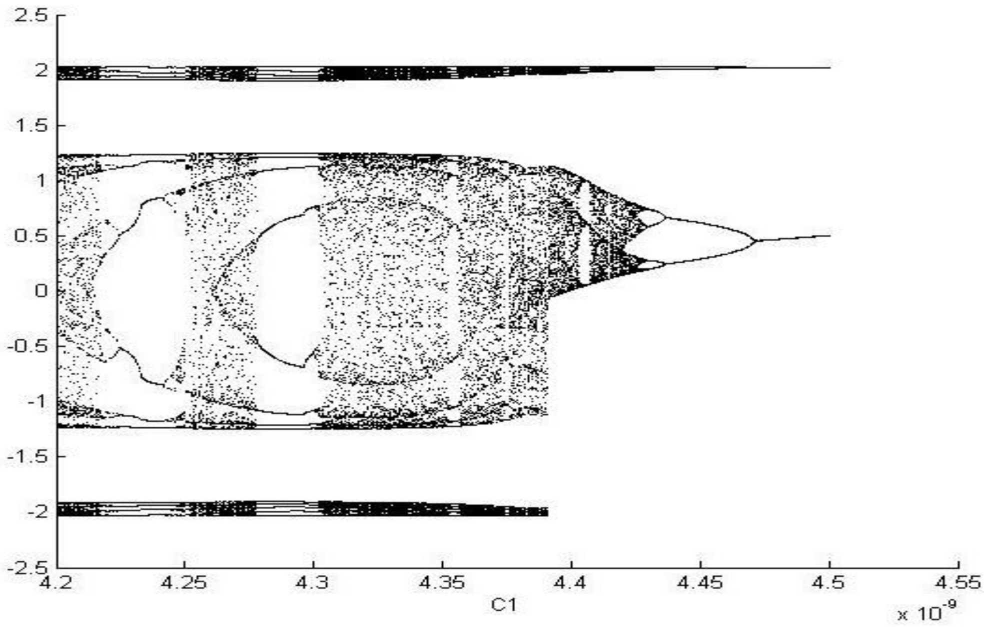
Assume that the trajectory h
N discrete points.

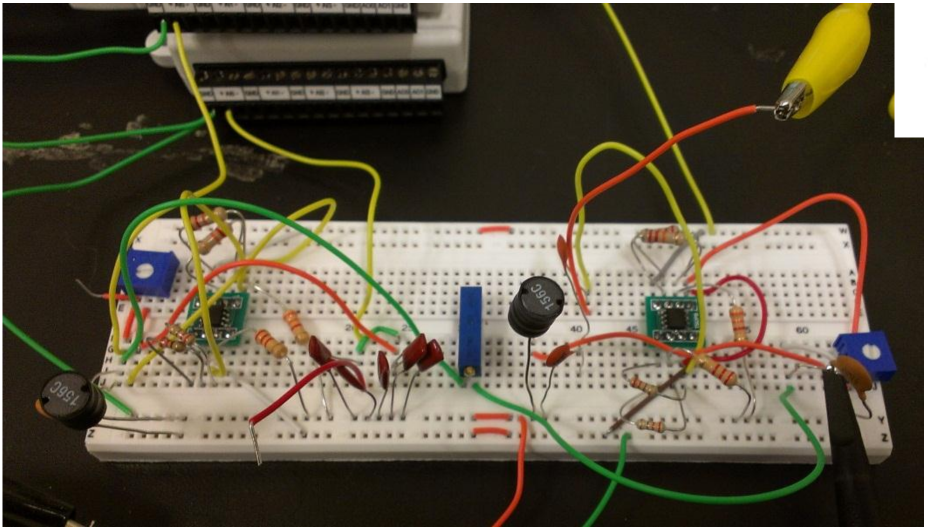




Outlook

- Period doubling of Chua's circuit
- Acoustically synchronized chaotic circuits





Thank you

Any questions?