Nonlinear Dynamical Response of Ferrofluid to Magnetic Field

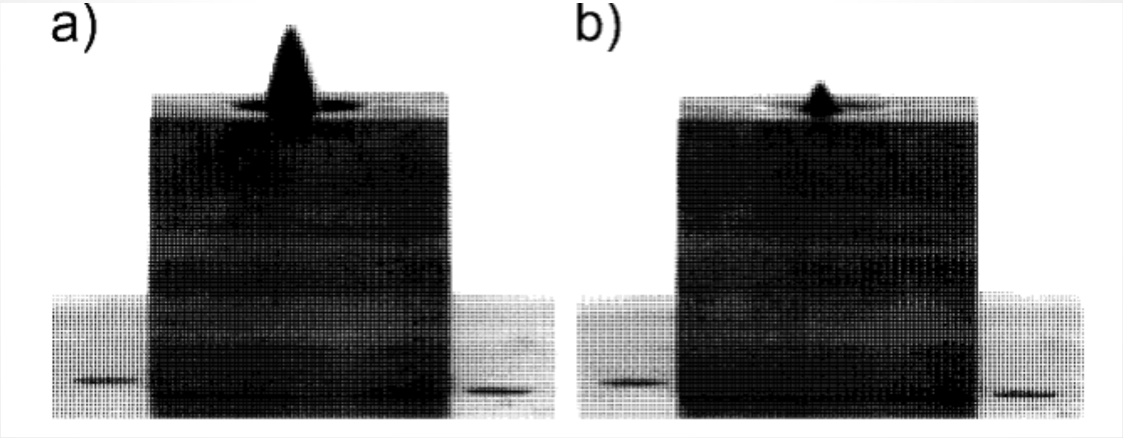
Kipp Schoenwald, Daniel Potter, Amir Hamid and Amin Agha

Previous Work and Objective

Mahr and Rehberg, "Nonlinear Dynamics of a Single Ferro-fluid Peak in an Oscillating Magnetic Field," 1998

> A subcritical bifurcation is modulated via an oscillating magnetic field, in which the oscillations can be harmonic, subharmonic, or irregular depending on the frequency and amplitude of oscillation.

Based on Mahr and Rehberg's work, varying the frequency and amplitude of oscillation is expected to give rise to various cyclical modes. The system is expected to exhibit period doubling.



Ferromagnetism

- The spin of the electrons are the main contribution to ferromagnetism. The dipole-dipole magnetic interaction between magnetic moments is expected to oppositely align two moments. However, this interaction is easily tampered with by thermal fluctuations. Also, at much shorter distances, the exchange interaction dominates, in which the parallel spin state (between unpaired electrons) is more stable than the antiparallel state. In this case, the spin-spin coupling between the electrons contributes significantly to the net magnetization of the material.

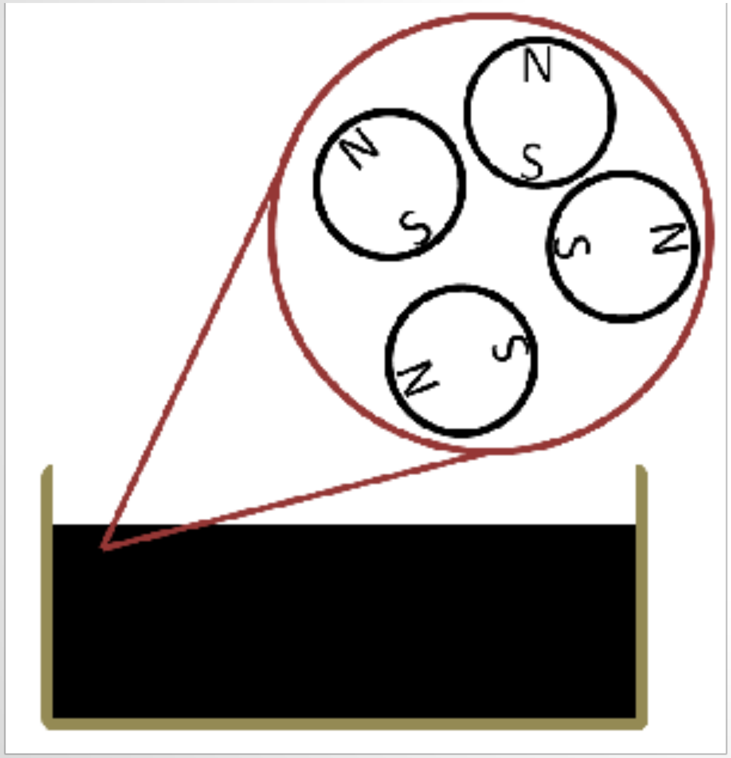
$$H(\sigma) = -\sum_{i\ j} J_{ij}\sigma_i\sigma_j - \sum_j h_j\sigma_j$$

Ferrofluid

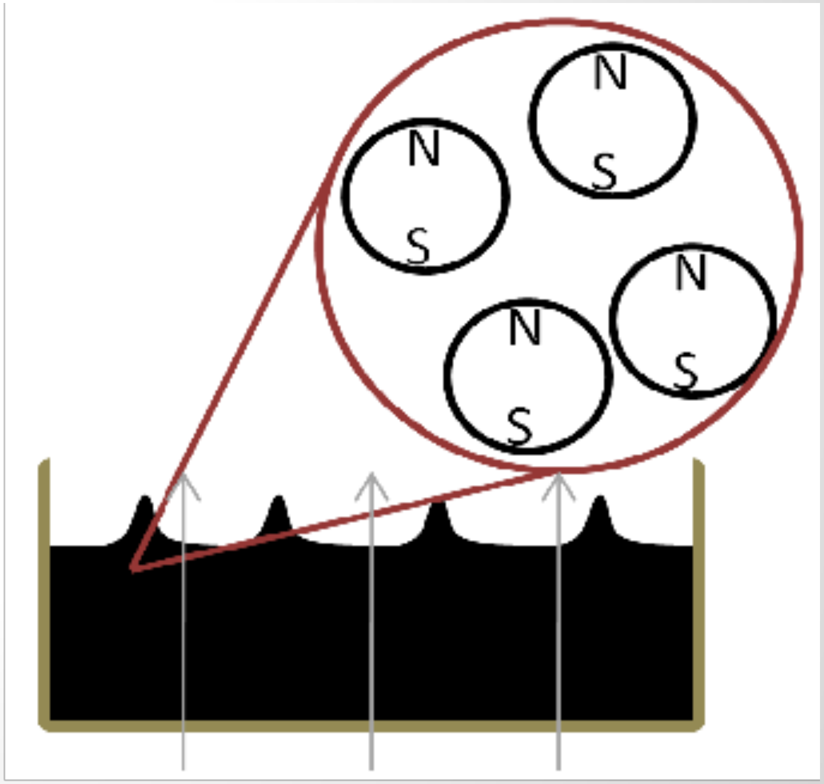
- Colloidal fluid mixture of oil and ferromagnetic nanoparticles (~10nm).
- The fluid becomes magnetized in the presence of a magnetic field. In deriving peaks, there are two important fluid properties: low viscosity and high magnetic permeability. Because these properties are inversely related, it is thus necessary to find a balance between the two.

Colloid behavior

Why many small peaks instead of one big peak?
- This is characteristic of magnetic mono-domains.

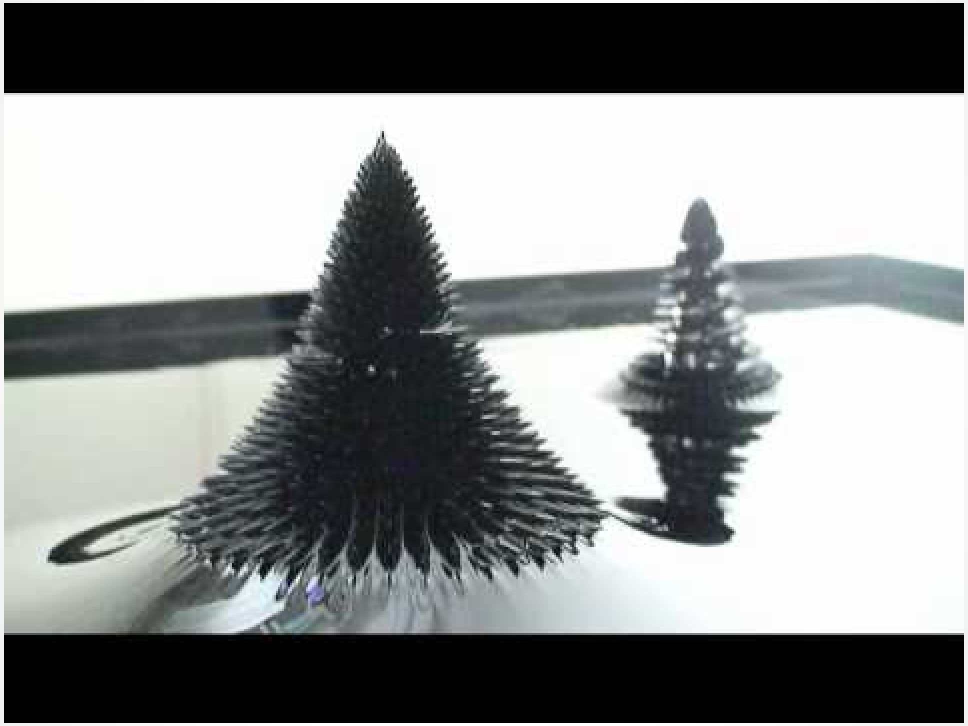


Without magnetic field



Magnetic field (H)

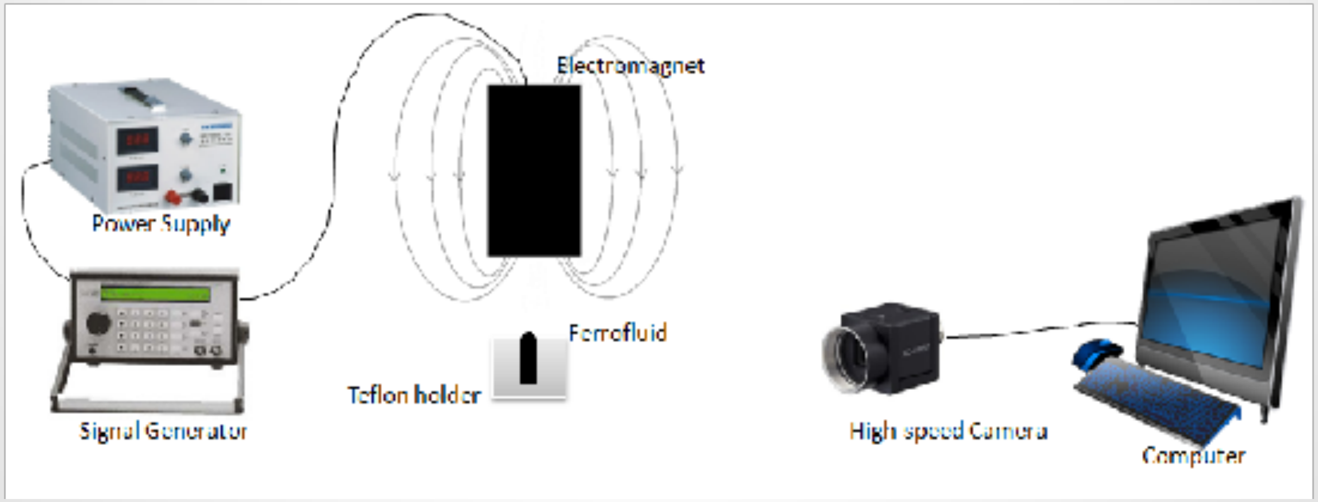
Ferrofluid in Action



The fluid forms peaks that are parallel to the magnetic field lines.

Experimental Setup: 2 Iterations

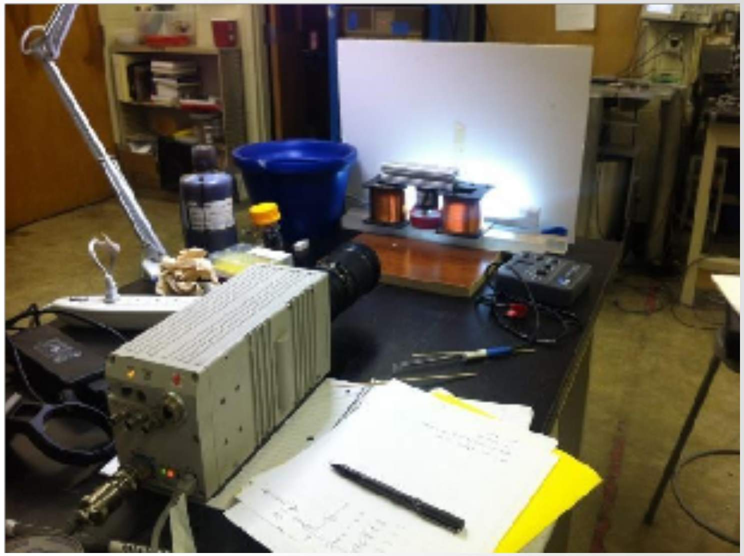
Coil Electromagnets:
Advantage: Uniform magnetic field
Disadvantage: Difficulty getting a sufficiently strong mangetic field
Round Island Pole electromagnet:
Advantage: Strong magnetic field (capable of 180lb pull force)
Disadvantage: Nonuniform field that varies linearly with distance from magnet.



Other Apparatus:
Tecron 7550 Power Supply Amplifier, F.W. Bell Model 5070 Gauss/Tesla Meter, High Speed Camera: Redlake Motion Xtra HG-LE, Pipette, Ferrotec EFH-1 Ferrofluid

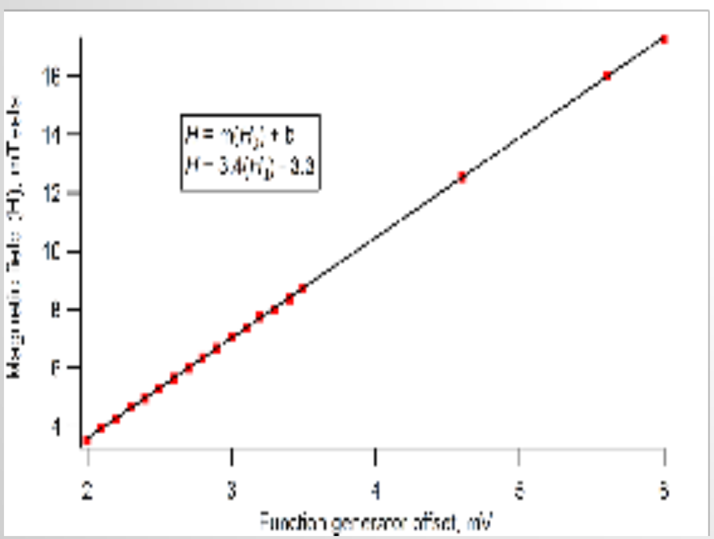
Experimental Setup

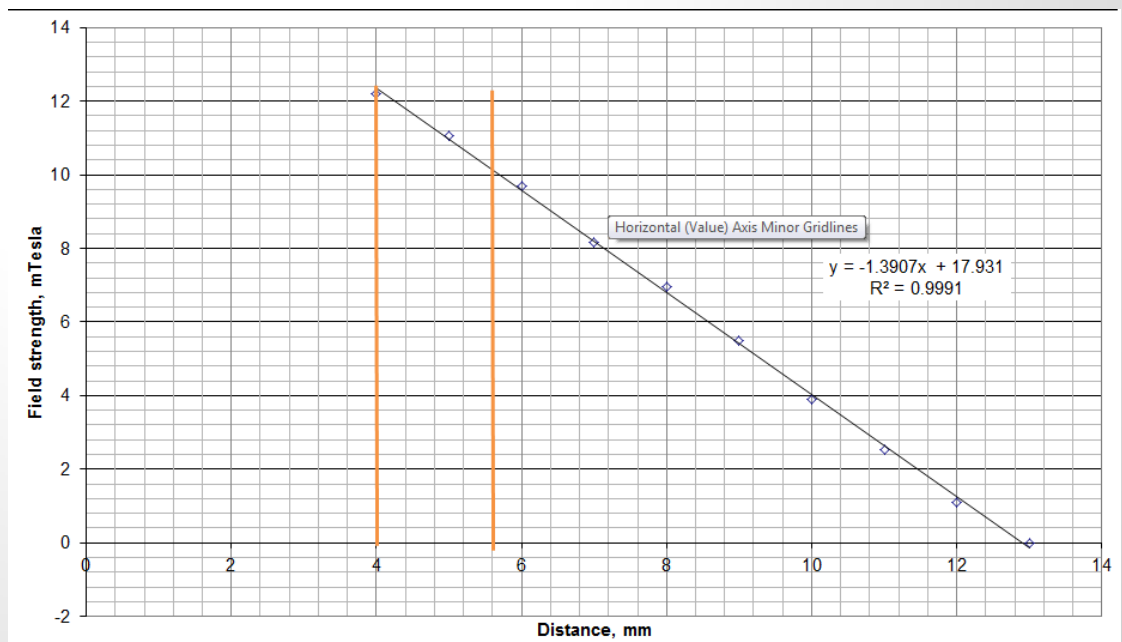
- Ferrofluid placed in holder (centered) under RIP magnet.
- Two Holders: 6.35 mm diameter pipette tip and 7.58 mm diameter fabricated Teflon holder
- Holder supported in sand in later trials for variable height adjustment
- White screen and light used to create contrast for motion capture of black fluid



Magnetic Field Characterization

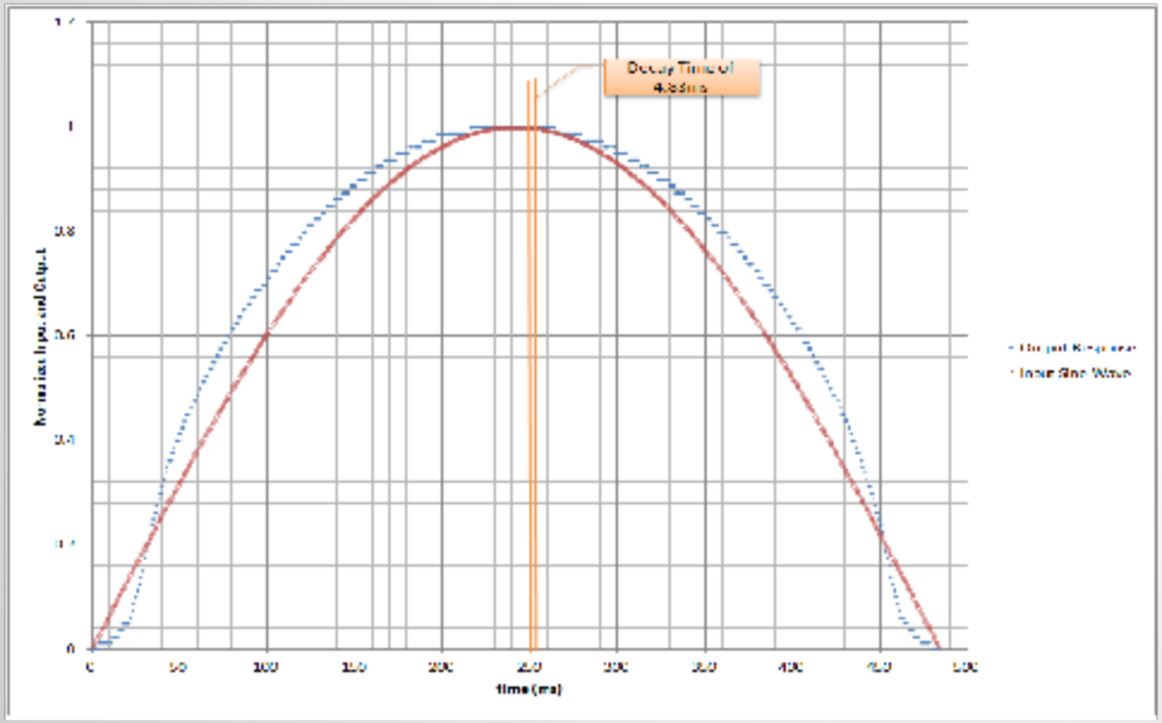
- The magnetic field was measured as a function of the input voltage with the Tesla Meter. Spatial Field variation was measured along the vertical axis up to 13 mm below the center of the magnet. A discrepancy of 0.01T over 1.0 cm was observed.

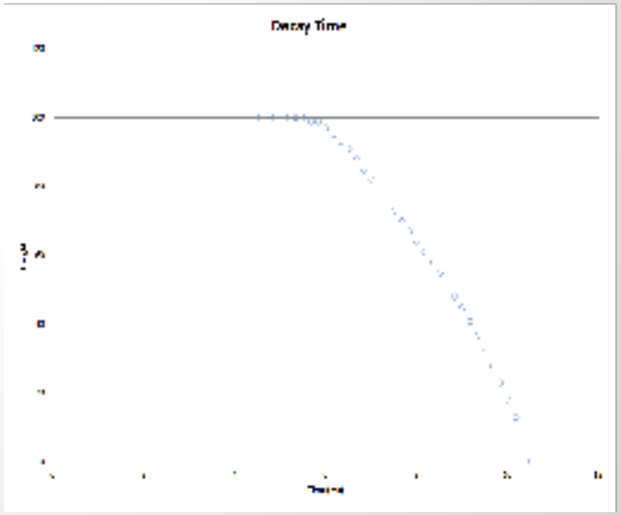




Decay Time

- Decay time = 4.83ms





Experimental Procedure

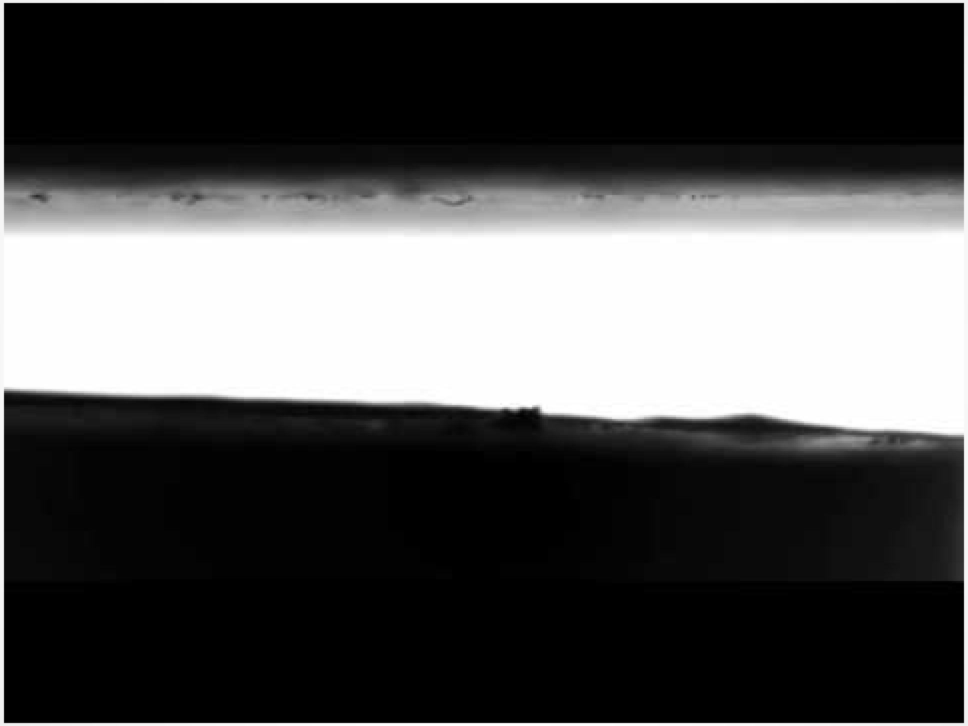
General: Vary the magnetic field and observe peak behavior using high speed camera

Driving Inputs Examined

- Static DC Field
- 1 Hz Triangular wave with Amplitude 3.0-5.2 V
- 7 Hz input with varying DC offset

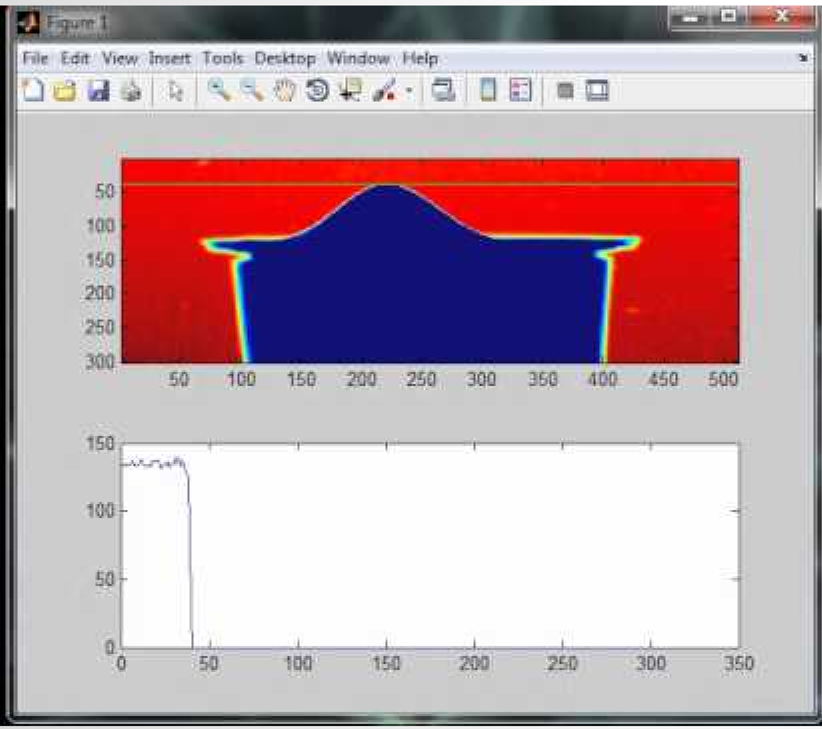
Image Processing performed in MATLAB

Single fluid peak: dynamics

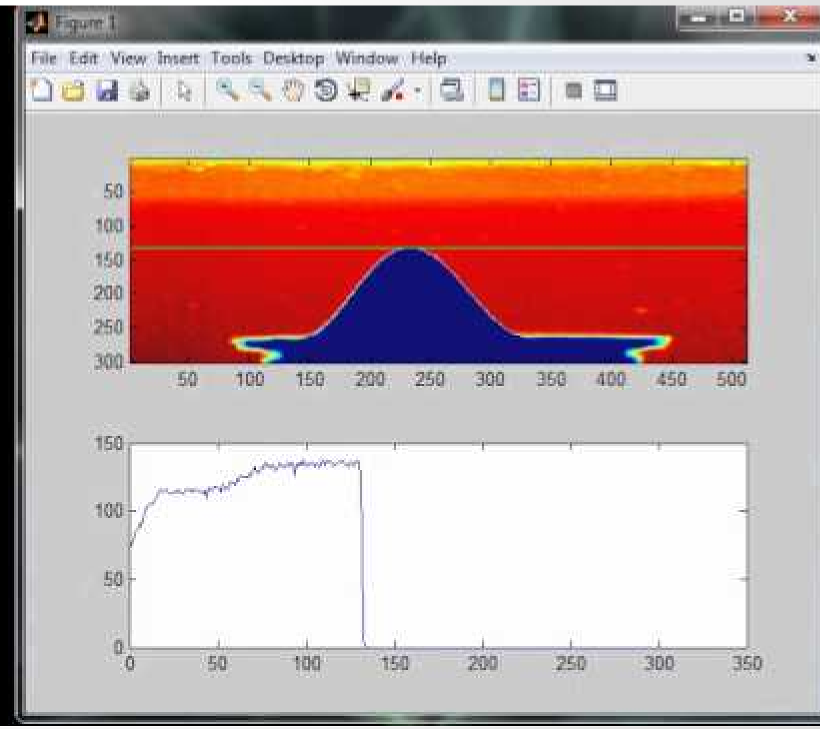


Single fluid peak: MATLAB Analysis

3.6 V triangle ramp



5.2 V triangle ramp



Analysis of Numerical Model

$$\ddot{h} + \beta\dot{h} - h + h^2 = \xi(t)$$

$$\xi(t) = \; driving\ frequency = \; H_o + \Delta H \sin(\omega t)$$

$$\dot{h} = \; v$$

$$\dot{v} = \; -\beta v + h - h^2 + \xi(t)$$

Analysis of Numerical Model

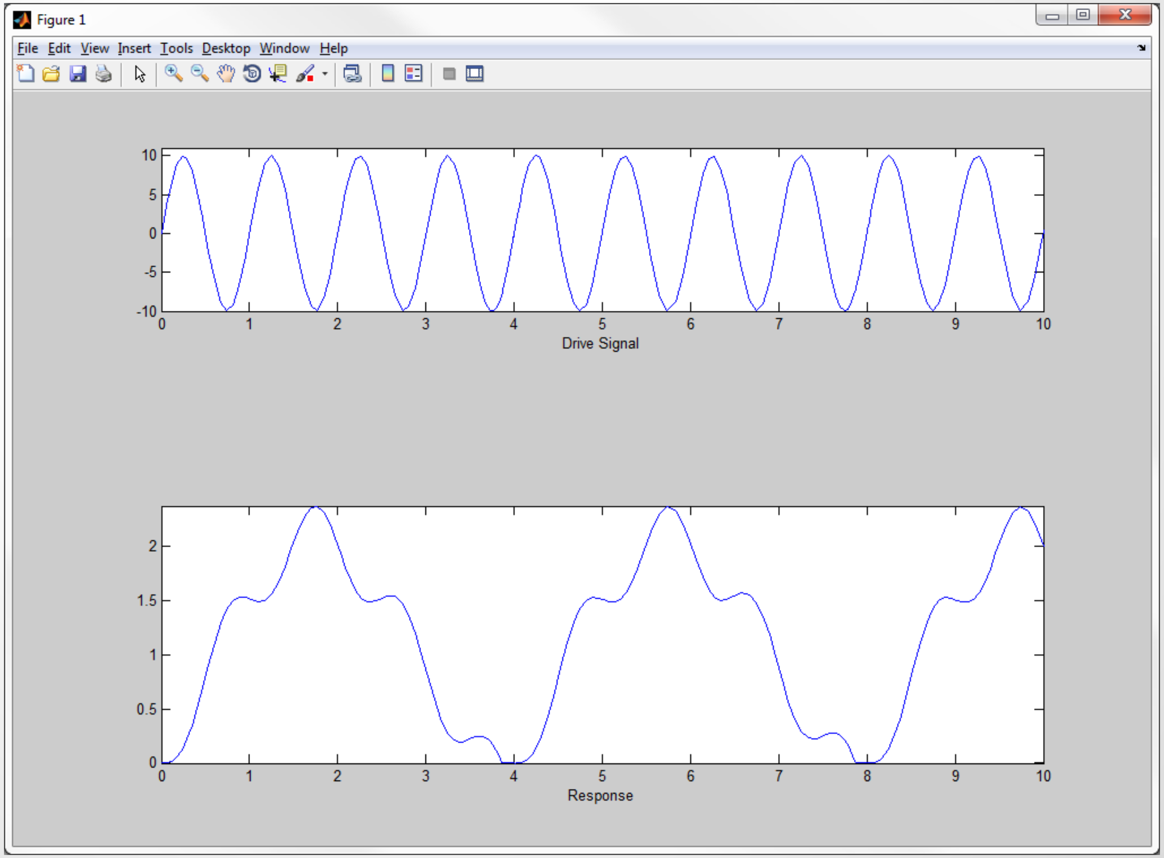
$$\xi(t) = \ driving\ frequency = \ H_o + \Delta H \sin(\omega t)$$

$$\beta = \ 0.1$$

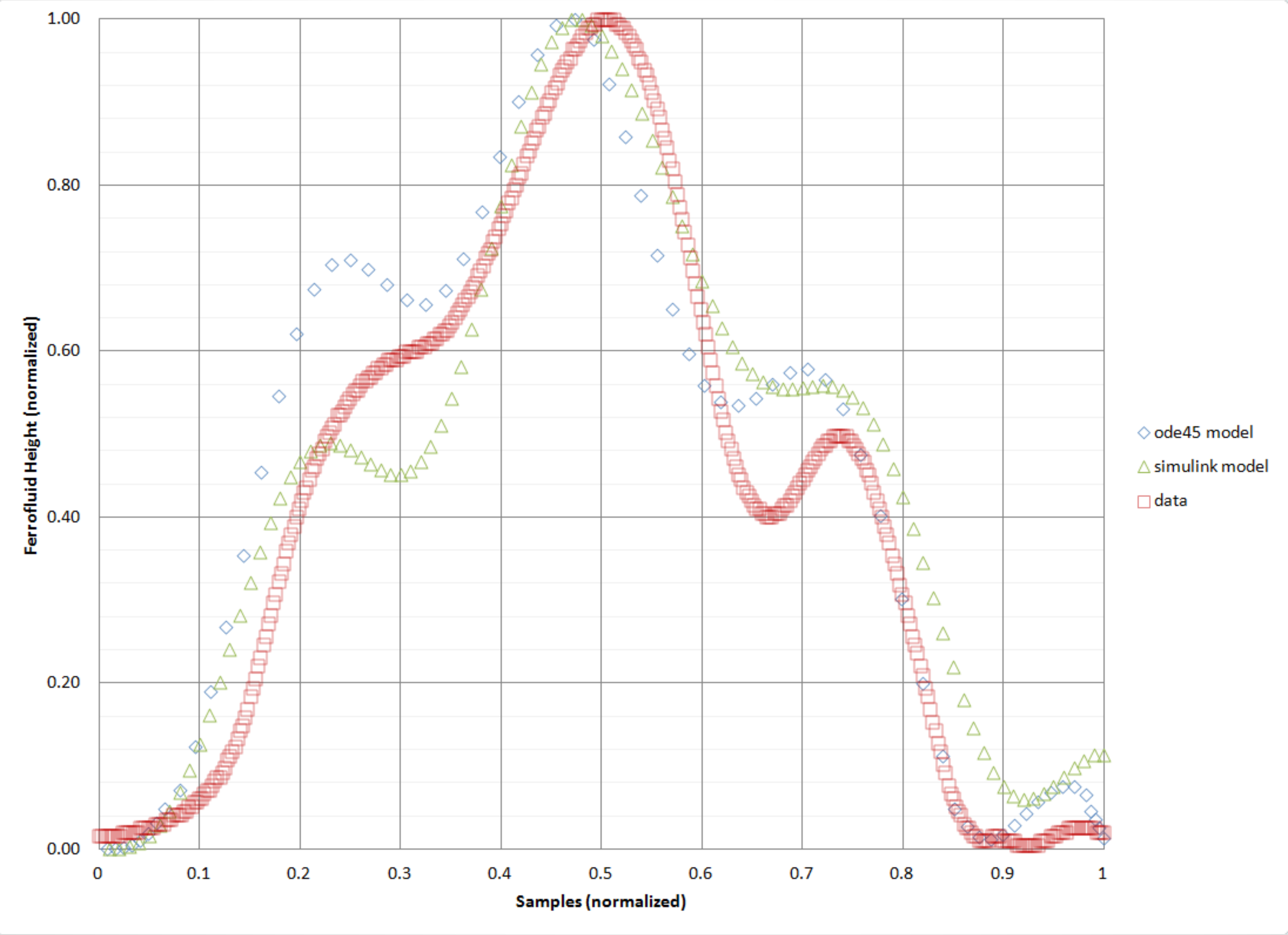
$$H_o = 0$$

$$\Delta H = \ 10$$

$$\omega = \ 1$$



Analysis of Numerical Model



Kipp Schoenwald

Numerical Analysis

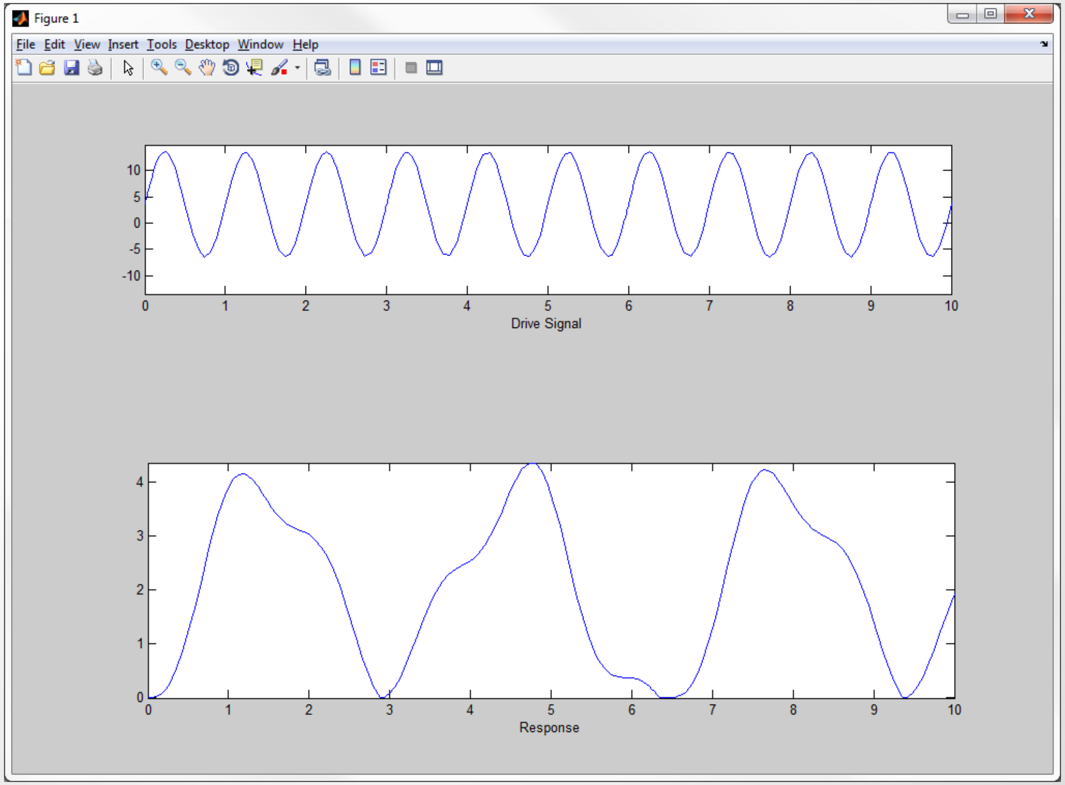
$$\xi(t) = \ driving\ frequency = \ H_o + \Delta H \sin(\omega t)$$

$$\beta = \ 0.3$$

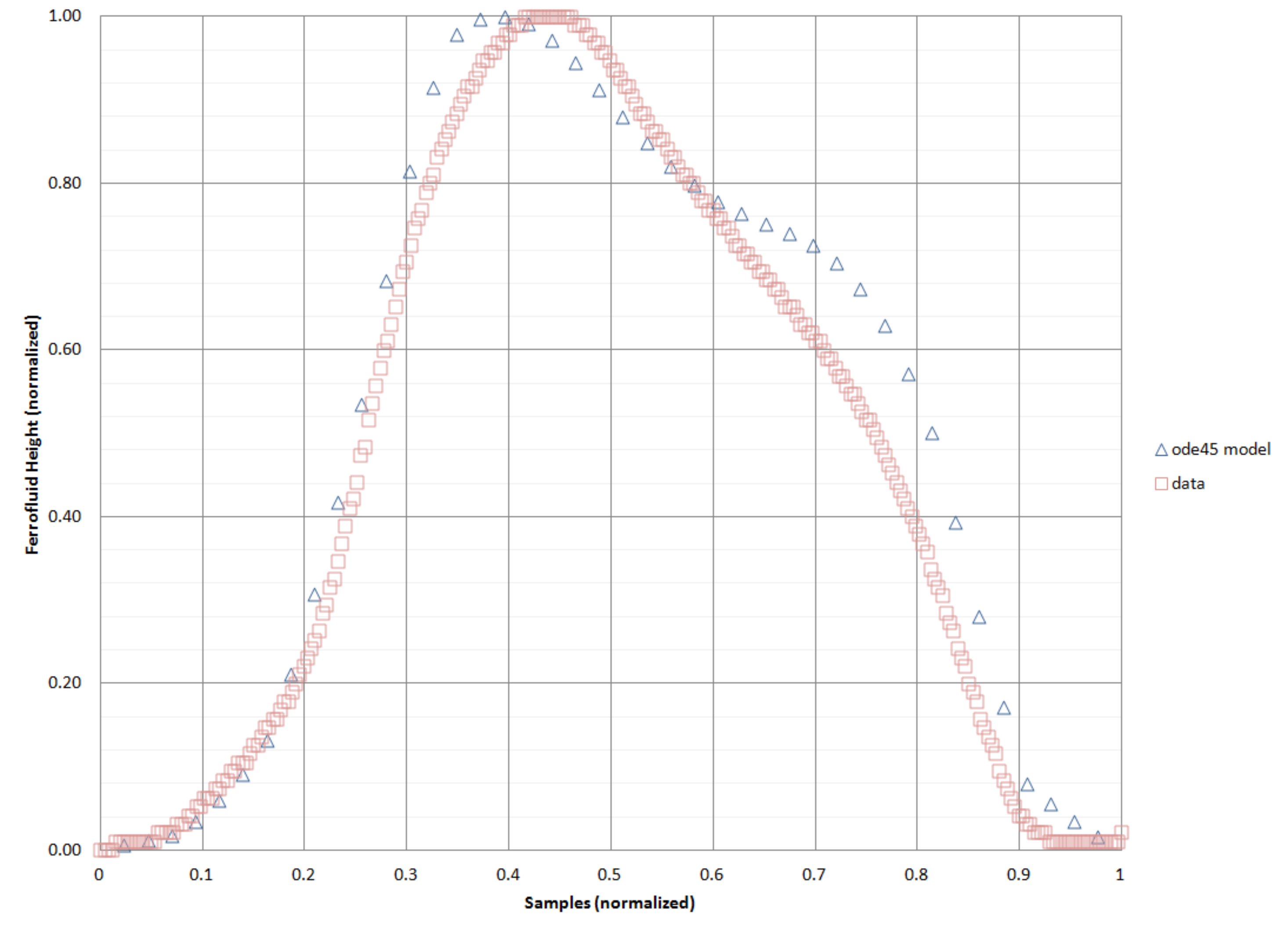
$$H_o = 3.55$$

$$\Delta H = \ 10$$

$$\omega = \ 1$$



Analysis of Numerical Model



Numerical Analysis

Modified Model

Though the model shows good correlation, the velocity term coefficient can not alone describe the entire fluid system.

Discrepency likely lies withing

fluid stiffness

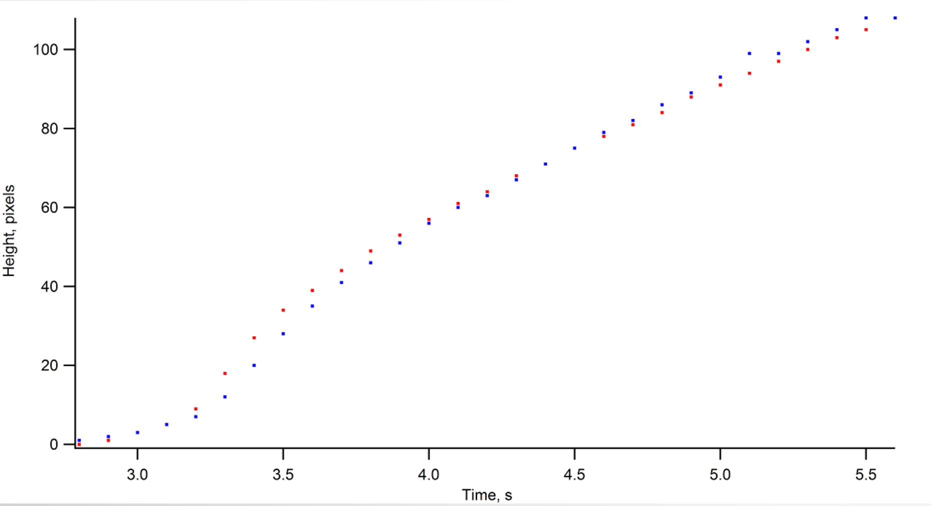
latency in particle alignment

$$\ddot{h} + \beta \dot{h} - h + h^2 = \xi(t), \quad where \qquad \beta = f(?)$$

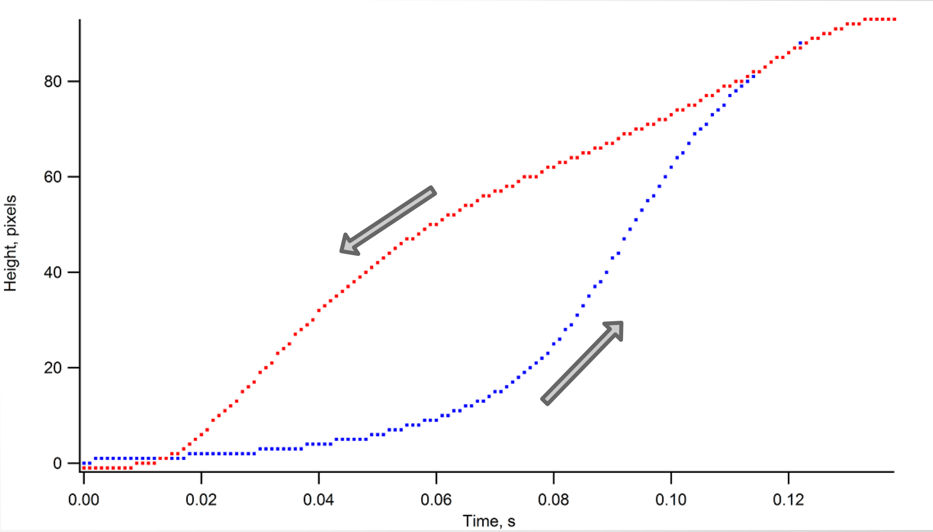
Characterizing the fluid

Fluid peak height vs magnetic field
Ramp up and down, showing hysteresis
Characteristic time of the fluid ~20 ms

Manual Triangle ramp up to 3.5V. No hysteresis.

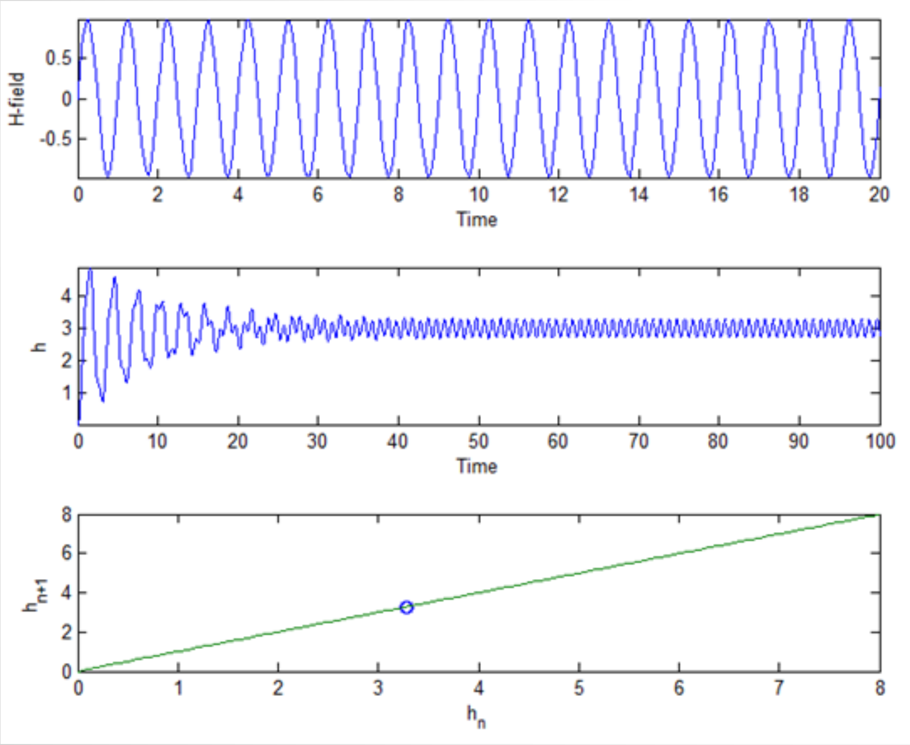


Triangle ramp, 1Hz up to 3.5V peak.

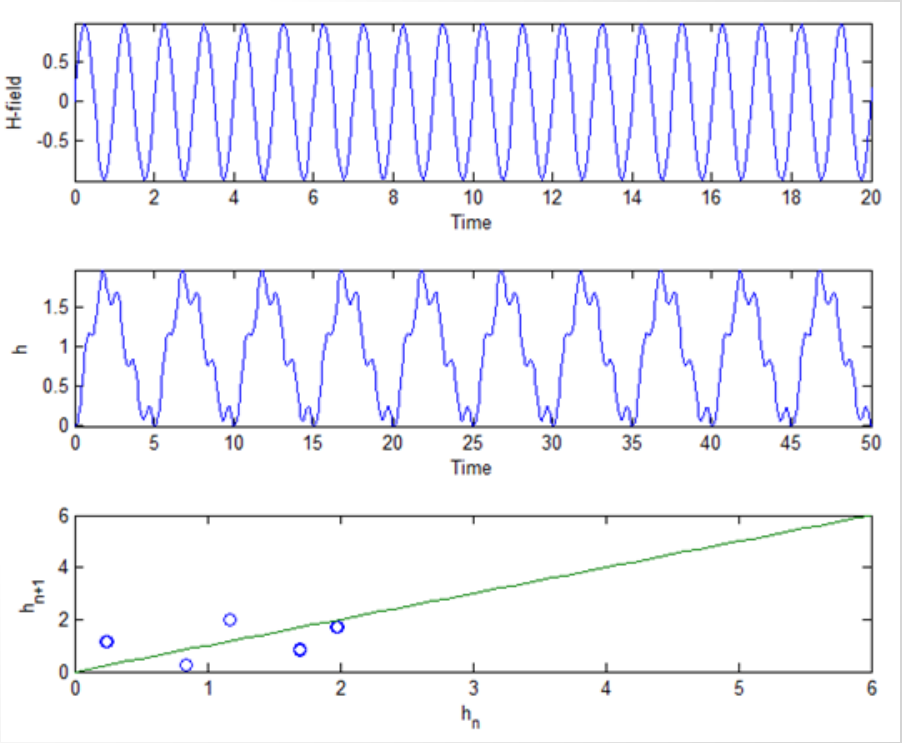


Model: Period changes

Drive function, peak height, and unimodal plot were analyzed for varying parameters.



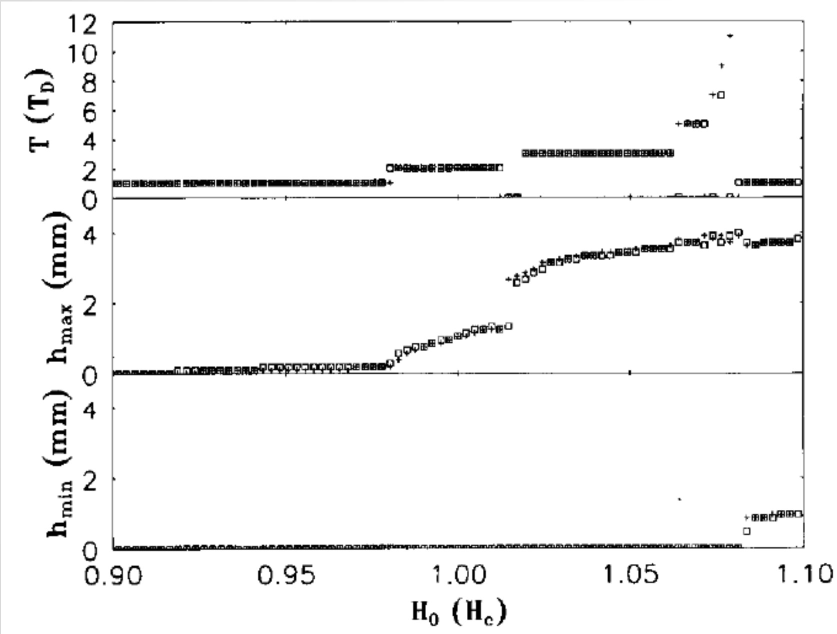
Two period



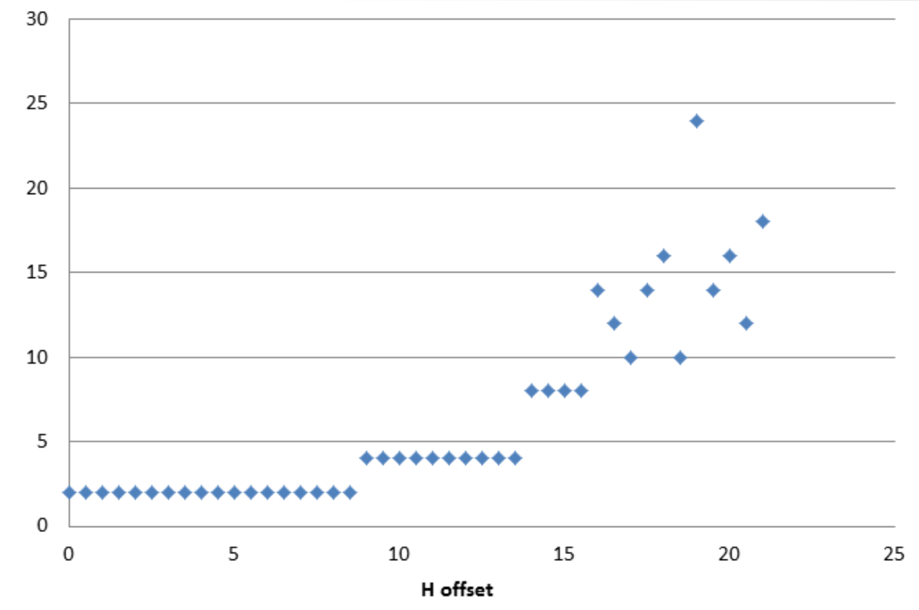
Ten period

Model: Verifying period changes

Period doublings show same general form.



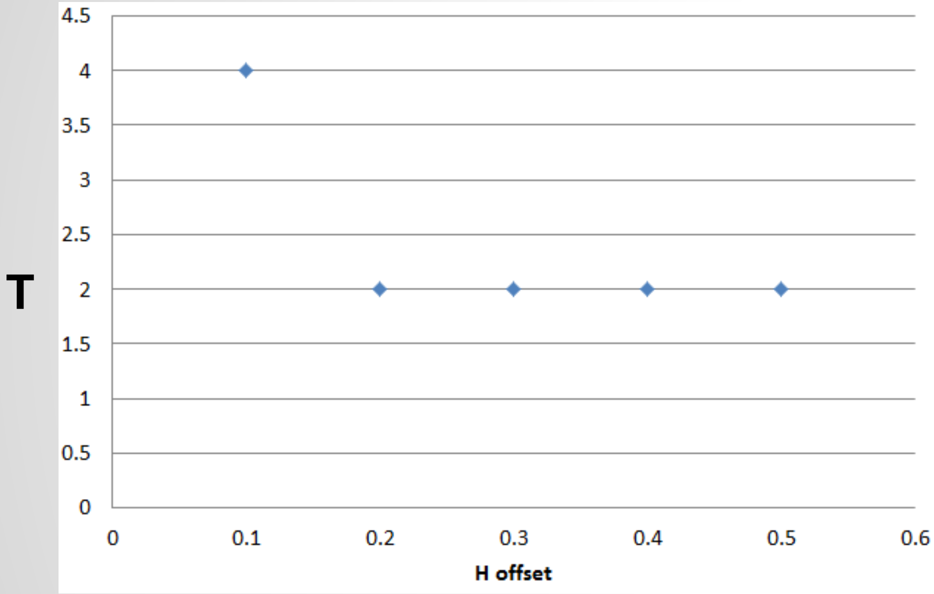
Mahr model



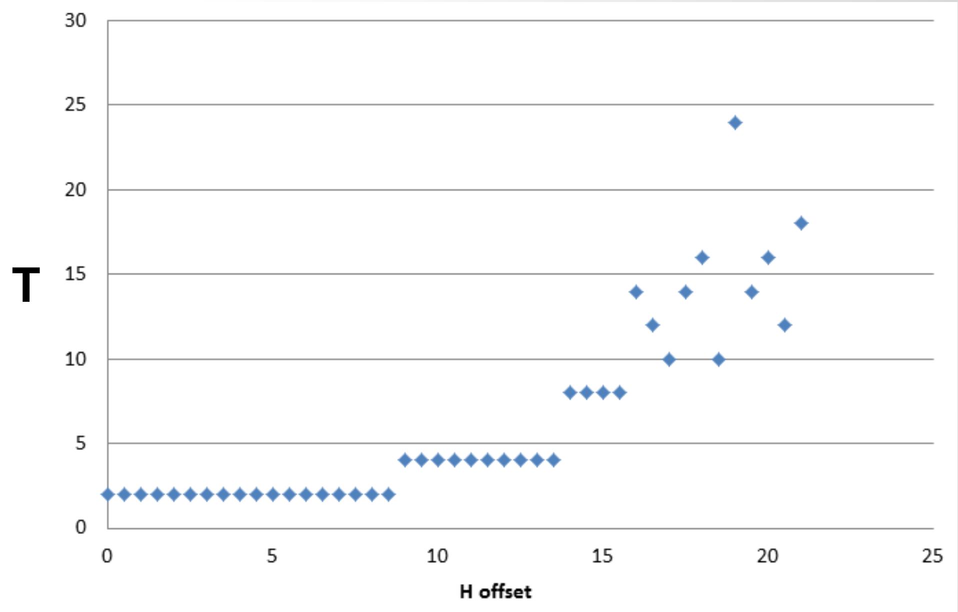
Mahr model reiterated in Matlab

Experiment vs Theory

Experimental

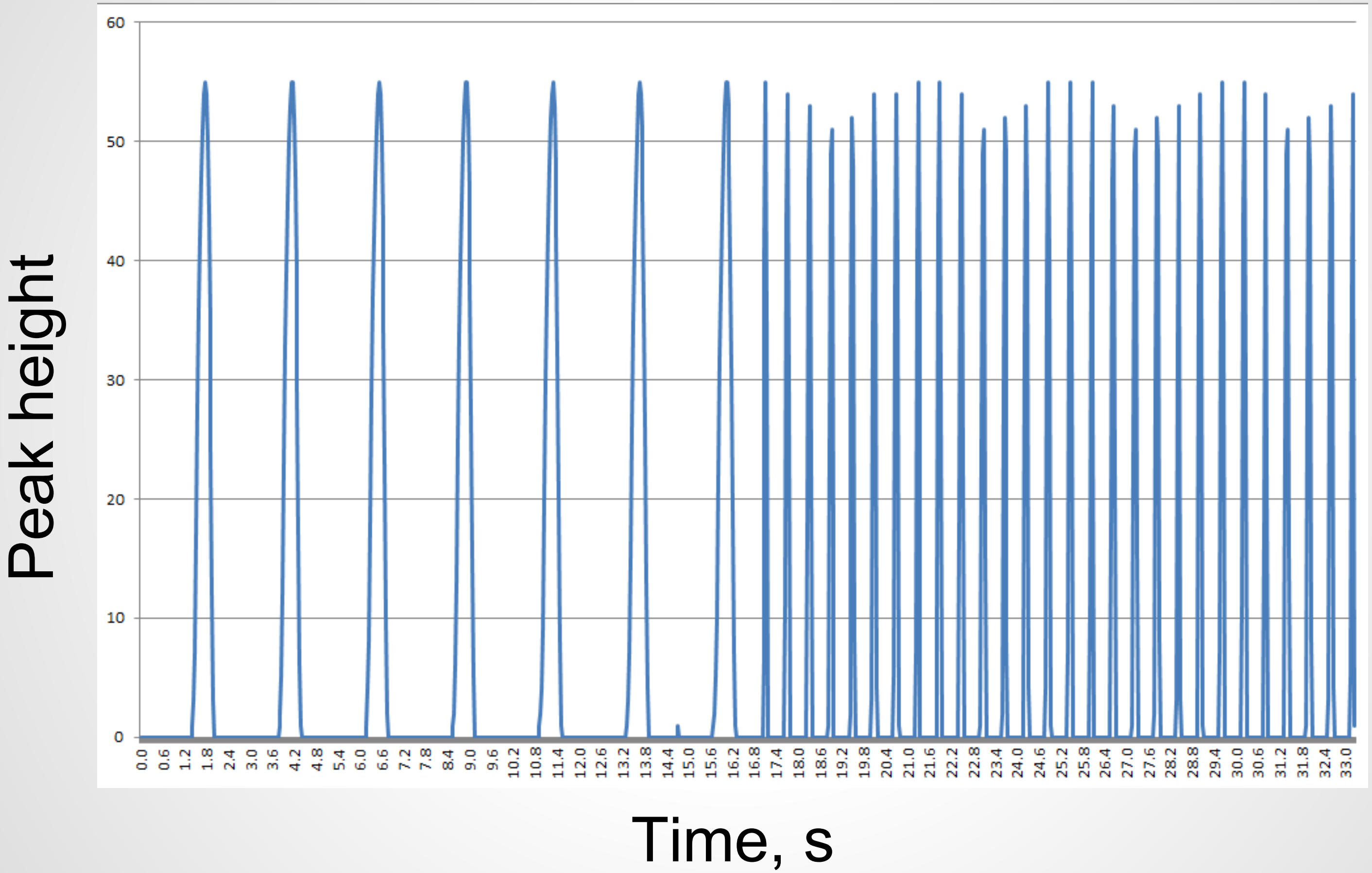


Mahr Model



At lower H offsets, the response periods generally agree.

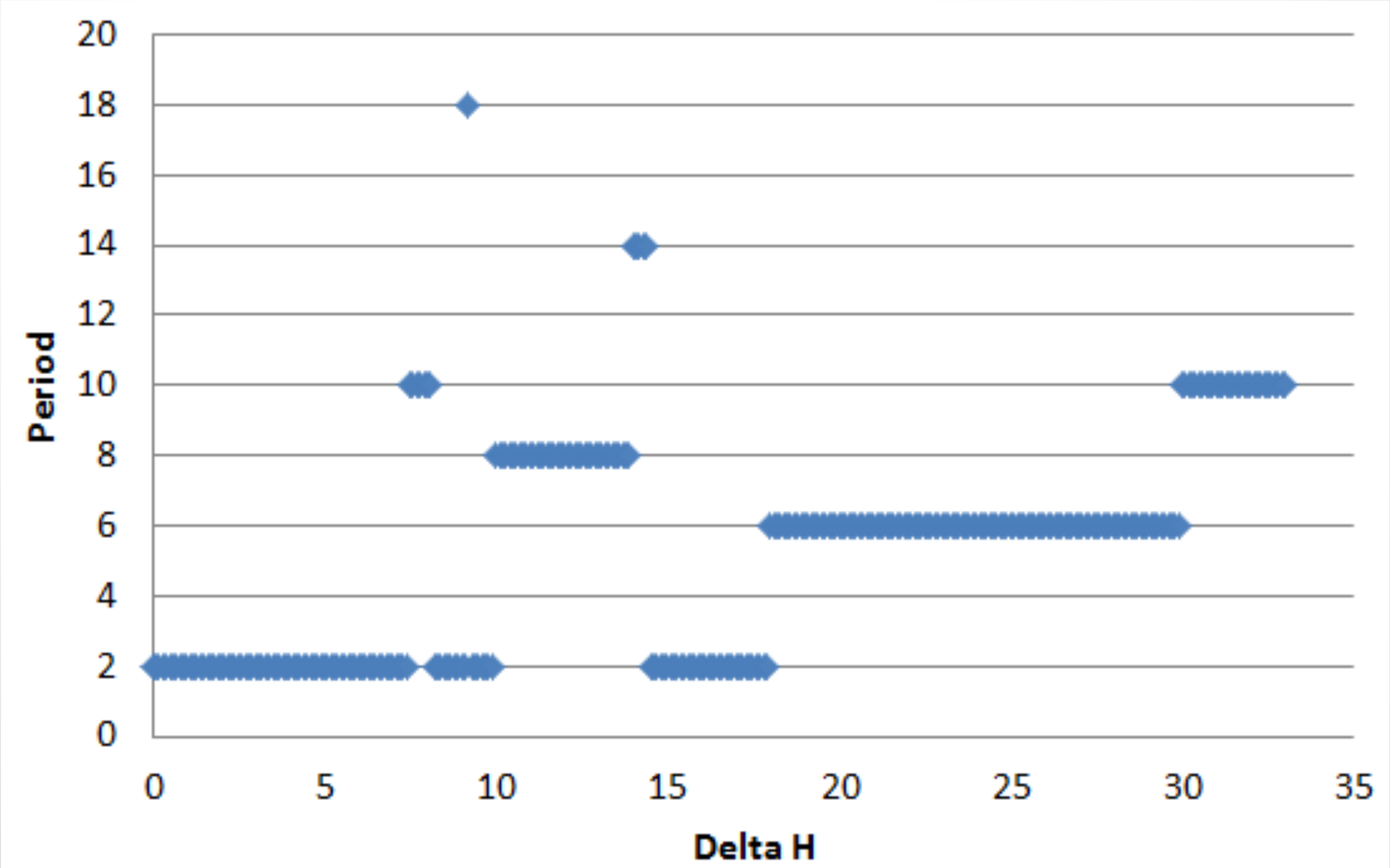
Video: Odd temporal phenomena



Frequency of peaks suddenly increases part way through video.
Max peak height is modulated over time.

Period changes as a function of H offset, delta-H, and frequency

The period fails to go chaotic even at high delta-H values!



Peak height vs H offset

We didn't see the kind of bifurcation reported by Mahr, but this is partly due to differences between viscosity and magnetization of our fluid and Mahr's.

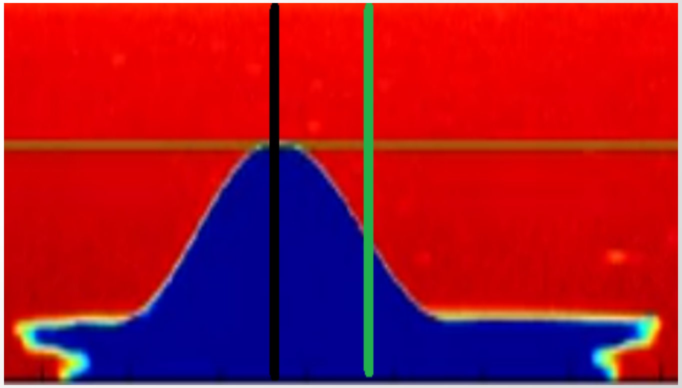
What about a frequency sweep? Exploring peak height vs frequency changes was difficult, as a rise in frequency meant a decrease in peak height.

Other unexpected behavior

Put this slide towards the end:

1) Sharp peak randomly appearing from fluid peak while doing slow manual ramp up.
2) Horizontal bifurcations show how the fluid moves to the side of the well, preferring this over rising in the center of the well.

Lateral bifurcation



Discrepancies between experiment and theory

1. Theory relies on uniform magnetic field (Helmholtz coil).
2. Different fluid from previous work.
3. Frame rate of video capture may have been accidentally altered.

Improvements:

1. Wider sweep of H offset.
2. Model to account for distance-dependence of H.