Synchronization of metronomes

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We examined the synchronization of two and up to nine metronomes placed on a moving base. We found that synchronization was always reached for 2, 3, 4 and 5 metronomes. For the rest of the cases synchronization was also possible but occurred a fraction of the times , and this fraction decreased with the number of metronomes. Average time to synchronization increased with number of metronomes in agreement with numerical results.

I. INTRODUCTION

The phenomenon of synchronization is found throughout nature and in many applications of different fields of science. Examples of synchronization include the flashing of firefly populations, the firing of pacemaker cells in the heart, applications of Josephson junctions, coupled fiber laser arrays, and perhaps even epileptic seizures. Many instances of synchronization in nature and applications in science become extremely complicated as soon as multiple variables or several coupled oscillators are introduced. However, the pervasive presence of synchronization in nature and science motivates work towards a deep understanding of such phenomena. So, as with any system that is to be scientifically understood, the simplest case is the best place to begin; for synchronization, the case of two identical, coupled oscillators is perhaps the simplest.

Research into the mutual synchronization of two identical coupled oscillators is believed to date back to the work of Christian Huygens in the 1600's, in which he discovered and investigated the synchronization of two pendulum clocks hanging on a common support beam.

Significant progress has been made in explaining simple instances of synchronization. In 2002, Bennett et al.[1] constructed an apparatus meant to recreate the setup decribed by Christiaan Huygens. The experimental and numerical results agree qualitatively with records from Huygens's research. Concurrently, Pantaleone[2] investigated a similar setup in which metronomes were placed on a common support which was free to roll across cylinders, allowing coupling through the low-friction horizontal translation of the platform. Pantaleone's theoretical analysis provided agrees with the experimental observations reported.

In 2009, Ulrichs et al.[3] reported agreement with Pantaleone's results in computer simulations. Additionally, these simulations were run for coupling of up to 100 metronomes and reportedly showed chaotic and hyperchaotic behavior. A paper by Borrero-Echeverry and Wiesenfeld[4], currently in the process of publication, describes a theoretical model of the N=2 case which encompasses the behavior of both types of oscillators often studied, clocks and metronomes, which often contrast in the phase of synchronization encountered.

Here we examine synchronization of metronomes using the same basic experimental setup as Pantaleone. We study the dynamics for 2 and up to 9 metronomes,and compare it to numerical simulations. We show that synchronization is possible even for a relatively large number of metronomes . We present the time of synchronization as a function of metronomes and show qualitatively agreement between experiment and simulation.

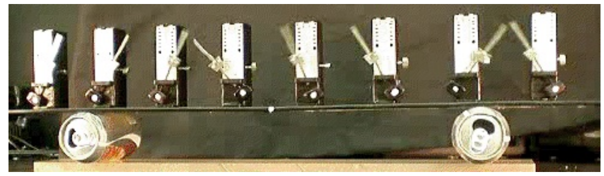

FIG. 1: Experimental setup

II. METHODS

A. Experimental setup

The experimental setup (Fig.1) consists of the metronomes resting side to side on a light board of mass 65.5 g, placed on top of two empty soda cans. The metronomes are Wittner Super-Mini-Taktell (Serie 880). Throughout the experiments we set the metronomes at the highest possible frequency of 208 ticks per minutes, which corresponds to 104 oscillations per minute. The mass of each metronome is 94 g. We placed the cans on top of a smooth platform to ensure that they moved evenly. We tracked the motion of the metronomes bob's and the platform using a high velocity camera. The software for tracking uses contrast. So, we tracked a white dot on the metronomes's bobs and platform, and painted the surrounding areas in black.

B. Time to synchronization

To calculate time to synchronization, we analyzed the horizontal displacement of the

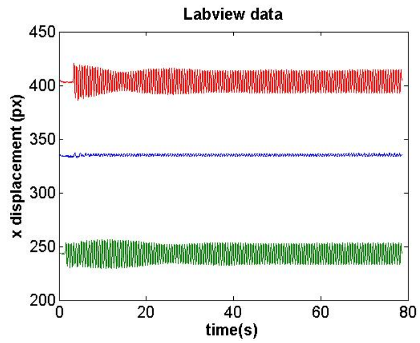

FIG. 2: Plot of horizontal displacement vs. time. The central curve corresponds to the displacement of the platform, the other curves represent the horizontal displacement of two metronomes.

metronomes's bobs as a function of time. Fig. 2 shows a time series for one of the runs with 2 metronomes. We studied 171 runs in total, going from 2 and up to 9 metronomes.

To calculate time to synchronization we looked at the peaks of the time series of the horizontal displacement. We considered the metronomes to be synchronized when the peaks of all the metronomes occurred within a small time interval for a given number of consecutive periods. The way we implemented this was as follows: first we choose one of the metronome as reference and look for peaks of the other metronomes within a two epsilon interval (one epsilon to the right and one to the left) of the peaks of the reference metronome, then we average the time of all these peaks, and finally we look at a two epsilon interval around this average time. If we find peaks for all the metronomes within the corresponding interval for 20 consecutive cycles, we consider the metronomes to be synchronized. We chose the epsilon to be 40 ms, which is somewhat larger than the average time step of the data. Because we are interested in the time to synchronization, one key aspect was the location of the initial time. To be able to locate the initial time easily we followed the following procedure: while holding the platform still, we give the metronomes, one by one, a random initial phase and angular velocity. Then we let go of the platform at the same instant we release the last metronome. This way, the initial time corresponds to the first peak of the last metronome we perturb.

C. Numerical simulation

We model the system as N ideal inverted pendulums, i.e. point masses connected to the platform by massless rods. The dynamics of the system is described by the set of (N+1) equations[1] :

$$\ddot{\phi}_j + b\dot{\phi}_j + \frac{g}{\ell} sin\phi_j == \frac{1}{\ell}\ddot{X} cos\phi_j + F_j \quad (1)$$

$$(M + Nm)\ddot{X} + B\dot{X} = -m\ell \frac{d^2}{dt^2}\left(\sum_{i=1}^{N} sin\phi_j\right) \quad (2)$$

There are N equations of the form (1),one for each metronome, where ϕ_j is the angle that the jth metronome makes with the vertical. The first term on the left-hand side of (1) comes from the inertia of the jth pendulum.The second term is the damping force and the third comes from the force of gravity. On the right-hand side is the interaction term between the pendulum and the platform, with X being the displacement of the platform and F_j the impulsive drive due to the internal mechanism of the metronome. Equation (2) is the equation of motion of the base . M is the platform mass, N is the number of metronomes, m is the mass of the pendulum, and B is the coefficient of friction.

To make up for the energy lost during the oscillations, energy is transfer to the metronome through a device called escapement. We introduce the action of the escapement into the model by applying a "kick" to the pendulum's bob when it is at the bottom of its swing. The kick consists of two parts: first, the angular velocity of the bob is reduced by a factor γ, then, it is given a fixed impulse in the direction of the motion[1].

We integrated the model numerically using MATLAB's ODE45, which is a solver based on an explicit Runge-Kutta (4,5) formula. To compare the results to the ones obtained experimentally, we generated 100 time series of the phase of the metronomes for each of the systems going from 2 and up to 9 metronomes.The initial conditions were drawn from a standard uniform distribution in the range $(0, \pi/4)$. We did this for two sets of the model's parameters. For the first set we used our best estimate of the parameters. We measured the parameters that could be easily measured (i.e. M,m,l). We deduced the damping force of the metronomes from the time it takes for a metronome to wound down. And we estimated the rest of the coefficients. For the second set of the parameters we changed the length l to get a better match to the natural frequencies of the metronomes in the experimental setup. For both set of parameters we determined the time to synchronization using two different criteria. The first method is the same used to calculate synchronization in the experiments. This way we can consistently compare numerical and experimental results. The second method is more stringent. The idea is to get a more exact measure for synchronization, which was precluded in the experimental case by noise in the data . For this, we considered the metronomes to be synchronized only when the peaks of the phase time-series of all the metronome occur within one time step in the simulation.

III. RESULTS

Fig.3 shows the average time to synchronization vs. the number of metronomes us-

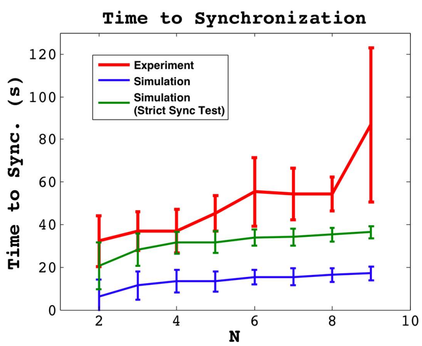

FIG. 3: Avg. time to synchronization vs. number of metronomes(N) for the experimental data and for the two different metrics for the simulation with the "best estimate" parameters.Error bars are the standard deviation.

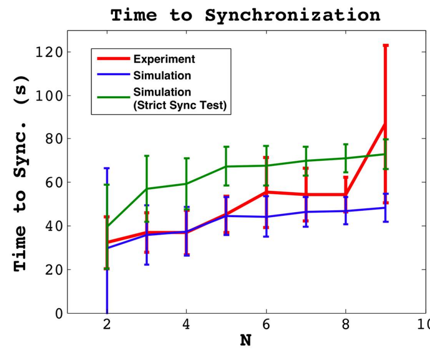

FIG. 4: Avg. time to synchronization vs. N for the experimental data and for the two different metrics for the simulation with the parameters scaled to match the frequency of the metronomes.Error bars are the standard deviation.

ing the best estimate of the parameters of the model. Fig. 4 shows the results when we scaled the length ℓ to match the natural frequency of the metronomes. As expected, we see that for all the cases the time to synchronization increases with the number of metronomes. Nevertheless, the upper trend isn't as pronounced as one might expect, considering how much richer the dynamics can be when increasing the number of dynamical variables.

The best agreement with the experimental data comes from the case where we used the parameters scaled to match the natural frequency of the metronomes and the same method for calculating time to synchronization.

Fig.5 shows the fraction of experimental runs for which synchronization occurred as a function of the number of metronomes. We see that increasing the number of metronomes decreases the probability of getting synchronization. This makes sense intuitively, but disagrees with the results from the simulations. In the simulations synchronization occurred for all the runs, except for close to 10% of the cases with two metronomes (Fig. 6).

IV. DISCUSSION

We found that synchronization is a common phenomena even for a relatively large number of metronomes. In the experimental cases where the metronomes were not observed to synchronize, a beating pattern was

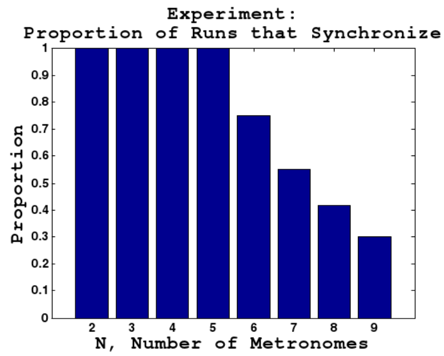

FIG. 5: Fraction of the runs that synchronize vs. number of metronomes for the experimental data.

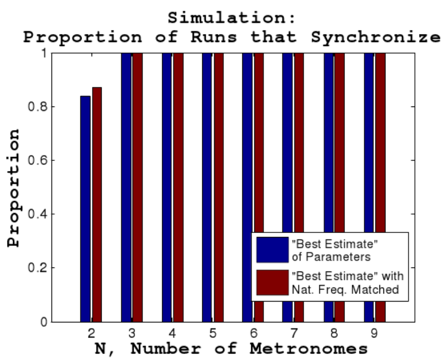

FIG. 6: Fraction of the runs that synchronize vs. number of metronomes for the numerical results.

observed . In this pattern, almost all the metronomes were synchronized, except for one or two metronomes that had a pattern consisting of intermittent periods of small and large amplitude oscillations. This pattern seemed to be stable for most of the cases. We tested the stability of this pattern in one particular case with 8 metronomes, and the pattern remained until one of the metronome's spring wound down. Nevertheless, it could be the case that the beating pattern was a transient behavior, and that the system needed more time that the one allowed by the experimental constrains in order to reach synchronization. The time limit set by the experimental setup could also be a possible explanation for the discrepancy, between simulation and experiment,in the fraction of runs that reached synchronization.

Our criteria for synchronization could be improved in several ways. First, we are only using the measured time of the highest horizontal displacement as the time of the real maximum of the horizontal displacement. But due to discrete nature of the time series, we know that this is not necessarily the case. We could improve our measure of synchronization, by improving the way we locate the maxima of the experimental data. This could be done by considering, not only the time for the highest value, but also the surrounding values weighted by the displacement. Additionally, in our study we are only looking for in-phase synchronization, and we are neglecting other interesting types of stable dynamics, like phase-locking, where the metronomes have the same final frequency, but don't oscillate in unison (e.g. anti-phase synchronization). Doing this will required changes to the experimental setup to obtain not only the displacements, but also the value of the angle. This would allow the calculation of instantaneous phase differences, and will permit the implementation of a more comprehensive metric for synchronization.

REFERENCES

[1] M. Bennett, M. Schatz, H. Rockwood, and K. Wiesenfeld, "Huygens's clocks," Proceedings: Mathematics, Physical and Engineering Sciences , 563–579 (2002).

[2] J. Pantaleone, "Synchronization of metronomes," American Journal of Physics **70**, 992 (2002).

[3] H. Ulrichs, A. Mann, and U. Parlitz, "Synchronization and chaotic dynamics of coupled mechanical metronomes," Chaos **19**, 43120–0 (2009).

[4] D. Borrero-Echeverry and K. Wiesenfeld, "Huygens (and others) revisted," Publication in progress.