Investigating Magnetic Plinko as an Analog to Ski-Slope Dynamics

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I. INTRODUCTION

The game Plinko on the Price is Right™ requires a participant to drop a puck vertically through a field of pegs such that it falls into bins at the bottom of the board. Each bin at the bottom contains a monetary value which is awarded if the puck lands in that slot. Simple probability theory shows that the distribution of possible final states of the puck should resemble a Gaussian distribution [1]. However, no work has been performed to map the final states to their corresponding initial conditions. It is initially posited in this work that basins of attraction for the final bins exist in the initial condition space. Certain areas of initial conditions will tend to the same final outcome.

Collision of the puck with a series of pegs resembles random behavior for a large number of pegs. This is due to the fact that small deviations in initial condition result in different reflection angles upon contact with a peg, as shown in Figure 1. This angular difference grows in subsequent collisions, and the trajectories will diverge. This prevents the establishment of basins of attraction because the system is too sensitive to initial conditions. Regions of initial conditions will result in trajectories that eventually cover the entire range of possible final states.

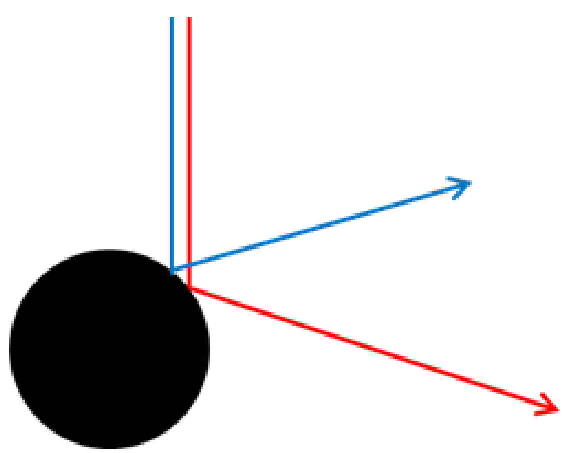

FIG. 1. Single collision schematic showing divergence of trajectories after collision.

While Plinko as shown on the Price is Right is a random process, there are similar types of scenarios wherein an object descends through a regular field of obstructions that alter its trajectory as it descends. One such scenario is ski-slope dynamics [2], as shown in Figure 2. In this scenario, a series of moguls are located on a slope, and a sliding object is allowed to descend under the force of gravity. As the sliding object interacts with the moguls, the trajectory is altered.

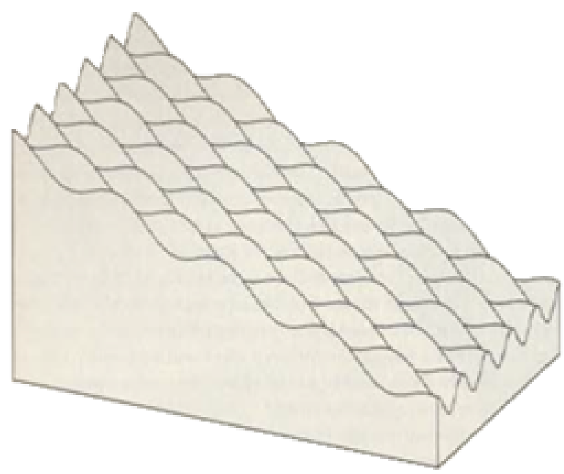

FIG. 2. Schematic of a ski-slope of moguls (adapted from Lorenz [2]).

Lorenz created a simulation of this problem that shows significant chaotic behavior. Tracking cross slope velocity as a function of cross slope location for a variety of downslope positions shows the formation of an attractor. This structure is shown in Figure 3, adapted from the results presented by Lorenz [2].

The peaks of the moguls are similar to the pegs of the Plinko board, in that any object approaching the peak will be repelled. Instead of a discrete collision as exists within Plinko, a ski-slope provides continuous force acting on the object. This creates a continuously varying trajectory which does not necessarily diverge from neighboring trajectories as in Plinko. Lorenz provides a simulation which sets a framework for looking at ski-slope dynamics, but does not have experimental data to support the model.

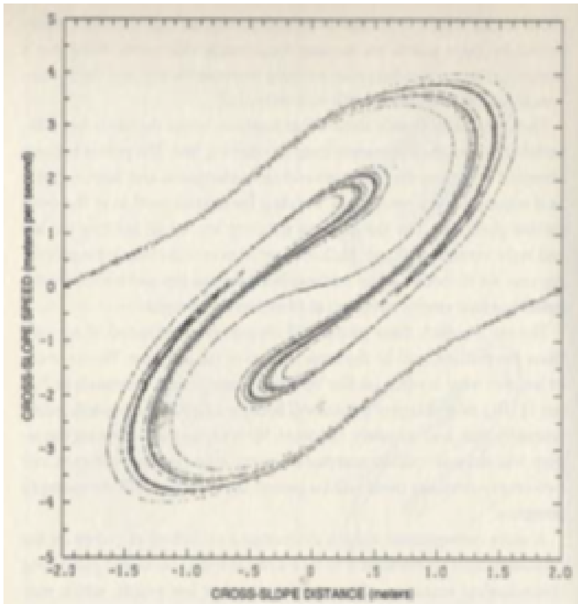

FIG. 3. Plot of for Lorenz ski-slope dynamics [2].

Combining Lorenz's observations with the original idea of investigating a Plinko board, an experiment and simulation have been developed to investigate the chaotic behavior of a magnetic Plinko board. Instead of discrete collisions from a peg board, magnets will be used to continuously alter the puck trajectory as it descends through the field. Rather than using raised pegs which could still provide collisions, the magnets are embedded within the Plinko board so as to provide a smooth, unobstructed surface for the puck to slide along. This is not a perfect analog to the Lorenz simulation, as in that simulation each mogul affects the trajectory without regard to the presence of other moguls. For a magnetic Plinko board, there will be some magnetic force associated with all of the magnets in the board, not just the nearest magnet to the current point in the trajectory. Tracking of the trajectories will provide data in support of the magnetic Plinko simulation to verify if the structures seen are physical in nature.

II. EXPERIMENTAL SETUP

There are three components to the experimental setup to obtain trajectory data for the magnetic Plinko scenario. First, the puck must be constructed so that it can slide down the Plinko board. The puck is made from Teflon to reduce the friction on the puck as it slides. The Teflon houses a magnet, as shown in Figure 4, to create the magnetic force on the puck as it slides through the magnet field. A white ball is attached to the top of the puck for ease of tracking with the camera system.

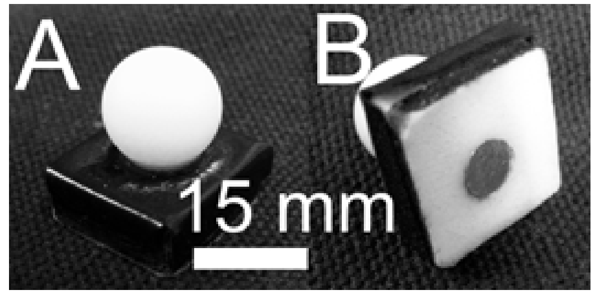

FIG. 4. Experimental magnetic puck.

Next, the puck needs a magnetic Plinko board to slide down. Rare earth magnets are inserted into a Plexiglas sheet that was predrilled to have a series of holes in a triangular formation with approximately 3 cm spacing. In this configuration, the magnets attract the puck, meaning that the magnet sheet represents the basins between moguls of the ski-slope dynamics problem. This sheet is found to create too strong of a magnet field by itself, as the puck would tend to attract to the magnets and stop or flip over and fall off the board as it passed. To reduce the strength of the magnetic forces while still providing a sufficient force to noticeably alter the trajectories, a thin sheet of plastic is added to the top of the Plexiglas. This serves to increase the distance between the Plinko board magnets and the puck magnet, which reduces the overall force. Figure 5 shows the board thickness, and Figure 6 shows the board overlaid with color coded regions corresponding to the initial drop regions for the experiment and the camera capture region. A total of 28 magnets distributed over 5 rows create the final magnet sheet. The plastic is black so that the ball on top of the puck is easily distinguishable.

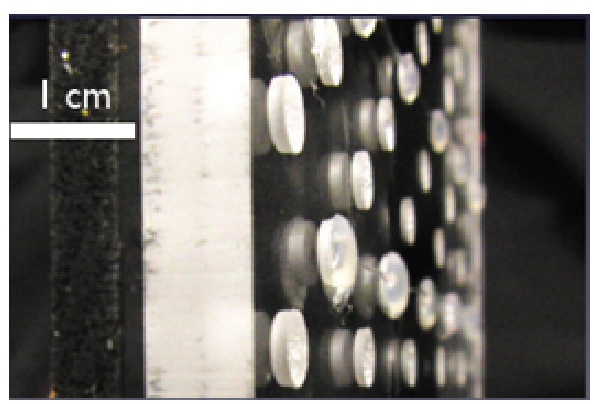

FIG. 5. Plexiglas magnet sheet with black sliding surface attached on the left.

Lastly, a camera system is required to capture the trajectories of the puck as it descends through the magnet sheet. An Allied Vision PIKE camera is connected to a computer and LabView is used for real time tracking and data collection of each run. LabView tracks the white ball on top of the puck, which shows up clearly against the totally black backdrop of the Plinko board. Trajectories are recorded in dimensions of pixels from the camera system.

It is quickly found that drops near the edge of the board have a possibility of falling off the edge of the Plinko board. Thus, runs are confined to initial drop locations near the center of the board. The drop region is shown in Figure 6 as a red box, and the camera field of view is shown with a white background.

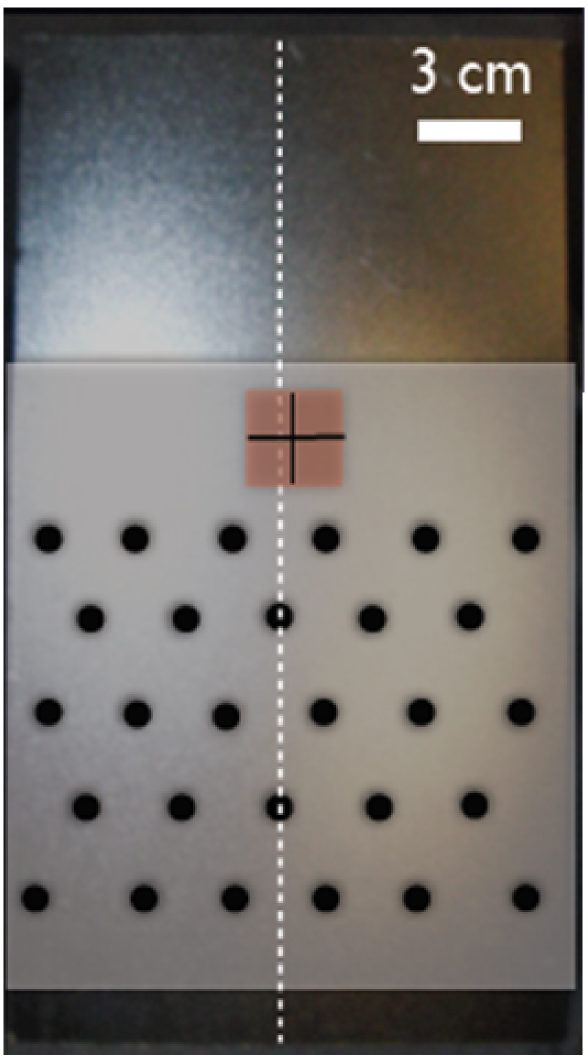

FIG. 6. Magnetic Plinko board showing initial drop region (red) and field of view of the camera (white).

The initial drop region is subdivided into four regions so that a clustering of runs can be more easily obtained. Runs with the puck starting in a similar region demonstrate the sensitive dependence on initial conditions determined by the proximity of the puck as it passes the magnets. The initial drop region is biased to one side since the magnet sheet should provide symmetric trajectories, and the effects of passing near a magnet on the top row should be examined in addition to trajectories which pass near the board's centerline. 100 drops are conducted within each region, for a total of 400 trajectories.

III. EXPERIMENTAL RESULTS

The trajectories for every run are shown in Figure 7, with X and Y coordinates plotted in pixels. The puck tends to want to follow paths which track either vertically between magnets or along diagonal lines of magnets. This is most likely due to these directions being the direction of the strongest magnetic force when the puck is passing a magnet. For the vertical path, the magnets on either side of the path will roughly cancel out in force, resulting in a net force down the magnet sheet. For the diagonal trajectories, there is again an approximate symmetry of the magnet positions relative to the trajectory that results in the off-diagonal magnets cancelling out each other's forces. Thus the net magnetic force is mostly directed along the diagonal magnet line. The oscillation in the trajectories is a result of the initial velocity vector direction causing a perturbation off the straight line path.

To investigate the shape of the attractor as seen in the Lorenz simulation, it is necessary to know the puck velocity at points along the trajectory. Since the LabView setup records time and position for the puck, a central difference is set up to calculate velocity of the puck at each point on the trajectory. To track the formation of the attractor, the velocity is mapped as the puck passes each row of magnets. The top row is designated as row 1, increasing to row 5 at the bottom row of magnets. Plots of cross slope velocity as a function of cross slope position are shown in Figure 12 and Figure 13.

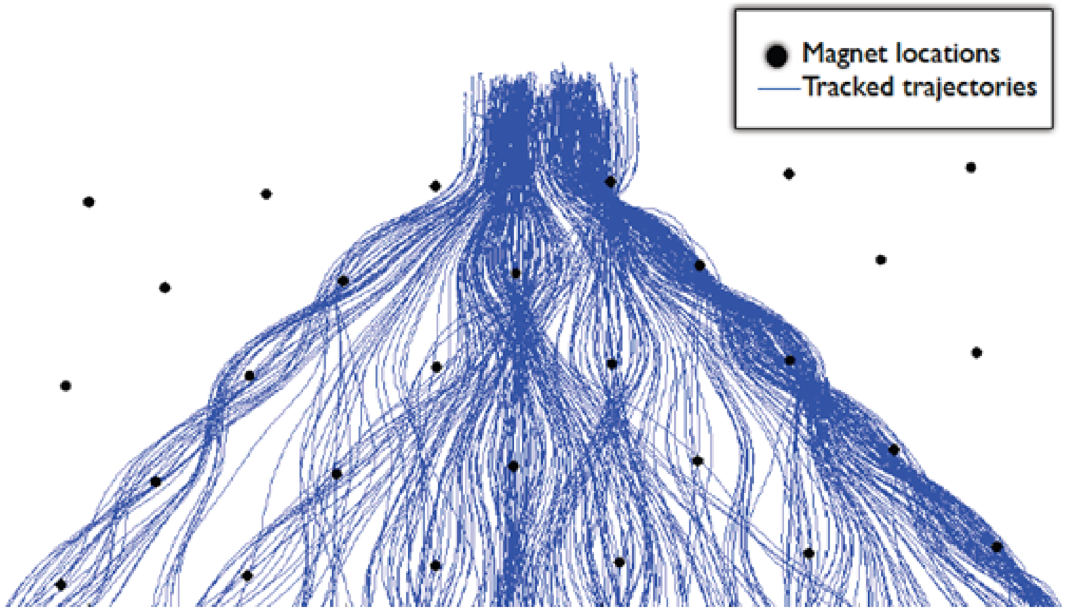

FIG. 7. Trajectory paths for each run with magnet locations shown as black dots.

IV. SIMULATION SETUP

To simulate the magnetic Plinko board, a MATLAB code has been written to integrate the equations of motion for the puck taking into account accelerations due to gravity, friction, and the magnets. The gravitational acceleration, shown in Equation 1, depends on the tilt angle of the board. The acceleration due to friction, shown in Equation 2, depends on the tilt angle of the board and the kinetic friction coefficient. The direction of this force is opposite that of the velocity vector but not dependent on the puck velocity. The acceleration from the magnets, shown in Equation 3, is modeled as a $1/r^2$ function, which should be valid for small magnets spaced far apart. This force is determined for each magnet and summed together to get the net magnetic force. A negative value of the coefficient represents an attracting magnet while a positive coefficient represents a repelling magnet. Each of these forces is written in the frame of the magnetic sheet, where X is cross slope position and Y is downslope position. The origin is set to be the lower left magnet on the magnet sheet.

$$\vec{a}_g = [0, -g \cdot \sin\theta] \tag{1}$$

$$\vec{a}_f = -\mu \cdot g \cdot \cos\theta \left[\frac{V_x}{V}, \frac{V_y}{V}\right] \tag{2}$$

$$\vec{a}_m = \sum_{i=1}^{N_{magnets}} \left(\frac{k}{{R_i}^2}\right) \cdot \hat{R}_i \tag{3}$$

In order to determine a reasonable coefficient for the magnetic force of each magnet, the simulation is run with a fixed initial drop height of 14 cm. The initial cross-slope position is varied from 0 to 7.5 cm, and the magnetic force coefficient is varied from -0.0005 to -0.04. The final cross-slope position, defined to be the location at which the puck passes the bottom row of magnets, is determined for the ranges of these two parameters. The contours of final cross-slope position are shown in Figure 8. Note that for a magnetic force coefficient less than roughly -0.005, the magnetic force saturates and trajectories converge to the same end state.

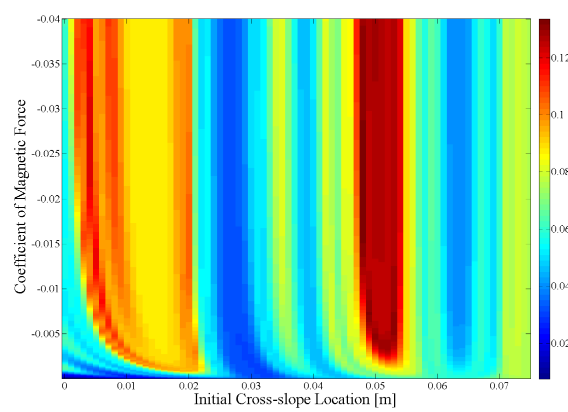

FIG. 8. Mapping of final cross-slope position to initial cross-slope position and magnetic force coefficient.

Table I shows the final values of the various parameters governing the motion of the puck used in the simulation. The coefficient of magnetic force is chosen as the value that shows the trajectories converging to similar final states.

Table I: Simulation parameters used to determine trajectories for magnetic Plinko.

Parameter	Value
Tilt Angle	18°
Gravity	$9.81m/s^2$
Kinetic Friction	0.15
Magnet Depth	$0.5cm$
Magnetic Force	-0.005

V. SIMULATION RESULTS

First, a sampling of trajectories is run in the simulation to ensure that the model is qualitatively showing the same results as are seen in the experiment, as shown in Figure 9. The trajectories show the expected behavior of preferring either a nearly vertical drop, or a path along a diagonal line of magnets. There are perturbations due to the other magnets, but the general trend agrees with that seen in the experiment.

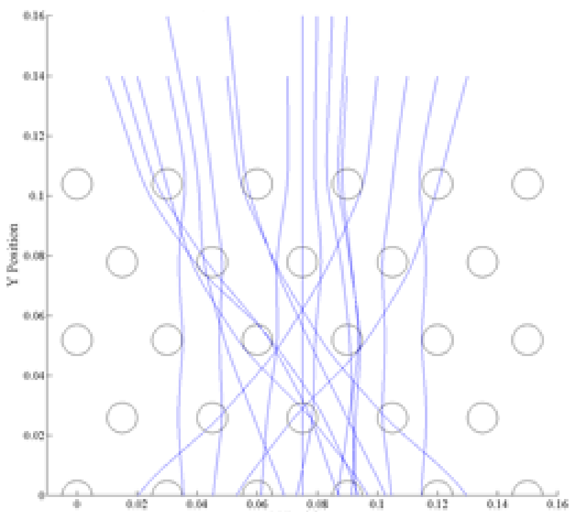

FIG. 9. Sampling of trajectories with the simulation parameters of Table I.

For a range of initial cross-slope positions and initial heights, clear basins of attraction exist when mapping the final cross-slope positions. These are shown in Figure 10. Since the mapping should be symmetric about the center of the Plinko board, only one half of the initial condition space is shown. Initial cross-slope varies from 0.5 to 7.5 cm and initial height varies from 12 to 16 cm. As the initial drop height increases, the puck has a greater speed when passing the first row of magnets. Thus, the trajectories are less affected by the magnets since the puck moves through the magnet sheet quicker. For drop heights just above the magnet sheet, the puck has not built up enough speed to shoot through the system relatively unperturbed, and more interesting dynamics exist. In particular the regions where a puck passes just to left or right of a magnet show a tendency to attract to the same final state along diagonal paths.

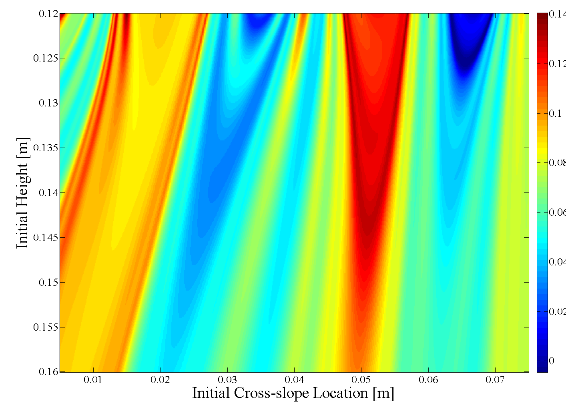

FIG. 10. Mapping of final cross-slope position to range initial cross-slope positions and heights.

The Lorenz simulation shows that an attractor forms in the mapping of cross-slope velocity to cross-slope position at a constant downslope location. Within the simulation, the velocity has been captured at each row of magnets and similar plots can be generated. Figure 11 shows the progression of this plot as the puck passes each row of magnets. The initial height is fixed at 14 cm, and initial cross-slope position is varied from 0 to 15 cm. The initial cross-slope position is discretized at intervals of 0.001 cm, which shows mostly continuous plots for the first four rows, but gets too sparse at row 5 to clearly see the underlying structure. A more refined set of solutions would need to be run to better determine the underlying structure. The mapping exhibits folding behavior from row to row, where the shape of the map folds on itself to create a chaotic structure. It is not expected that this map will exactly resemble the Lorenz simulation, as the physics of the problem are different, but the formation of an attractor does seem to agree in principle. A larger sheet of magnets may better match the Lorenz results by providing a longer distance downslope for the attractor to form.

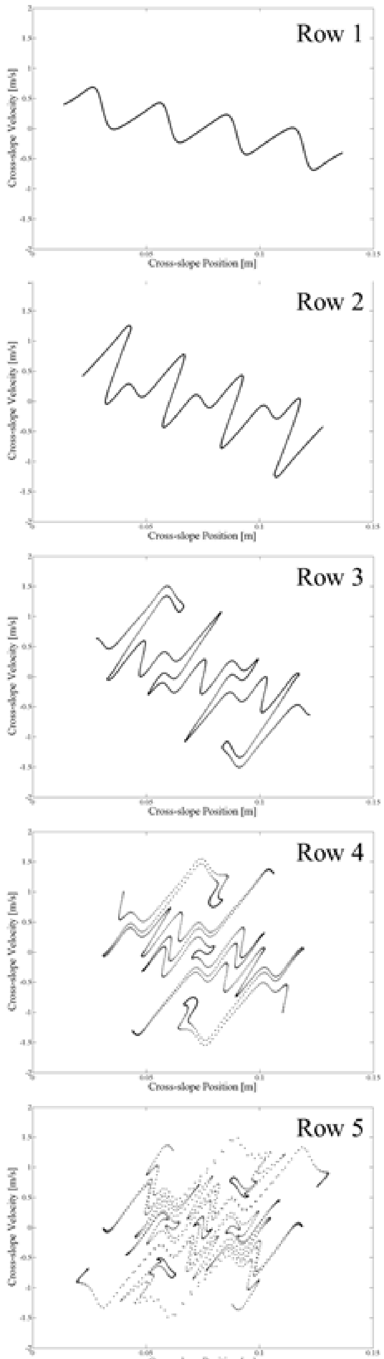

FIG. 11. Progression of attractor structure from top magnet row (1) to bottom (5).

While the magnetic Plinko simulation agrees qualitatively with the results seen in the Lorenz simulation, its agreement with the experimental results must be examined. Since the experiment did not run a full sweep of initial cross-slope positions at a fixed height due to concerns about the puck leaving the magnet sheet, a full structure comparison is not possible to obtain. However, the limited experimental runs do provide some basis to support that the simulation is accurately modeling the experimental setup. Figure 12 shows plots of cross-slope velocity as a function of cross-slope position for Row 1 in both the simulation and experiment. The box on the simulation plot indicates the corresponding range of cross-slope locations to the experimental plot. A similar structure exists for both plots, indicating that at least for the first row of magnets, the simulation compares favorably with the experiment.

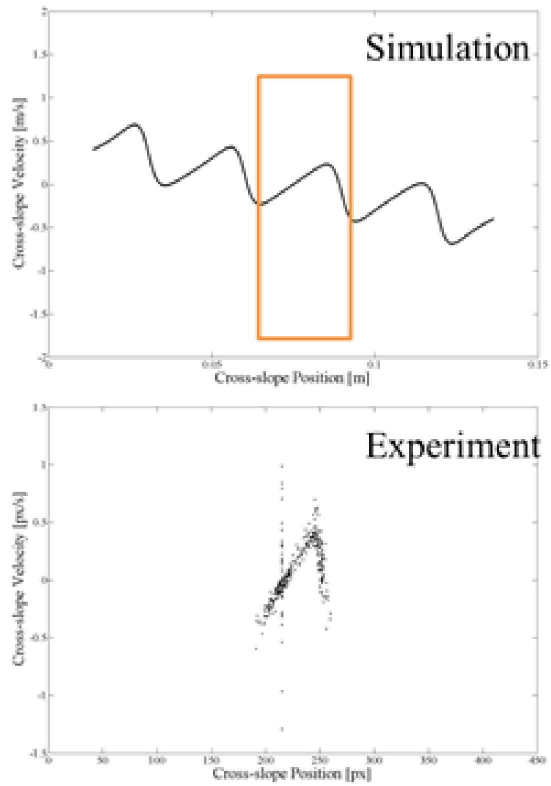

FIG. 12. Comparison of simulation with experiment for puck passing Row 1.

Performing a similar comparison at row 4 or row 5 is not possible due to the limited number of experimental data points. The comparison for row 3 is shown in Figure 13. The scarcity of experimental points still makes it difficult for an exact comparison, but some of the features of the simulation structure appear to exist in the experimental results. There is a rise and fall of the mapping which tends to follow a similar form between the two plots. The experimental results do show a positively sloped trend compared to the negatively sloped trend from the simulation, but it may be that the experimental results only show a subset of the interior of the simulation structure. For example, just the central region of the simulation plot shows a neutral or slightly positive trend, which may be more in line with the experimental results.

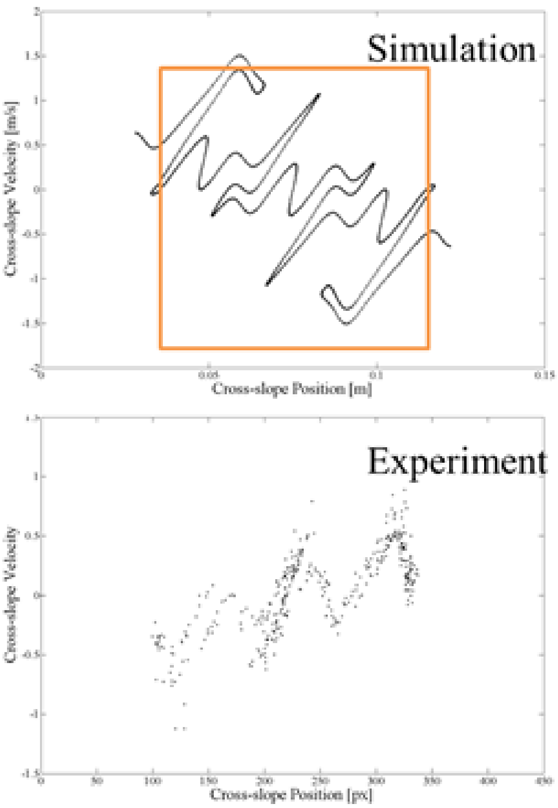

FIG. 13. Comparison of simulation with experiment for puck passing Row 3.

VI. CONCLUSION

A magnetic Plinko board exhibits characteristics which are consistent with results seen for a ski-slope dynamics problem. Clear basins of attraction form for a magnetic puck dropping through a sheet of magnets, particularly when the puck initially passes close to one of the magnets. Mapping cross-slope velocity against cross-slope location as the puck travels down the Plinko board shows the formation of a folded structure similar to that seen in Lorenz's ski-slope dynamics simulation. Experimental results and simulation results of the magnetic Plinko board are in qualitative agreement under the assumptions made for the values of modeling parameters in the simulation. Inconsistencies arise due to physical errors and uncertainties in the Plinko board, location of magnets, and determination of relevant parameters that limits direct quanitative comparison. Limitations in the experimental setup due to the puck faling off the side of the Plinko board prevented acquisition of data for initial conditions located far off center, which also limited the ability to validate the simulation results with the experiment. What qualitative data was taken shows an attractor structure that is similar in shape between the simulation and experiment, indicating that the simulation is a good model for the puck traveling through the magnet sheet.

To further investigate the chaotic structure of magnetic Plinko, more experimental data points would need to be taken. Determining a method to allow for a wider range of initial conditions would allow for a more thorough construction of the attractor as the puck passes each row of magnets. Within the simulation, running a finer resolution sweep of initial conditions would better fill in the structure of the attractor for magnet rows further down the board. Improving the models for the magnetic force would also potentially affect the attractor shape. A series of rare earth magnets will create a complicated magnetic field that is not being modeled in the current simulation. Some of these underlying magnetic field interactions may better explain the tendency of the puck to travel along certain paths as it descends through the magnet sheet.

[1] Biesterfeld, A. *The Price (or Probability) Is Right.* University of Colorado at Boulder. Journal of Statistics Education. Volume 9, Number 3 (2001).

[2] E. Lorenz. *The Essence of Chaos.* Jessie and John Danz lectures. UCL Press, 1995.