The Completely Inelastic Bouncing Ball

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N. Arora, P. Gray, C. Rodesney, J. Yunis
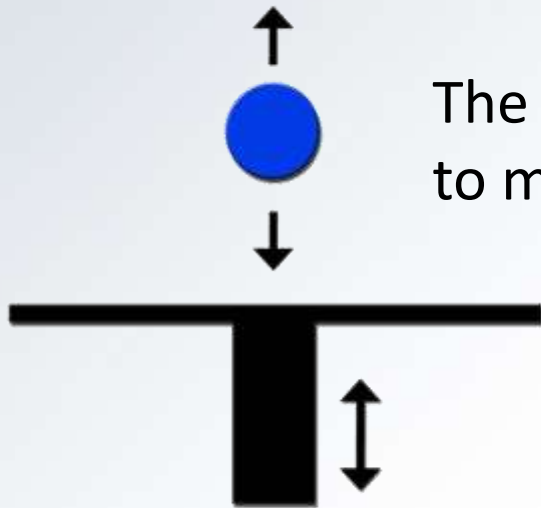
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Completely *Inelastic* Bouncing Ball?

- Can there be an inelastic bouncing ball?

YES!

The ball is constrained
to move vertically

The plate is driven periodically
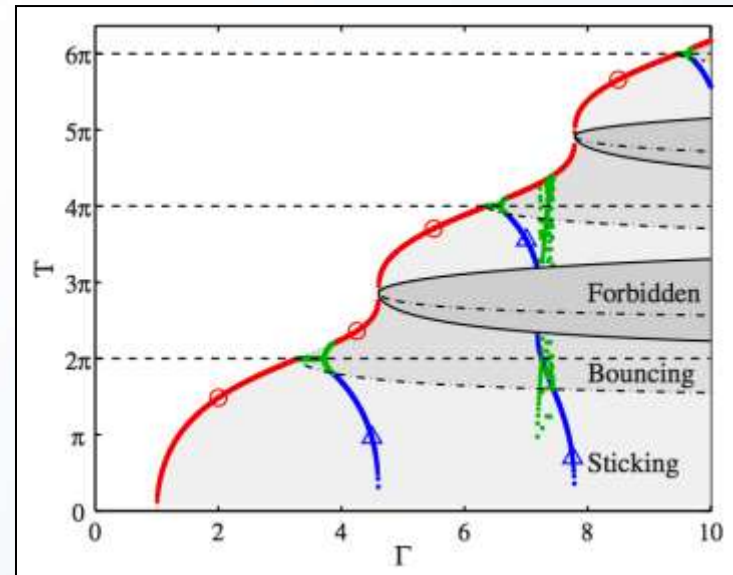in the vertical direction

The ball can "take off" when the acceleration of the plate
exceeds the acceleration due to gravity

Background

- Mehta and Luck (1990)

  – Partially resolved the motion

- Gilet, et. al. (2009)

  – Presented a complete analytical resolution of the inelastic bouncing ball on a sinusoidally forced plate.
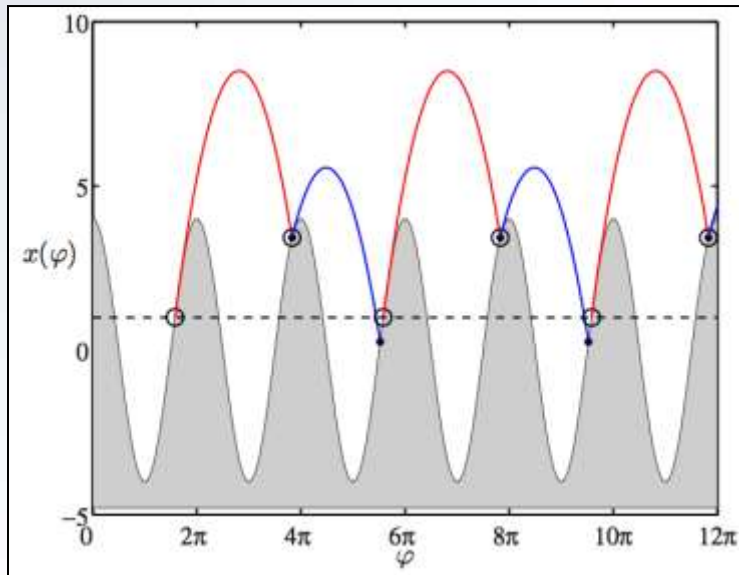
Governing equations

Position of plate   **Time**   **Driving Frequency**

$$x_p(t) = A[\sin(\omega t) + \sin(c\omega t)]$$

Amplitude   **Ratio of Frequencies (integer)**

Time When the Ball Takes Off

$$x_b(t) = x_p(t_0) - x_p(t) + \dot{x}_p(t_0)(t - t_0) - 1/2g(t - t_0)^2$$
$$x_b(t) = A[\sin(\omega t_0) + \sin(c\omega t_0) - \sin(\omega t) - \sin(c\omega t)] + \ldots$$
$$A[\omega\cos(\omega t_0) + c\omega\cos(c\omega t_0)](t - t_0) - 1/2g(t - t_0)^2$$

$$\ddot{x}_p(t) = -A[\omega^2\sin(\omega t) + c^2\omega^2\sin(c\omega t)] \leq -g$$
$$\frac{A\omega^2}{g}[\sin(\omega t) + c^2\sin(c\omega t)] \geq 1$$

Threshold for Takeoff

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Non-Dimensionalized Equations

Phase:

$$\varphi = \omega t$$

Generalized Position:

$$\chi = x\omega^2/g$$

Reduced Acceleration:

$$\Gamma = A\omega^2/g$$

New set of governing equations:

$$\chi_p(\varphi) = \Gamma(\sin(\varphi) + \sin(c\varphi))$$

$$\chi_b(\varphi) = \Gamma[\sin(\varphi_0) + \sin(c\varphi_0) - \sin(\varphi) - \sin(c\varphi)] + \ldots$$
$$\Gamma[\cos(\varphi_0) + c\cos(c\varphi_0)](\varphi - \varphi_0) - \ldots$$
$$1/2(\varphi - \varphi_0)^2$$

New takeoff condition: $\Gamma[\sin(\varphi) + c^2\sin(c\varphi)] \geq 1$
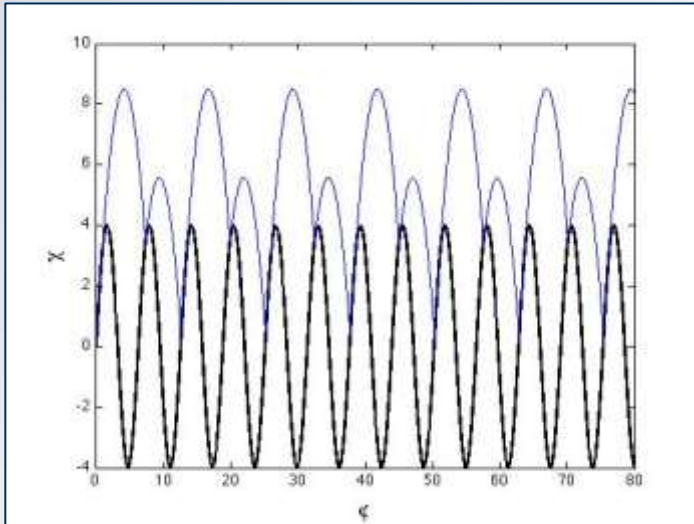
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Time of flight

Time of flight is defined as: $\quad T \equiv \varphi_i - \varphi_0$

Flight time is determined by solving numerically when F=0:
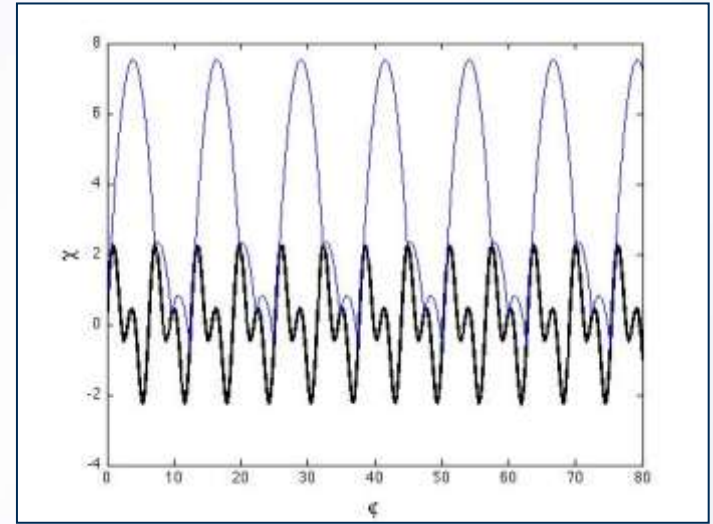
$$F \equiv 1/2 T^2 + \Gamma[\cos(\varphi_0) + c\cos(c\varphi_0)]T - \ldots$$
$$\Gamma[\sin(\varphi_0)(1 - \sin T) + \sin(c\varphi_0)(1 - \sin cT) - \cos T \cos(\varphi_0) - \cos(cT)\cos(c\varphi_0)]$$
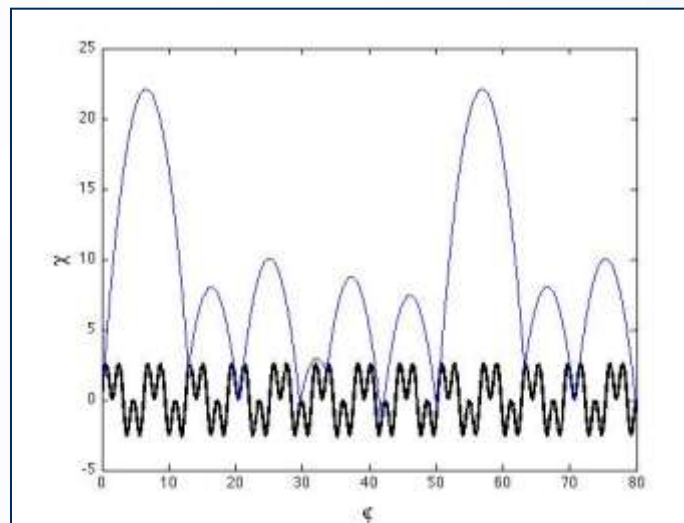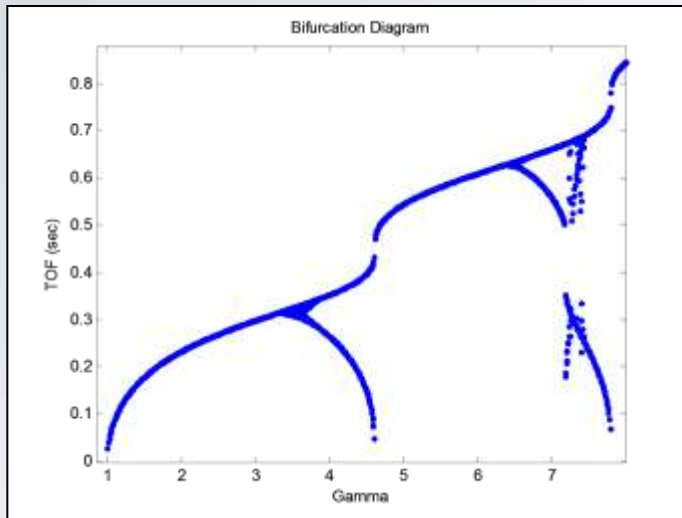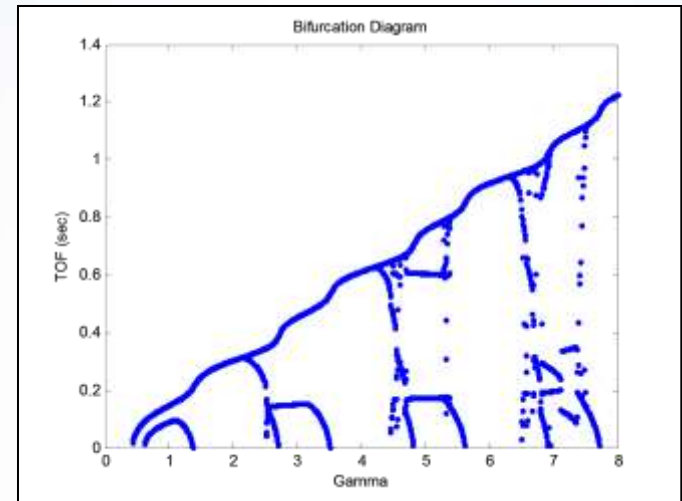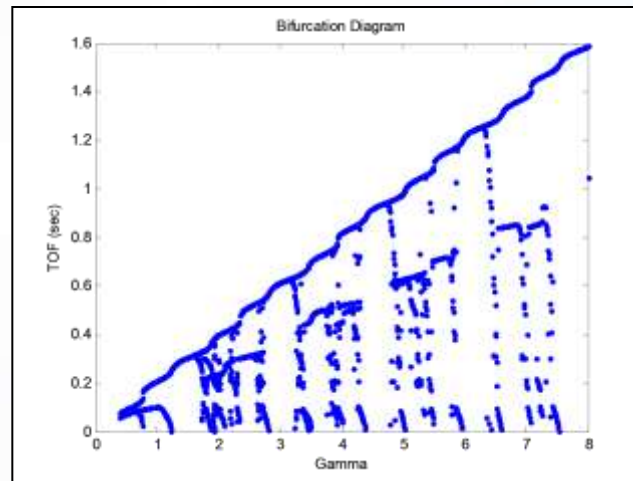
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Computational models

C=1

C=2

C=3

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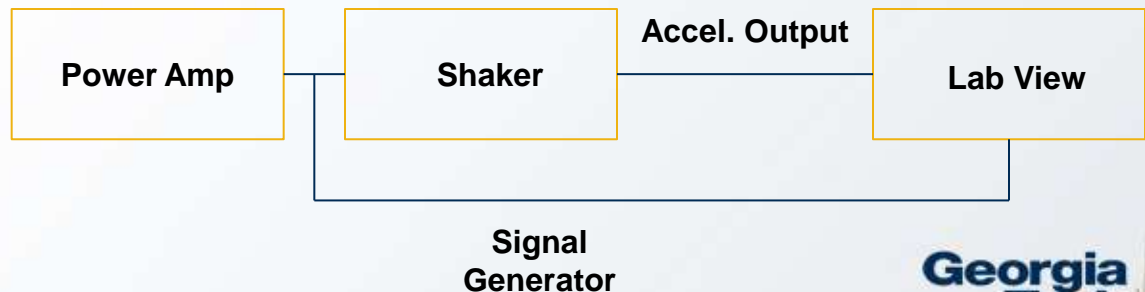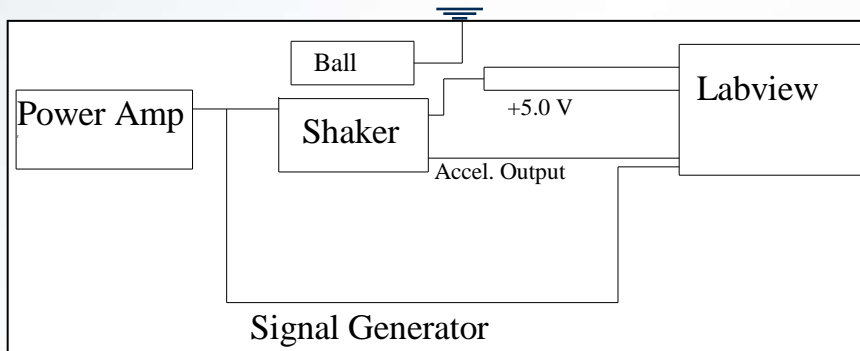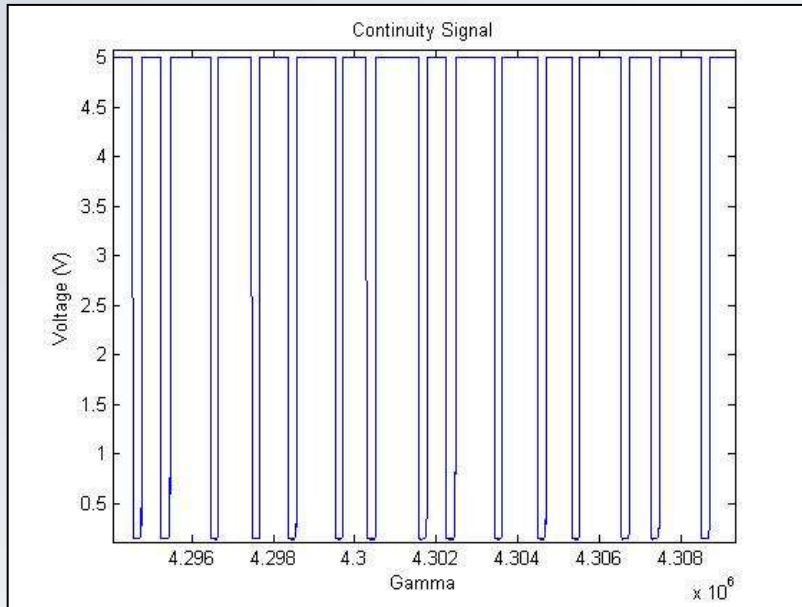# Computational models

C=1

C=2

C=3

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Initial Setup
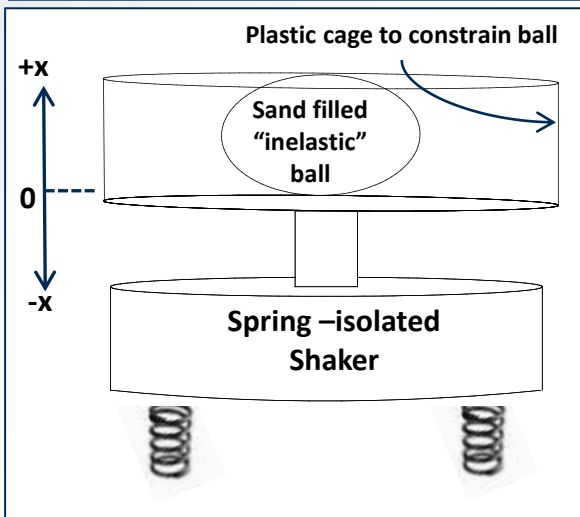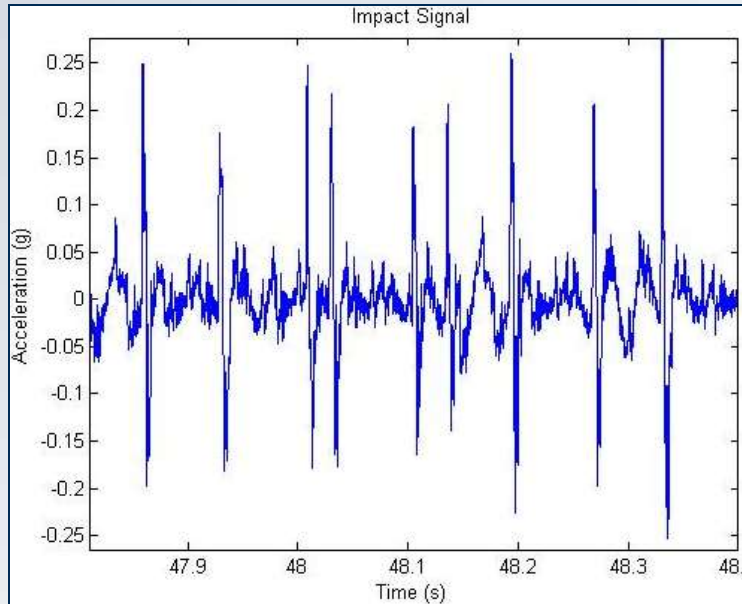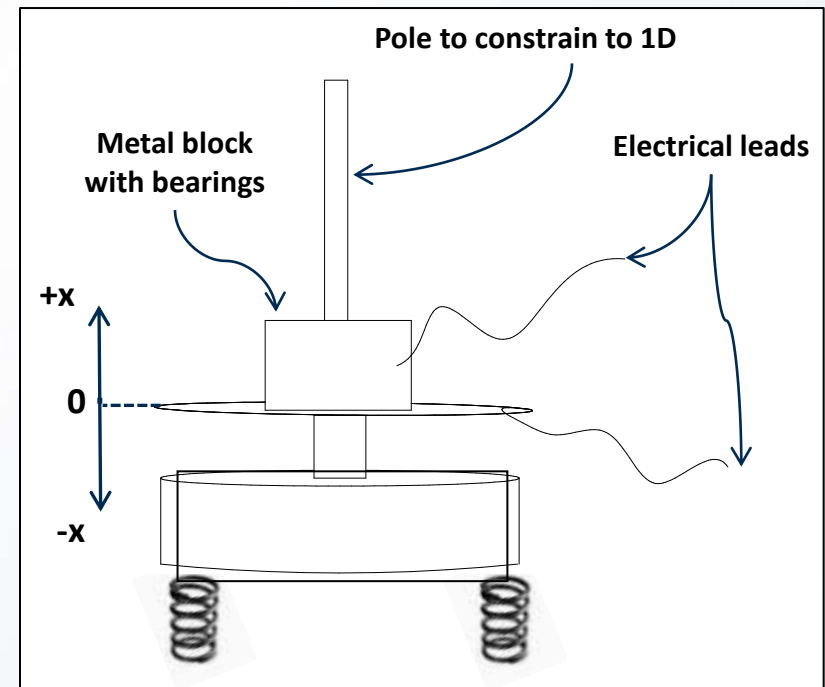

- Position data provided by accelerometer on platform
  - Ball takes off when Fplate > Fg
  - Ball lands when accelerometer detects impact
- Problems:
  - Noisy accelerometer signal
    - When ball lands while plate is descending, there may be little additional acceleration
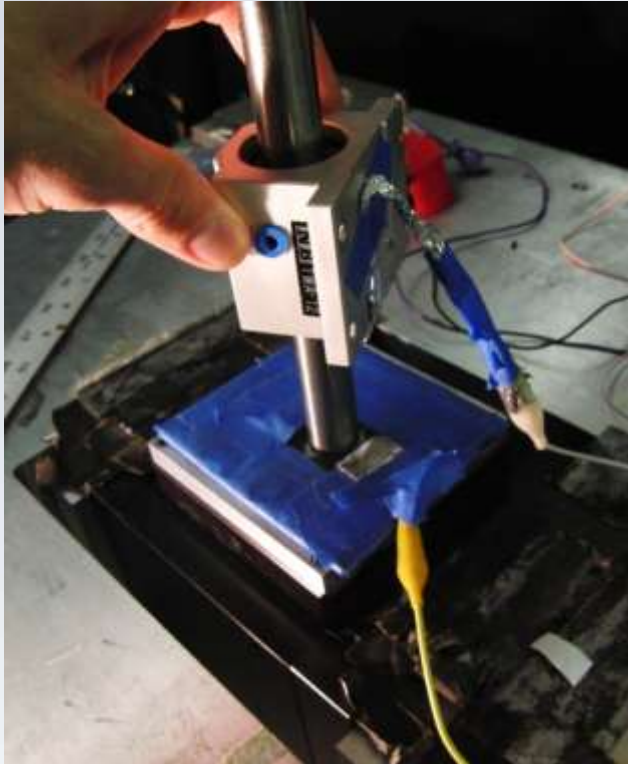 - Ball not constrained to 1D movement

Plastic cage to constrain ball

+x

Sand filled "inelastic" ball

0

-x

Spring –isolated Shaker

Power Amp

Shaker

Accel. Output

Lab View

Signal Generator

9

Reconfigured Setup

Continuity Signal

- Position data provided circuit contact (tin foil)
  - Ball takes off when v= 5.0 V
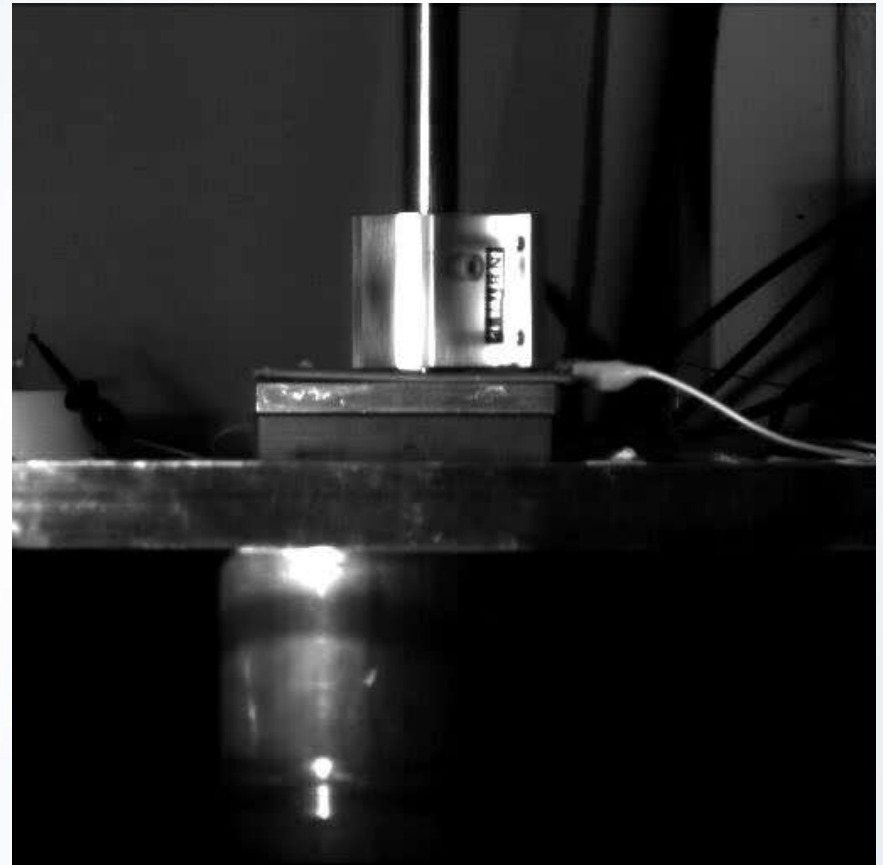 - Ball lands when shorted to ground
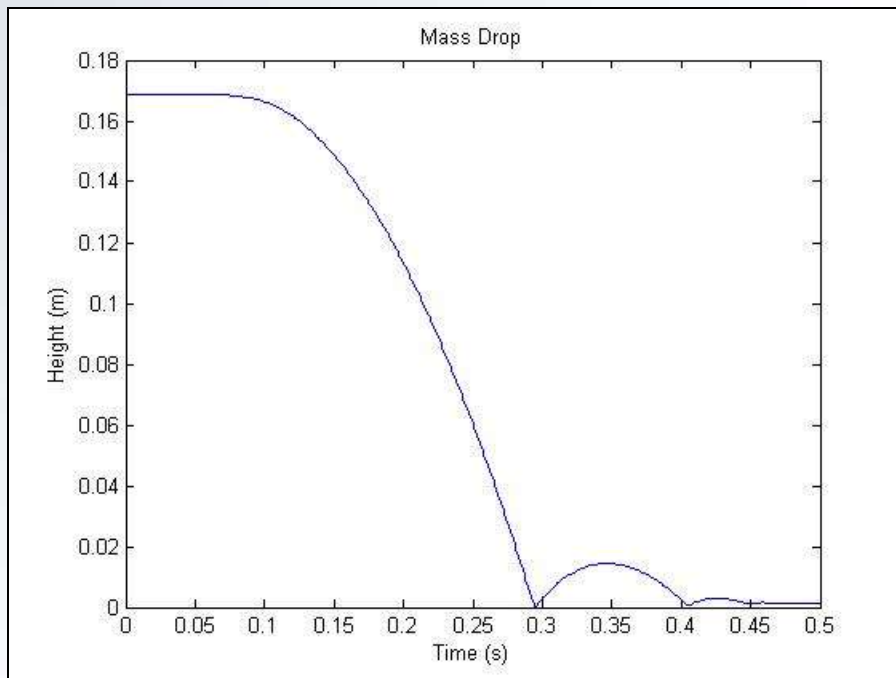- No noise in flight time signal

Pole to constrain to 1D

Metal block with bearings

Electrical leads

+x

0

-x

Power Amp

Ball

Shaker

+5.0 V

Labview

Accel. Output

Signal Generator

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Reconfigured Setup

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Issues with Setup

- Friction
- Twisting of mass
- Not truly inelastic
- Shaker oscillation at high Γ

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Friction

- Coulomb Friction due to ball sliding over the post
  - Experimentally determined the frictional acceleration by a drop test
  - Constant frictional acceleration of ~0.3 g found via fitting a parabola
  - Friction modeled as a constant force acting in the opposite direction to the relative velocity of the ball and plate
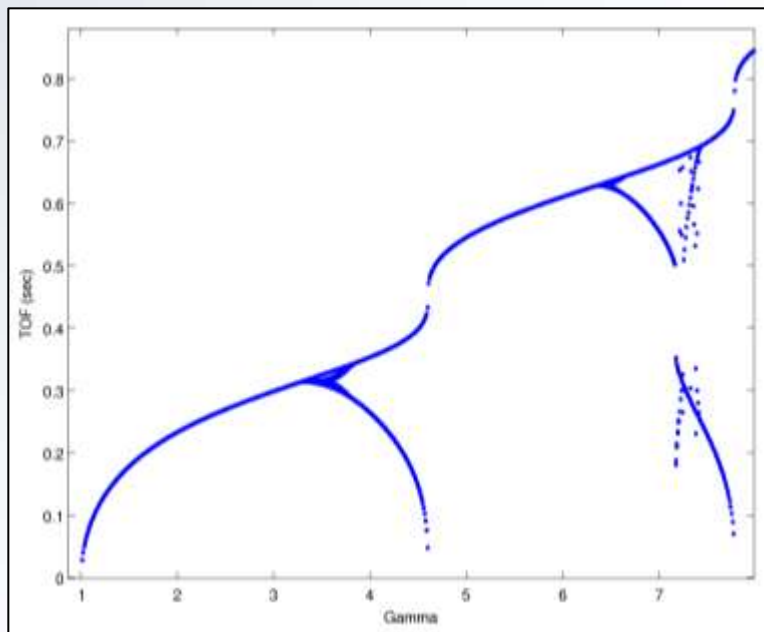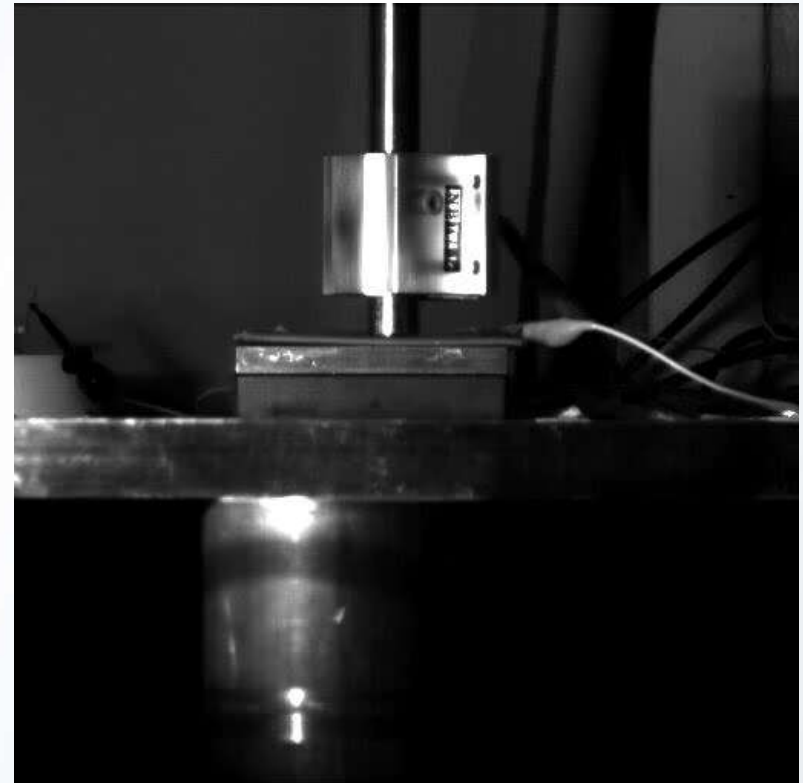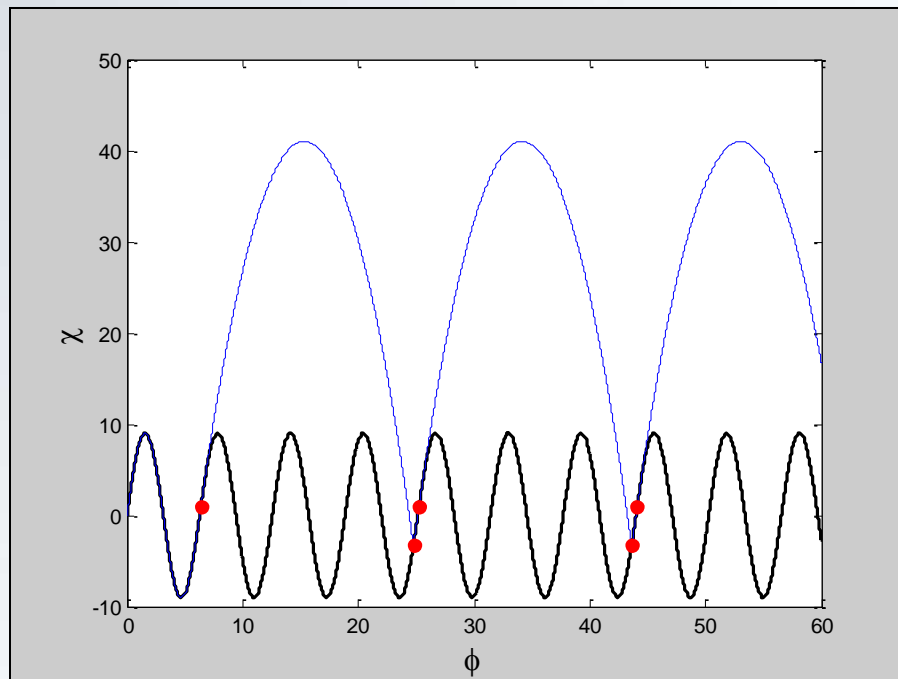

Without friction
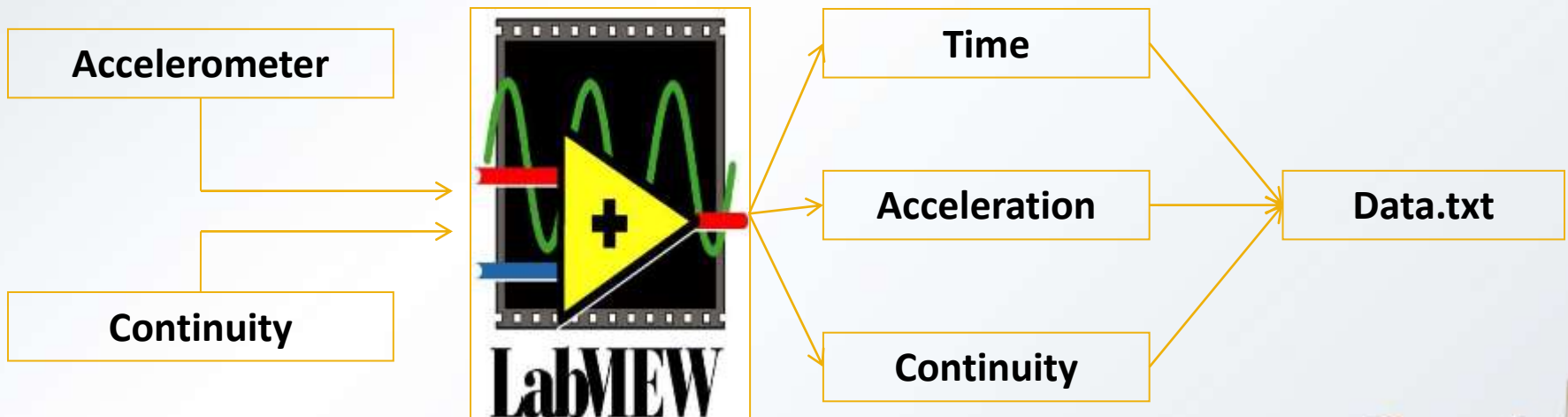
With friction

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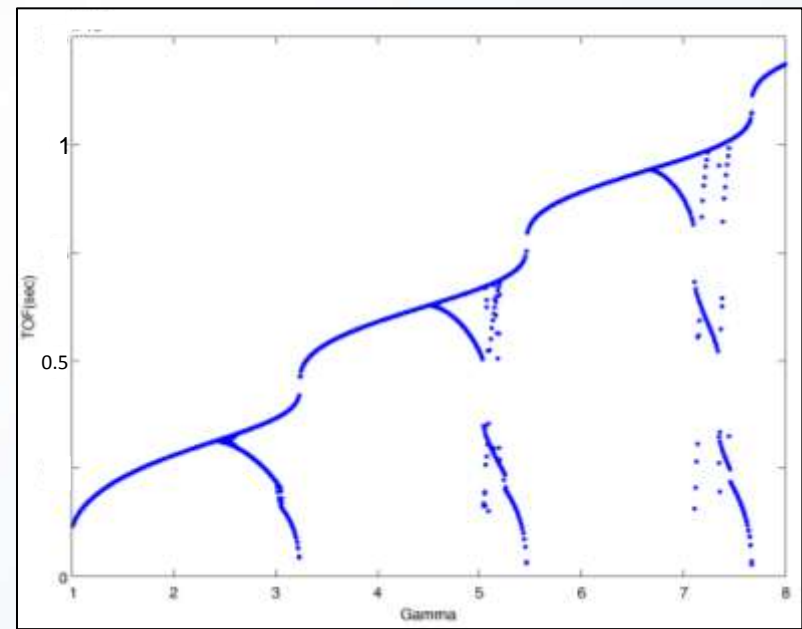
Whoa! Looks like they match!

Data Acquisition

- Data was taken at a sample rate of 20 KHz
- Two channels were read in to LabView
  - Acceleration
  - Voltage from continuity sensor
- Data output in tab delimited format to text file with two additional columns
  - Time
  - Acceleration due to the impact of the ball

Data Acquisition

- For a given frequency, gamma was varied by ramping up the amplitude of vibration to generate accelerations from 1g to 8g.

- Lab View took very small steps to increase the amplitude lingering at each amplitude
  - The number of periods to wait before increasing the amplitude was given by the user.
  - Values from 5 to 60 were experimented with.

- Each step represents a part of the bifurcation diagram.
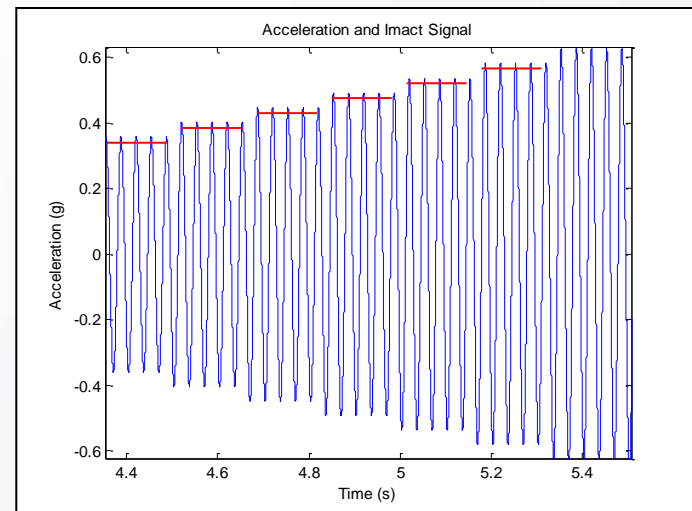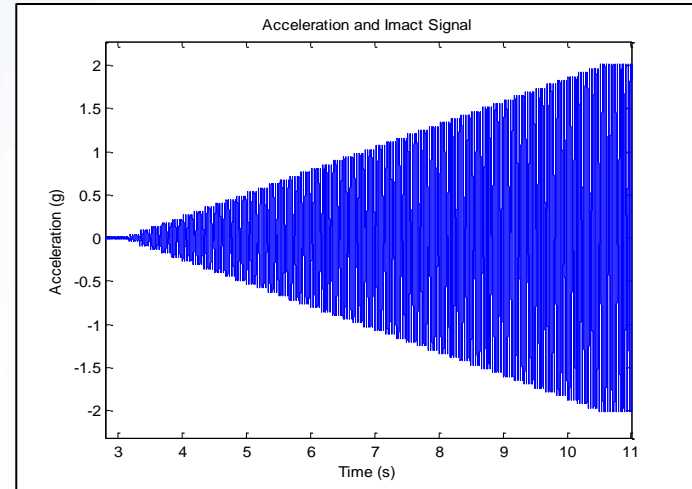
Data Acquisition

- ## Matlab was used to visualize and analyze the data.
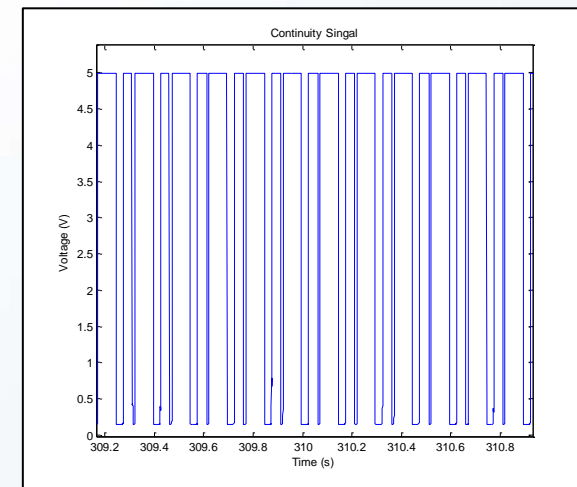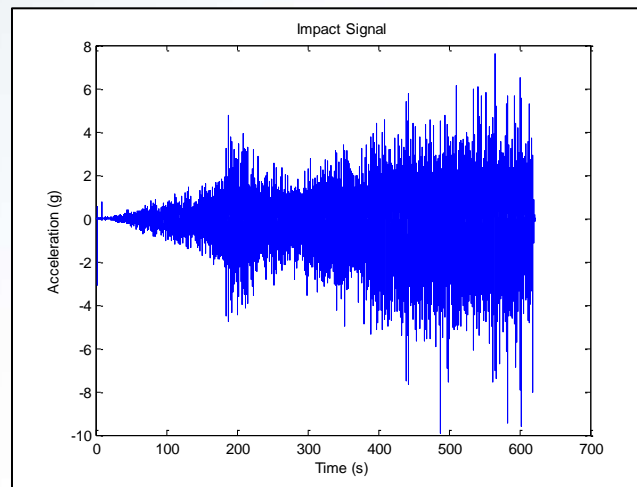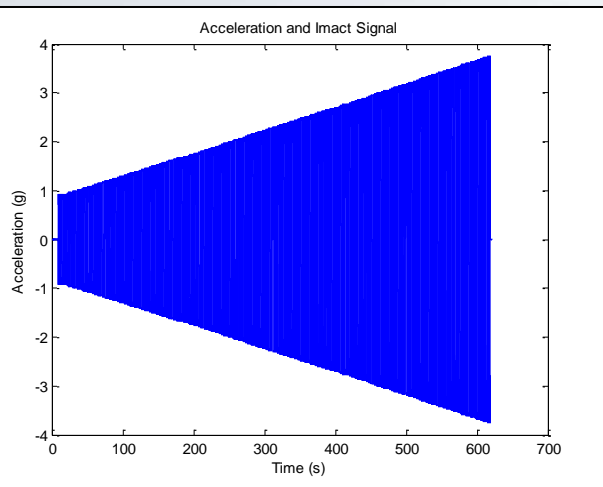
- The "reshape" command was used to reshape the data such that each step of constant amplitude could be analyzed independently.
- TOF for determined for each constant amplitude group
- The first TOF values were ignored steady state response was considered and not the transient response.
- Plotting the TOF values for each amplitude gives the bifurcation diagram.

Bifurcation Diagram

Full Bifurcation Diagram
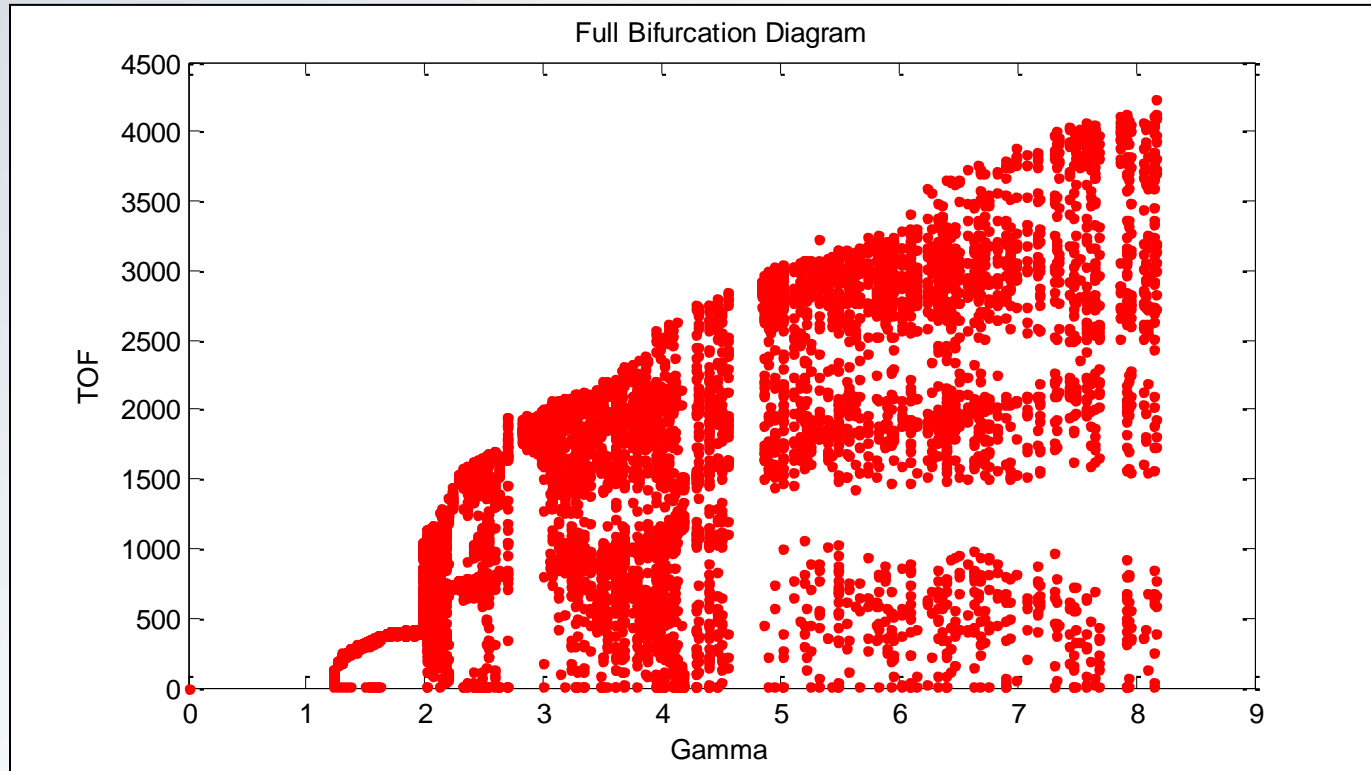
- The full bifurcation diagram ranges from 1 to 8g.
- Contains regions of period doubling bifurcations as well as forbidden regions

Data Analysis

- To further investigate and visualize hidden patterns, a return diagram (N+1 diagram) was created for each section of the bifurcation diagram.
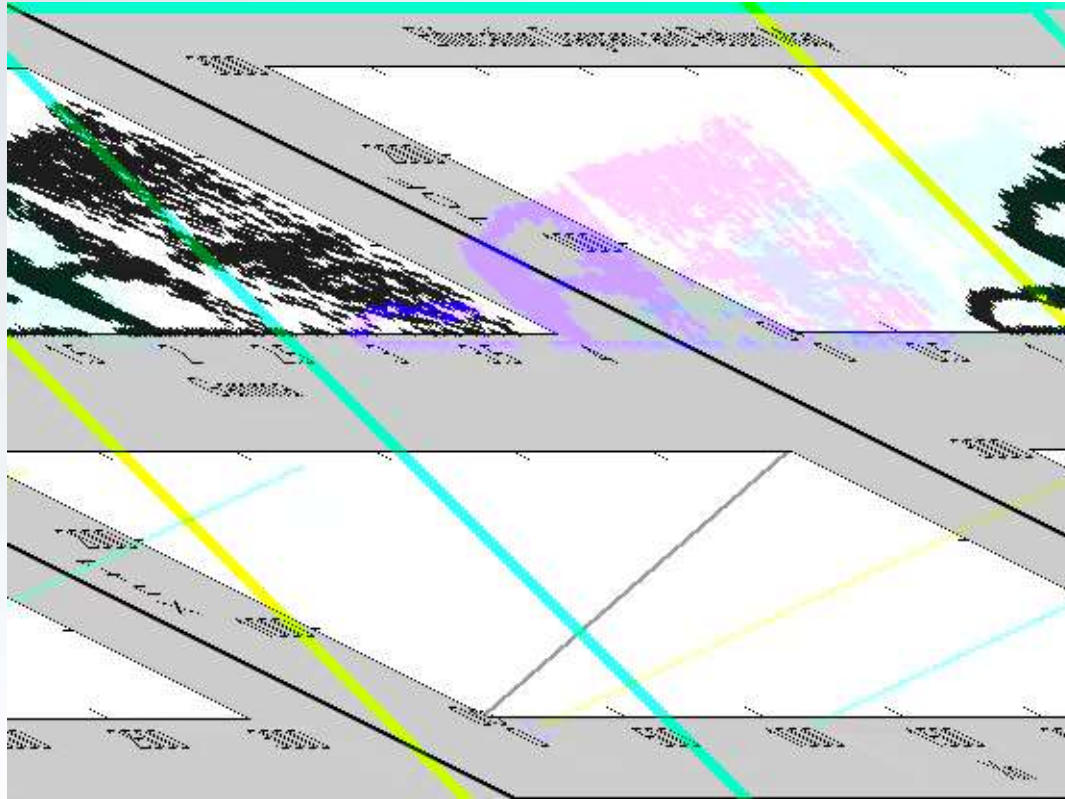
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Data Acquisition

- More data was taken at lower gamma values due to reduction in flight times introducing less error into the test.

Data Acquisition

- To further investigate and visualize hidden patterns, a return diagram (N+1 diagram) was created for each section of the bifurcation diagram.

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Data Acquisition

- TOF values from the measured bifurcation diagrams were compared to the published results and model predictions.

- Measured TOF values were seen to be significantly lower than the predicted values.

- General shape of bifurcation diagrams seem to be similar
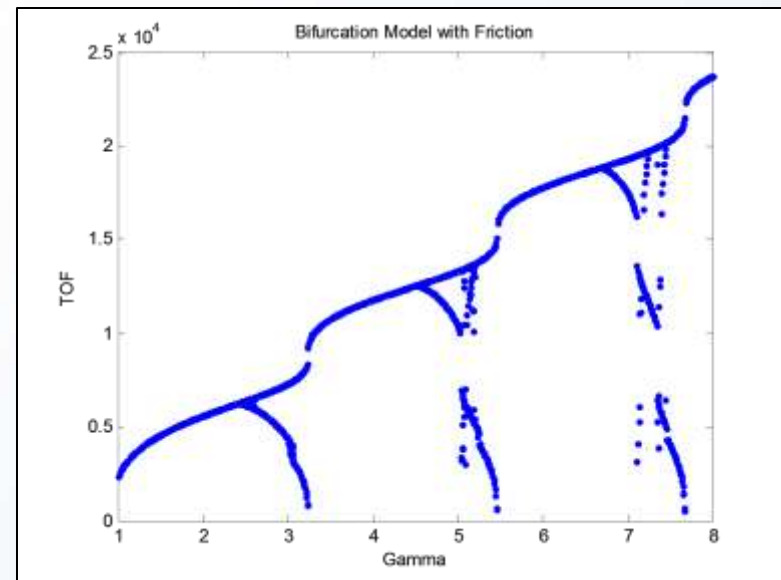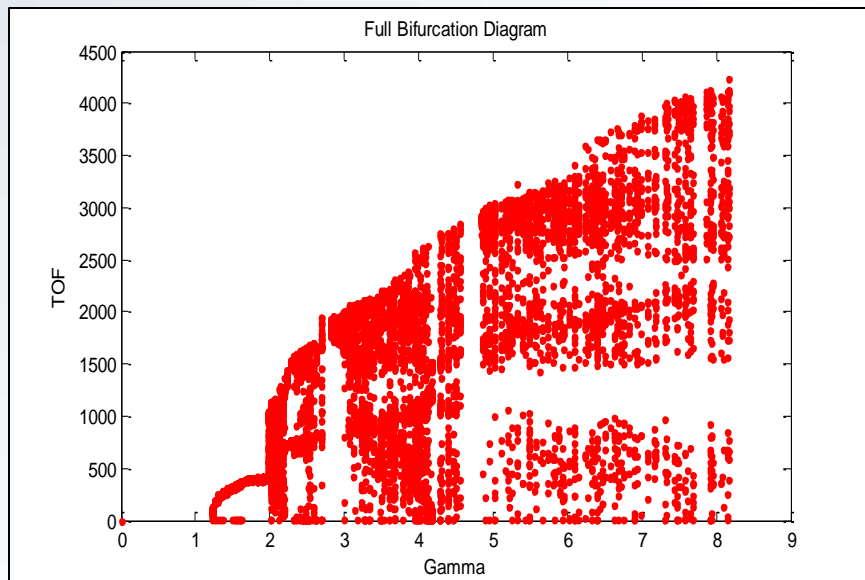
Other Sources of Model Error

- Elasticity in the bouncing ball
- Plate idle feedback, altering the time of flights especially during high g inputs
- Uneven collision between the shaker and the ball during the impact
- Ball rotation observed at higher g inputs may be attributed to the torqueing previously discussed

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Conclusion

- Successfully studied the behavior of a completely inelastic bouncing ball on a vibrating plate subjected to a sine based input forcing function

- Attempt has been made to model the Coulomb friction in the theoretical model.

- Experimentally validated the bifurcation pattern, the pattern is closely matched but the actual time flights are off.

- Discrepancy in absolute values attributed to possible sources of error like friction, plate idle feedback, torqueing impact of the ball and elasticity of the bouncing ball.

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Future Work

- Implement a variety of input forcing functions for experimental analysis

- Improving the experimental setup by correcting torqueing at impact and minimizing the plate idle feedback.

- Modeling of the elasticity of the "ball" used for this current experiment.

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Questions