Synchronization of LED fireflies using finite line of sight

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The synchronization of fireflies has been well studied over many different models. In previous studies, models assume that the fireflies are globally coupled, or that each firefly can see every other firefly[1]. In the wild fireflies can demonstrate this synchronization over a forest[2] in where it would not be possible to see every firefly in the network and instead face physical limitations of boundaries and distance. Using a Peggy LED board, this puts forth a model that accounts for these constraints by setting the adjacency matrix for the network by setting a periodic boundary radius for each LED. The effects of time to synchronize are measured against this radius and variations in the coupling strength to determine a predictive model for the time scale to synchronize the population.

I. INTRODUCTION

It has long been known that fireflies can synchronize with each other such that they flash simultaneously[3]. This behavior is viewed accross forests for large populations of fireflies. More recently it has begun to be examined as to why fireflies have developed this synchronization strategy. While the flashing of individual fireflies has been known to be connected to mating, Avila et. al demonstrated that synchronization furthers this goal[4]. Female fireflies will only respond with a flash when they see a set of periodic flashes at the recognized frequency of their species. If males were to flash out of phase from each other, the female would not recognize the aggregate sum of flashes as being of her species and would not respond in turn with a flash. The fireflies would then not attract each other and the mating strategy would fail. Instead, the fireflies synchronize their flashes to entrain to a particular frequency and produce the desired female response.

While fireflies have been known to synchronize, this is not the only example in nature of a network of coupled oscillators. Many of the same fundamental principles apply among these various systems[1]. Pacemaker cells in the heart, nerons in the brain, circadian pacemakers, and insulin-secreting cells in the pancreas all are examples of coupled biologic oscillators. Among other organisms synchronization phenomena can be observed such as with the chirping of crickets. Even with humans women are known to synchronize their menstrual cylces.

To better understand the synchronization behavior in fireflies in particular many different models have been proposed. Without being able to directly measure the firing function that an individual firefly uses an only observing the flash at the ned of the period, it is unknown what the exact model of firing should be. Ultimately models are chosen such that the resulting behavior gives something similar to the observed synchronization behavior of fireflies. In truth, the exact model depends partly on the species of firefly[5]. Some fireflies, such as the Photinus pyralis, only demonstrate transient synchrony that can best be described by integrate and fire methods that only advance phase such as the one used be Strogatz[?] . Other species such as P. cibellata demonstrate many possible frequencies where it is assumed that their phase may advance or delay depending on their stage in the cycle[2]. A model such as the Kuramoto model can describe that behavior[6]. Lastly some asian species of fireflies such as the P. malaccae, Pteroptx tener, and Luciola pupilla can entrain with zero phase lag in what is called perfect synchrony[5]. There are even some models that propose that female fireflies follow different coupling laws than males of the same species[4].

These proposed models each put forth some physically or biologically unrealistic assumptions. The Kuramoto model in particular uses some questionable assumptions[6]. In the Kuramoto model, the phase of each oscillator updates according to the equation:

$$\frac{d\theta_i}{dt} = \omega_t + \frac{K}{N}\sum_{j=1}^{N} sin(\theta_j - \theta_i)$$

where θ_i is the phase of the firefly, ω_t is its frequency, K is the coupling strength, N is the population size, and θ_j is the phase of another coupled oscillator. The problem with this equation is that it allows for fireflies to update continuosly, knowing the instantaneous phase of a neighbor without it even needing to fire. Kuramoto aside, all these proposed models are tested in cases of globally coupled oscillators and do not take into account sight limitations that fireflies face in nature.

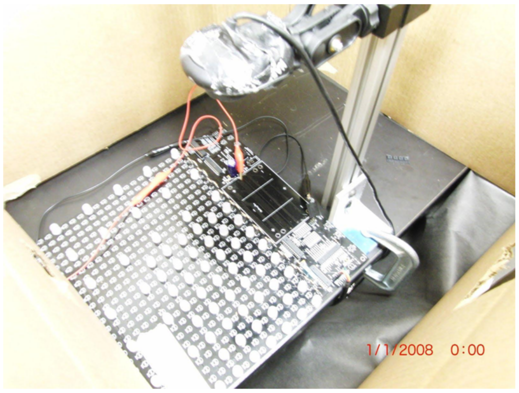

FIG. 1. Experimental design in light-blocking box

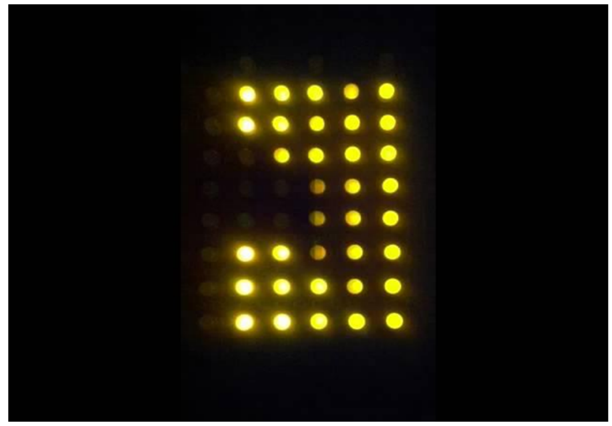

FIG. 2. Captured image from webcam

II. METHODS

Fireflies are modeled with LED's being placed on a Peggy 2.0 LED board in an 8 by 6 array. A webcam is entrained over the selected region and processes live video to identify whenever an LED fires. These images are processed in a Matlab program that updates each simulated firefly according to the model and passes output commands to Arduino. Arduino intefaces directly with the Peggy board and the fireflies are updated once more.

Given that there is a difference between male and female fireflies in their response to flashes, it is assumed that all of the modeled fireflies are male. The model used to dictate the behavior of the fireflies is an updated version of the Kuramoto model. Instead of updating continuously the model is instead updates only when a firefly has just fired. In addition an adjacency matrix is added to the model such that the new Kuramoto equation is as follows:

$$\frac{d\theta_i}{dt} = \omega_t + \frac{K}{N} \sum_{j \subset fired} A_{ij} sin(\theta_j - \theta_i)$$

where A_{ij} is the adjacency matrix for the network. The adjacency matrix is given values of 1 for connected fireflies and 0 for those that are not connected or when $i = j$.

The adjacency matrix is determined using a radial line of sight that is loops across boundaries to generate a symmetric matrix. The distance is determined by the unitless position of each firefly on the 8x6 grid. Each firefly effectively draws a circle of a particular radius about itself and any fireflies that fall within that circle are considered to be within sight and connected. With periodic boundaries, the circle is allowed to wrap around boundaries so that each firefly is connected to the same number of fireflies. Furthermore, the line of sight method is also symmetric where fireflies are mutually connected.

Radii.png

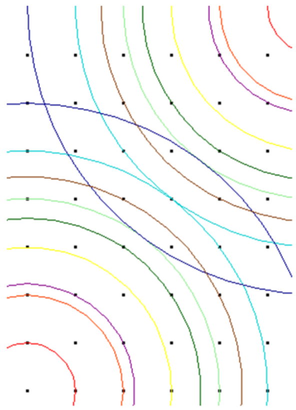

FIG. 3. Circles of various radii drawn around one node with an example reflection accross the diagonal boundary

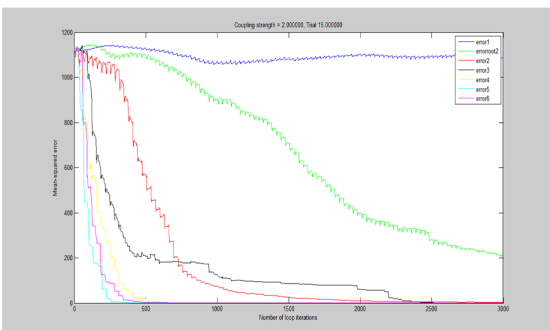

FIG. 4. For set coupling constant $K = 2$, mean-squared error with respect to number of loop iterations for different radii.

III. RESULTS

In varying line of sight for different adjacency matrices A_{ij} and the coupling constant K the time scale to synchronization could be observed. For a typical run, the mean squared error between the different phases θ would be measured with respect to the number of loop iterations as demonstrated in figure 4.

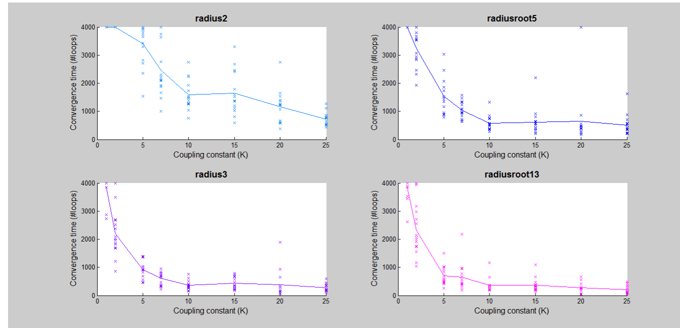

FIG. 5. Average of different starting conditions, τ vs K for different radii.

A threshold can be applied to this data such that an iteration time for synchrony τ can be pulled out, with τ defined as the iteration number when the mean squared error between the phases dropped below 1. With τ defined it is possible to plot different synchronization times with respect to different values of K and different adjacency matrices, as shown in figure 5.

Upon further processing the data, an equation can be developed that will predict the average time it will take to synchronize given the coupling strength K and and the number of connections each individual firefly has as determined by A_{ij}. It can be seen in figure 6 that there appears to be an exponential fit to this data. This would suggest that

$$\tau(K, N_k) = a(N_K)e^{b(N_k)K}$$

where N_k is the number of connections and a and b are parameters with respect to N_k.

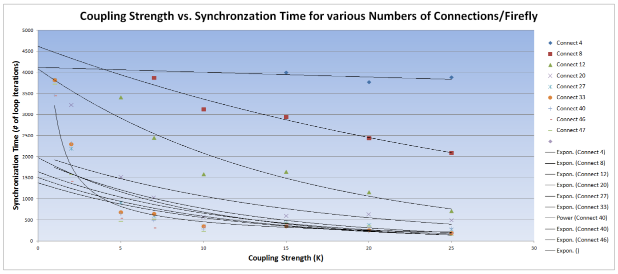

FIG. 6. τ vs. K over various number of connections with an exponential fit

Passing through some final fitting those parameters can be found such that

$$\tau(K, N_k) = \frac{18800}{N_k^{0.67}} e^{\frac{-K}{1000}(1.3N_k+42.5)}$$

.

IV. DISCUSSION

While this is not a perfect fit and a K approaches 0 one would expect τ to approach infinity it is nonetheless a predictive model for the timescale to synchronize within the parameter range. Within this set timescale to synchronize decreases as the coupling constant increases and as the network of fireflies becomes more interconnected.

In this model periodic boundary conditions for the line of sight radius were used in order to make the adjacency matrix symmetric and have the same number of connections for each individual firefly. Without those conditions it was found that the synchronization time was deeply affected by the starting states of the fireflies with a large variance from those initial conditions. A more exhaustive study could run over a larger set of trials to reduce the significance of the variance.

V. CONCLUSION

In proposing a new model for firefly connectedness it can be seen that how fireflies are connected greatly influence their time for synchrony. As obstacles become denser or sight becomes more impaired from natural conditions it would significantly increase the time it takes to synchronize for a firefly population, potentially breaking it from its mating rituals.

Further studies may look into larger populations of fireflies and take into account variance in their natural frequency. With more iterations in the experiment and taking over a larger set of data the predictive model for the time scale to synchronize can be enhanced and improved to predict firefly coupling over a larger set of situations.

REFERENCES

[1]R. Mirollo and S. Strogatz, “Synchronization of pulse-coupled biological oscillators,” SIAM Journal on Applied Mathematics **50**, 1645–1662 (1990).

[2]A. Moiseff and J. Copeland, “Firefly Syn-

chrony: A Behavioral Strategy to Minimize Visual Clutter," SCIENCE **329**, 181 (2010).

[3]J. Buck, "Synchronous rhythmic flashing of fireflies," QUARTERLY REVIEW OF BIOLOGY **13**, 301–314 (1938).

[4]G. M. Ramirez Avila, J. L. Deneubourg, J. L. Guisset, N. Wessel, and J. Kurths, "Firefly courtship as the basis of the synchronization-response principle," EPL **94** (2011), 10.1209/0295-5075/94/60007.

[5]B. Ermentrout, "An adaptive model for synchrony in the firefly pteroptyx malaccae," Journal of Mathematical Biology **29**, 571–585 (1991).

[6]S. Strogatz, "From kuramoto to crawford: exploring the onset of synchronization in populations of coupled oscillators," Physica D: Nonlinear Phenomena **143**, 1–20 (2000).