"Nonlinear Pattern Formation on Faraday Waves: a One Dimensional Study"

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We propose to study the nonlinear pattern formation of Faraday waves. This is basically an experimental study of surface waves excited by vertical vibrations contrasted to existing theories, e.g. the amplitude symmetry equation. In our study, we will discuss the importance of the viscosity of the fluid used in each setup, the importance of the form of the excitation wave, the importance of the shape of the closed vessel, and the importance of the free-surface boundary condition. Stability boundaries, wave amplitude, and perturbation characteristic time of decay will be analyzed.

I. INTRODUCTION

The principle in this experiment is to vibrate a container with liquid vertically at a fixed frequency and amplitude. These waves consist of a standing wave pattern that occurs in a free fluid surface under excitation. At low vibration amplitudes, the liquid moves as a solid body, but at high vibration amplitudes, surface waves start to form. For instance, When a layer of fluid is oscillated up and down with a sufficiently large amplitude, patterns form on the surface, a phenomenon first observed by Michael Faraday in 1831[1]. The standing waves form a pattern on the surface because the amplitude of the excitation exceeds a critical value. The excitation number is defined as $(A - Ac)/Ac$, where the vibration amplitude is denoted by A, the onset amplitude for the waves is denoted by Ac. This excitation number will define the characteristic of the pattern formation.

A collection of different wave modes compete and which pattern wins depends on the excitation number, the frequency, and the liquid viscosity. It is worth noting that all the modes have the same wave length, and that the wave length is independent of the vibration amplitude. In addition, the patterns observed are approximately independent of the boundary conditions for large enough containers; however if the container is small, the shape of the boundary will interfere and may destroy the pattern formation. The patterns range from squares, hexagons, triangular, and 8-fold quasi-periodic depending on the excitation frequency fluid properties (viscosity, surface tension, and density) Even though this phenomenon has been studied since its discovery by Faraday, a full understanding of the subject has not yet emerged.

The formation of Faraday waves has been studied in several physical context, including convective fluids, nematic liquid crystals, nonlinear optics, biology, and, recently, in Bose-Einstein condensates[2]; where the mechanisms of pattern selection was investigated using the tools of symmetry and bifurcation theory. Faraday waves in Non-Newtonian fluid was also recently studied. It was found that it is possible for holes to be created inside fluids such as cornstarch when undergoing vertical oscillation[3]. Forming at a critical driving shaker acceleration and frequency, these holes are unusual in that their motion is not directly dependent on the Faraday waves, because the holes form far away from each other in the medium and tend to merge if they get to close to each other. Zhang and Viñals[4] have derived and equation that describe Faraday-wave amplitudes in the weakly damped and infinite depth limit. These equation has been tested in the weakly viscous and large depth limit and it provides a good agreement between this theory and experiments under these conditions. This theory predicts that the preferred patterns are the ones in which the Lyapunov functional is minimized; which predicts for most excitation frequencies a square pattern for low viscous fluids. But for a narrow frequency band, hexagons and other quasi-periodic patterns are predicted. The experimental observation are: at high viscous dissipation, the observed wave pattern above threshold consists of parallel stripes; at lower dissipation, patterns of square symmetry are observed in the capillary regime of large frequencies. At low frequencies, higher symmetry patterns have been observed like hexagonal, and eight- and ten-fold quasi-periodic patterns. However, many of the experiments use shallow viscous layers of fluid to counteract the presence of high frequency weakly damped modes that can make patterns hard to observe. (A map of patterns vs. frequency is know[5]. Current research aims at the study of this nonlinear pattern formation at a finite distance from threshold. It is now known that the evolution of the wave pattern near onset is variational, and its parameter space behavior is small in most fluids and vanishes with the viscous damping parameter. Non-variational corrections are anticipated to be large near onset, and play a important role in Faraday-wave instabilities and on the transition to spatio temporal chaos.

II. METHODS

The experiment station includes three main parts: (Please see Fig 1)

I. A shaker with attached accelerometer; a digital camera connected to a computer; and a stroboscope.

II. A signal generator; an amplifier; and a digital oscilloscope.

III. A sample preparation station with a fluids; vessels; beakers;

The shaker is expensive and delicate. we must set the amplifier gains to minimum before turning on the system, gradually turning up the gain knob during use. The input frequency should never exceed 130 Hz. The range of interest for Faraday waves for our system is $10Hz \leq f \leq 100Hz$. If you see and hear the typical features of beats instead of a harmonic mode of vibration, immediately set the amplitude to zero with following adjustment of the driven frequency, just to shift the mode of vibration away from the dangerous state.

This experiment is designed to reproduce the results of Binks and W. van de Water[2] and contrast the results with Zhang and Viñals[4,6] prediction of the amplitudes of the standing waves. We expect a fair agreement between our experiment and the theory. In

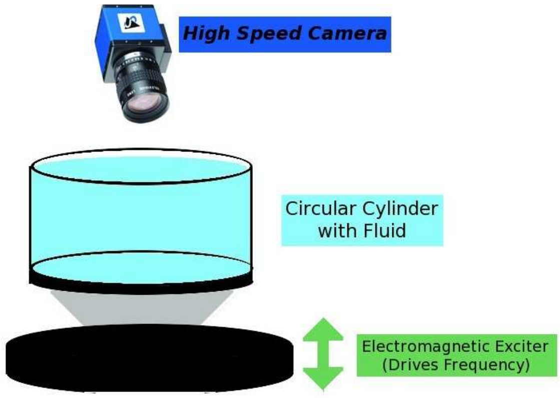

Figure 1. Experimental setup.

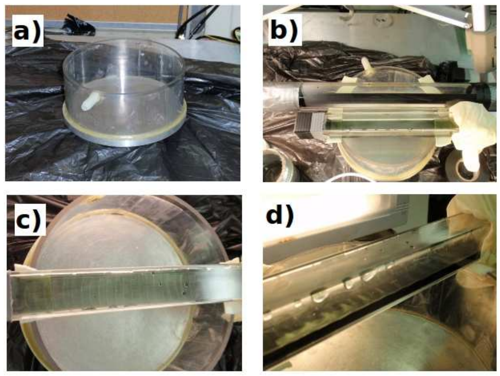

Figure 2. Different container shape representing the boundary conditions.

addition, our experimental study will move a step further and not only investigate the theory for different fluids, including a non-Newtonian fluid, but for different oscillatory patterns (square, sinusoidal,and triangle) and the importance of the boundary condition for relatively small container. To begin, we will build a basic experimental setup in Dr Goldman's lab using an open container that will be subjected to vertical sinusoidal oscillations, which periodically modulate the effective gravity and capture the amplitude by using a high speed CCD camera. The camera will be placed above the container to take snapshots of the standing waves and the frames per second of the snapshots must be greater than the oscillation period of the standing waves. By using the sinusoidal oscillator, we will also construct our trial waves, which will have the form of square and triangular.

We will calibrate our system by measuring the driving amplitude A that exceeds the critical threshold Ac and cause an standing-wave instability; this occurs with temporal frequency that is around one half of the driving frequency as predicted by Faraday. After we have estimates of acceleration at which a pattern first appears, we will be able to observe how these patterns change as the driven amplitude is slowly increased. The characteristic spatial wavelength of the standing-wave pattern can be estimated by the dispersion relation (k) of the fluid and the amplitude of the waves can be measured by a laser.

After the building process, we will first reproduce classical results on pattern formation of water and oil to validate our experimental setup. Since the patterns form by water and oil are well documented, we will observe surface standing waves in water and oil with different driven frequencies and amplitudes to verify that our setup is working properly. Experiments will begin with small depth h=3 mm and increase the depth to about 1.5-2 cm. Then we will begin increasing slowly and carefully the driving force amplitude and stop increasing the amplitude when the resonance is strong enough and begin to increase frequency. This part must be done very slowly as every new pattern cannot become stable very fast. Second we will compare patterns obtained and amplitudes with classical results as a function of excitation frequency, for at least two different working fluids, and one being Non-Newtonian; specifically we will study the amplitudes and patterns with theory as a function of driving frequency. By using the Non-Newtonian fluid, we will test if the theory is consistent across different fluids. Third we will also discuss the effects on patterns for different excitation wave forms like sine, square, triangular. Finally, we will study the boundary conditions of the wave-amplitude formation. For this propose, we will use different container shape like cylinder, and box.

III. RESULTS AND DISCUSSION

All the experiments are performed in the following manner; the frequency is fixed and the amplitude is varied going from 0 to beyond its critical value (Ramp-up) and returning to 0 (Ramp-down). The critical value is where the first Faraday Instability appears. We work with Water, Oil, and a Non-Newtonian fluid with different container shape. Different excitation signal, Sine, Triangular, Square are used. The range of frequencies are between 30Hz to 110 Hz.

In Fig 3, the bifurcation diagram for water at $30Hz$ is plotted, where the A/A_{max} is plotted vs acceleration. In the Ramp-up process, we observed that below a critical acceleration value (≈ 0.4), there is no any formation of Faraday wave. After passing the critical value, the oscillation of the amplitude increases very rapidly from a low to a high value, showing that there exists unstable states between them, locking the system in a bistable behavior. we also observed that the system has hysteresis. In Fig 4, all the bifurcations diagrams are plotted. They all show consistent result presenting similar trend and a hysteresis loop; however the critical acceleration form Ramp-up and Ramp-down are shifted as the frequency of the oscillation is increased, and at high oscillation both Ramp-up and Ramp-down will have similar critical acceleration, shrinking the window between them. At $110Hz$, we notice that there is a lot of noise in the diagram. This diagram may have some artifacts since at high frequency the length of the rectangular box is small and boundary conditions are introducing waves in the horizontal direction and that makes irregular and noisy the measurements of the amplitude. In Fig 5, we use multiple cycles of ramp-up and ramp-down. The principal idea here is to perceive if the system is consistent with multiple realizations. As we see the whole system is trapped around the hysteresis loop; which means that the process is repeatable and not very sensitive to the initial condition; but to the memory of the previous state.

$$\tau_o \frac{\partial A}{\partial t} = \xi_o^2 \frac{\partial^2 A}{\partial x^2} + \epsilon A - g|A|A^2 \qquad (1)$$

$$\tau_o \frac{\partial A}{\partial t} = \epsilon A - gA^3 - kA^5 \qquad (2)$$

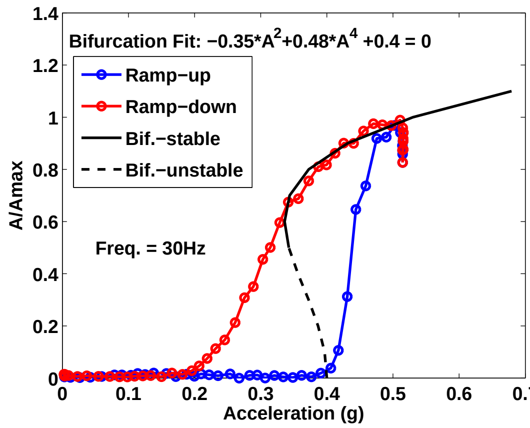

Figure 3. Bifurcation diagram for water at $30Hz$

In Fig 6 are plotted the coefficients of the the Ginzburg-Landau equation [equ. 1] with an added fifth order term (A^5) [equ. 2] where the $\partial^2 A/\partial x^2 = 0$ since there is no dependence on x. For Faraday waves, the theory predict that g and k have to be negative and positive, respectively. In fact the cubic term

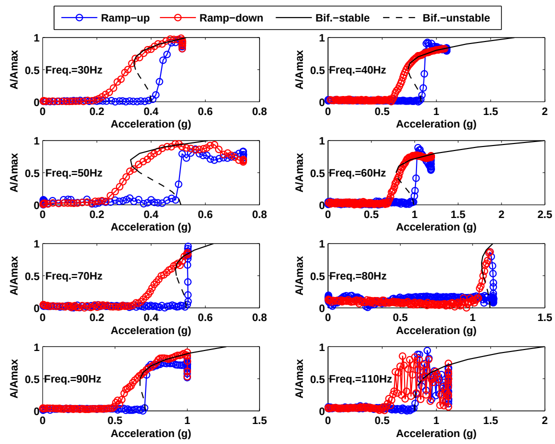

Figure 4. Bifurcation diagram for water from $30Hz$ to $100Hz$

lends a helping hand in driving the trajectory out to infinity; this effect needs a stabilizing term, (A^5), that balance the 2 efetcs[7]. Our results agrees very well with this predictions. For all the range of frequencies, it is found that g is negative and k is positive; and that they balance each other because they are in the same order of magnitude being k slightly larger than g in absolute value. In all bifurcation figures, hysteresis of the Faraday instability is observed when the system is excited beyond the critical acceleration at a given frequency, indicating a sub critical bifurcation. Amplitude equations up to the 5th order were needed to predict the hysteresis boundary and fit our data. These calculations neglects the rotational component of the flow. As indicated in, the lowest order contributions to the cubic damping coefficient are of the same order for both irrotational and rotational components of the flow. Hence the latter cannot be neglected in a nonlinear theory, even in the limit of small dissipations.

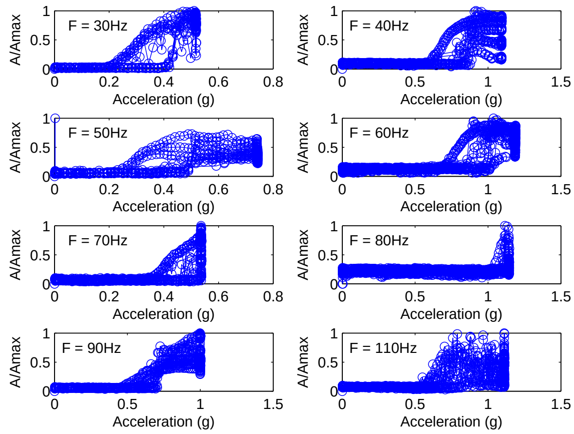

Figure 5. Multiple realization of the Bifurcation diagram for water from 30 Hz to 100 Hz. All the diagram multiples Ramp-up and Ramp-down

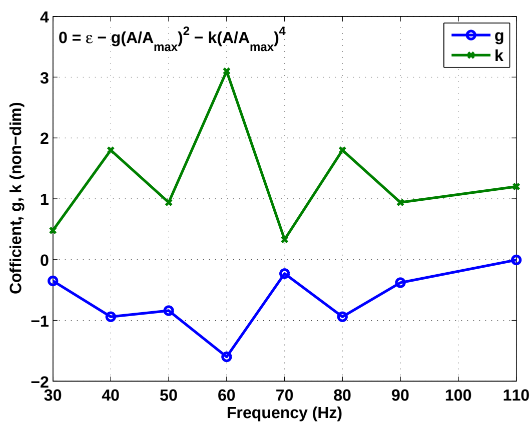

Figure 6. Coefficients for the Ginzburg-Landau equation.

In Fig 7, the A/A_{max} is plotted vs acceleration with a perturbation tap while the mea-

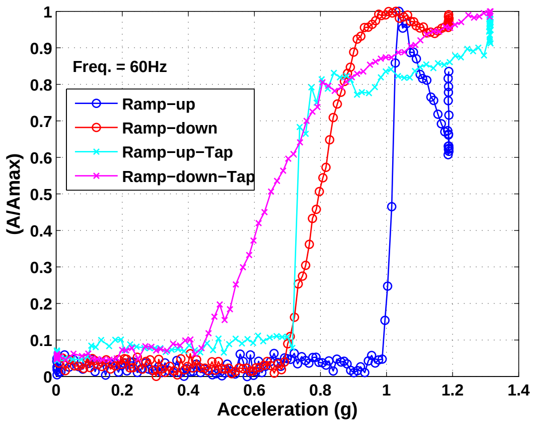

Figure 7. Bifurcation tap test at 60 Hz .

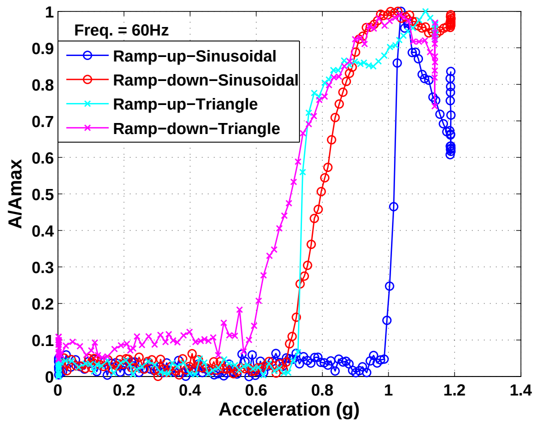

Figure 8. Sinusoidal wave vs. Triangular Wave.

surements are acquired. This is performed to know if the system is stable or can easily turn into chaos for any perturbation. The tap is applied the whole measurement in the ramp-up and ramp-down. We observe that the tap easily move the system from one state to another; meaning that the critical acceleration is shifted down and at a early stage goes up and down form one to another. It is worth noticing that the tap does not change the the overall behavior because the curves have basically the same shape; but the critical acceleration is shifted. It also seems that the shift is basically constant for the process going up and down respect to the system without any external perturbation.

We also investigate the dependency of the amplitude oscillation and the critical acceleration respect to the input signal. Here three signals are studied, sinusoidal, triangular and square. In Fig 8, only the results of the sinusoidal and triangular wave input are plotted because the square wave produce multiple waves with different amplitude and wave length that make very difficult to analyze. We clearly see that when a triangular wave is applied the window between the critical acceleration of the ramp-up and ramp-down process increase and both shifted up. This means that the system becomes more stable when the triangular wave is the input, because the first Faraday instability appears at higher acceleration for both ramp-up and -down process. This input signal also change the dynamics of the system since the curves are basically different. Both ramp-up and -down process transit from one state to another very fast as is shown in their sharp slope; but in the sinusoidal the transition is slower because the slope is blunt.

To complete our systematic examination, we include in our study a Non-Newtonian

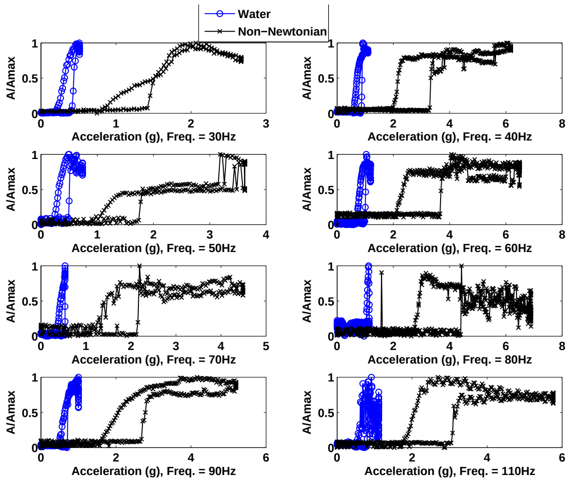

Figure 9. Newtonian vs. Non-Newtonian

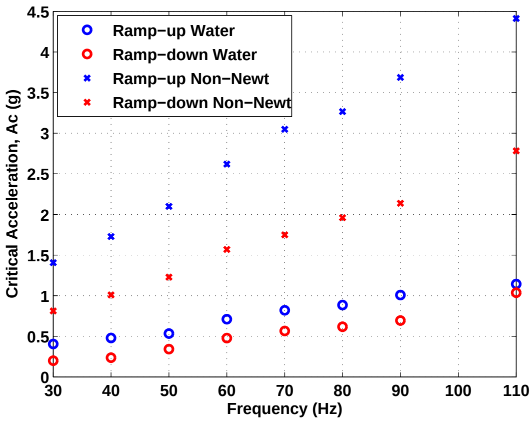

Figure 10. Critical Acceleration vs. Frequency (Potential New Findings)

fluid to asses the dependence of the fluid on the bifurcation diagram. As it is know that Non-Newtonian fluids have the property that its viscosity is not constant. This means that the critical amplitude and frequency required for Faraday waves is higher and up to a point the number of possible Faraday waves will decrease or stop all together above certain of frequencies and amplitude. In Fig 9, a comparison between the bifurcation diagram for Newtonian and Non-Newtonian fluid are plotted in the range of 30 Hz to 110 HZ. As it is expected the whole bifurcation diagram is shifted up; having a higher critical acceleration. It is important to notice that most of the Non-Newtonian fluid gets saturated and shows a hysteresis where the window seems to be constant at every frequency in the studied range. The ramp-up process is also mark for a sharp slope and the ramp-down process for a blunt one; similar to the Newtonian fluid; suggesting that this is unique characteristic of Faraday waves that is not dependent on the fluid viscosity. In overall, we observe similar behavior in both Newtonian and Non-Newtonian, having similar dynamics.

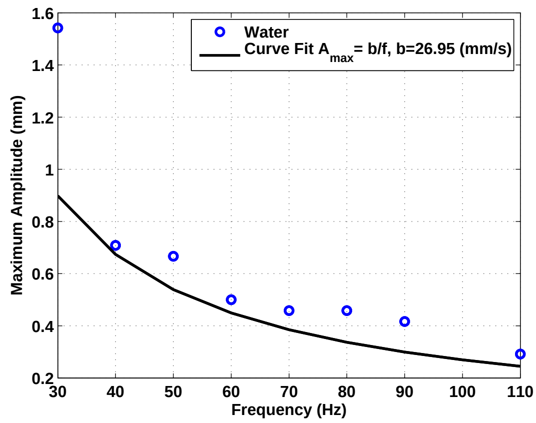

Figure 11. Maximum Amplitude vs. Frequency (Potential New Finding)

The following Fig 10,11,12 are obtained by data analysis. The maximum amplitude are

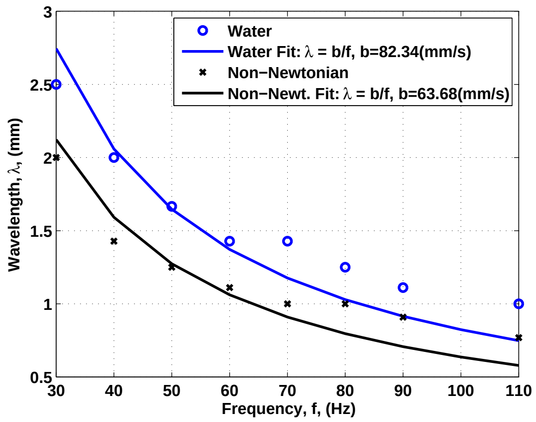

Figure 12. Wavelength vs. Frequency

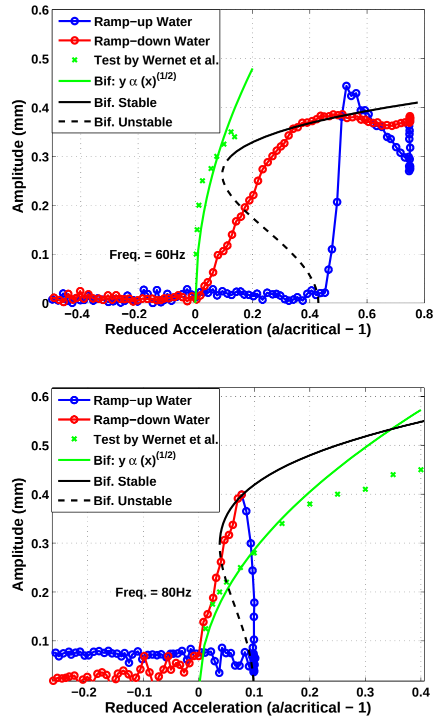

Figure 13. The upper Figure, Amplitude vs. Reduced Acceleration at 60Hz, and the lower Figure, Amplitude vs. Reduced Acceleration at 80Hz.

calculated as the distance between peak in a sinusoidal wave, and the wave length is measure form the Fourier Transform of the signal. The critical acceleration is read from bifurcation diagram for up and down process. The Fig 10 shows the relationship between the Critical Acceleration and Frequency. Surprisely, it displays a linear relationship for both Newtonian and Non-Newtonian with the same slope for down and up process. as it is expected, the slope for Non-Newtonian is higher then for Newtonian because of the viscosity. To the best of our knowledge, there is no theory that predicts this behavior; thus it could be a potential new finding that can be corroborated by theorist. The Fig 11 shows the maximum amplitude on each specific frequency. We observe that the maximum amplitude decays exponentially as the frequency is increased which means that the systems undergoes a transition from an ordered to a chaotic behavior. This could be possible because at higher frequency there are so many waves that interfere each other and cancel out their effect causing a total decrease of the amplitude. We could not find any theoretical approach that predicts this behavior. This is also a new finding. The fig 12 evidence the dependence of the wavelength respect to the frequency. Our results suggest that the Wavelength is inversely proportional to the frequency for both Newtonian and Non-Newtonian fluids; this result agrees well with the theoretical prediction of Binks et al[5]. This enforces that our experimental data is consistent since it supports the theoretical prediction.

Finally we contrast our results with the result of Wernet et al[8] in Fig 13; although this is a different experiment that has different fluid; boundary conditions, etc; we observe that our results are not only in the same order of magnitude but also in a good agreement.

IV. CONCLUSION

In summary, the main goal of this project is to study the nonlinear behavior of the pattern formation on the Faraday experiment. The problem is formulated for general contrasting the theory with experimental results. Although, we focus on a few particular cases, where only few parameters are measured and controlled, our experimental design, at the same time, can provide new insights since there is still a lack of full understanding of this phenomenon. For the chosen working fluids, water and mixture of water with cornstarch, and this experimental setup, Sub-critical behavior of Faraday waves is observed. Different input signals generate different wave forms, with different critical accelerations, and amplitudes. A confirmation of wavelength $\approx 1/freq$ is observed for Newtonian and non-Newtonian fluids. Maximum amplitudes are in overall agreement with experimental results from Wernet et al. Finally in our experiments potential new findings are found. The critical accelerations seem to vary linearly with frequency for both Newtonian and non-Newtonian fluids and maximum amplitudes tend to scale with $\approx 1/freq$ for increasing frequency. To the best to our knowledge, there is no theory that predicts this behavior.

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