Variable Length Spherical Pendulum: The Astrojax

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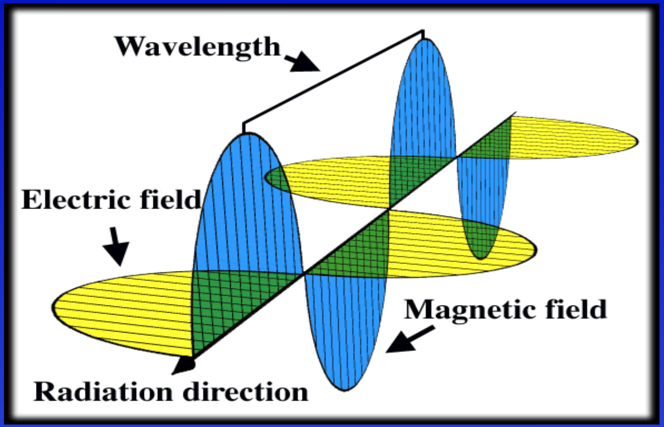
Introduction

- An oscillation is defined, in physics, as a regular variation in magnitude, position, etc. around a central point
- Oscillators are a massive subset of dynamical systems
- Oscillations are responsible for the sights you see and the sounds you hear



Pendulums

- While oscillators are a large subset of dynamical systems, pendulums are a large subset of oscillators.
- The most basic definition of a pendulum is a mass suspended from a pivot so that it can swing freely.
- There are many types of pendulums, both 2D and 3D.
- Insofar as construction is concerned, most pendulums are very, very simple. However, their simple structure belies the massively complex dynamics of more involved systems.
- Only the most basic pendular systems can be solved analytically. The most well known is the 2D, single pendulum for small oscillations.

Pendulums (contd.)

- There are many, many kinds of pendulums, some more complex than others.
- As the degrees of freedom increase, so too does the resultant complexity of the dynamics.
- The goal of our research was to investigate the dynamics of the Astrojax

The Astrojax

- The Astrojax is a simple toy consisting of three bobs in series on a string
- We chose to investigate such a system because its simplicity veils its truly complex and chaotic behavior, which intrigued us
- The Astrojax is a perfect example of a system that allows the general public to view the onset of chaos

Prior Work

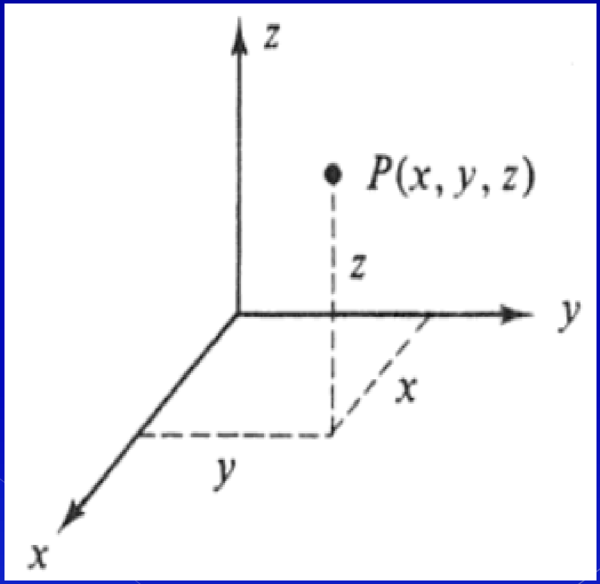
- Interestingly enough, we were able to find only one other paper dealing with the Astrojax specifically
- Our research and Dr. du Toit's differ in the fact that we focused on sinusoidal forcing, using one of the bobs on the end of the Astrojax to force the rest of the system. Dr. du Toit simply simulated the motion of the entire system with some initial conditions

Experimental Setup

- Our experiment was two-fold.
- We chose to investigate the forcing of the Astrojax system via both a robotic arm and a human arm.
- While the method of forcing differed between the two, the method of data capture and analysis remained the same.

Data Capture

- We placed four OptiTrack motion capture cameras around the Denso robotic arm downstairs
- The robotic arm had no way to hold the "top" Astrojax ball, so Colin designed a holder, and we 3D printed it.
- We covered the two masses we wished to observe in reflective tape to allow the cameras to track their motion
- We recorded our takes in MOTIVE, OptiTrack's proprietary software, and exported the data to Excel, which we then analyzed with the help of MATLAB



Robotic Forcing

- We normalized the speed of the robotic arm
- Starting from rest, we had the robotic arm force the system with a specific amplitude in one or multiple directions.
- We recorded multiple instances of the same forcing schemes

Human Forcing

- We covered Andy’s arm in black cloth to keep it from showing up in the cameras, put a glove on his right hand, and then taped an OptiTrack marker on the top of his hand
- Andy then forced the Astrojax in three different manners
  - Horizontal
  - Vertical
  - Butterfly
- We obtained multiple recordings of these forcings also

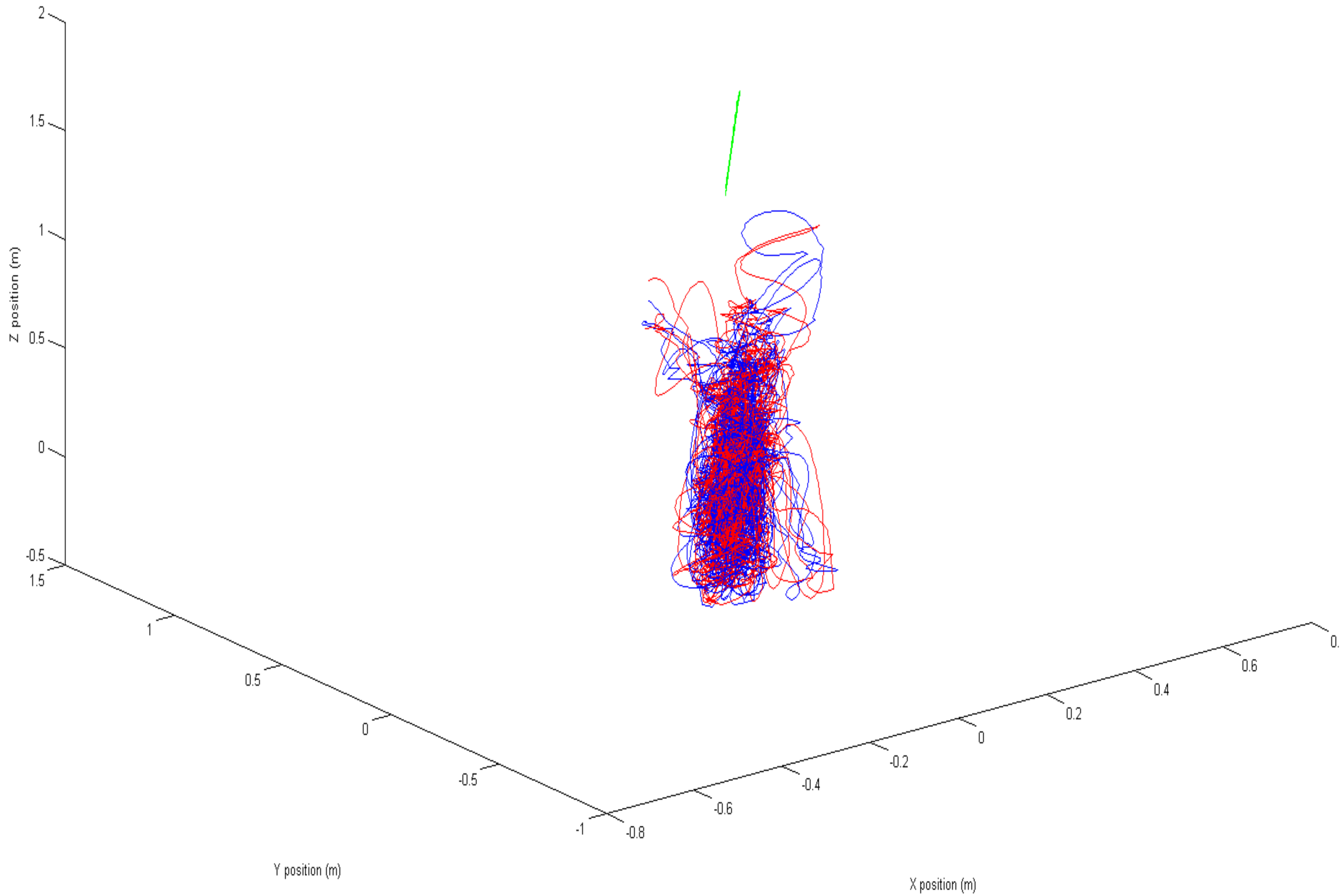
Results (Robotic)

- The motion we observed from the forcing via the robotic arm were truly chaotic
- For low amplitudes, the two balls would not separate and thus simulated a rigid spherical pendulum
- As the amplitude increased, splitting occurred more frequently, and chaos quickly emerged

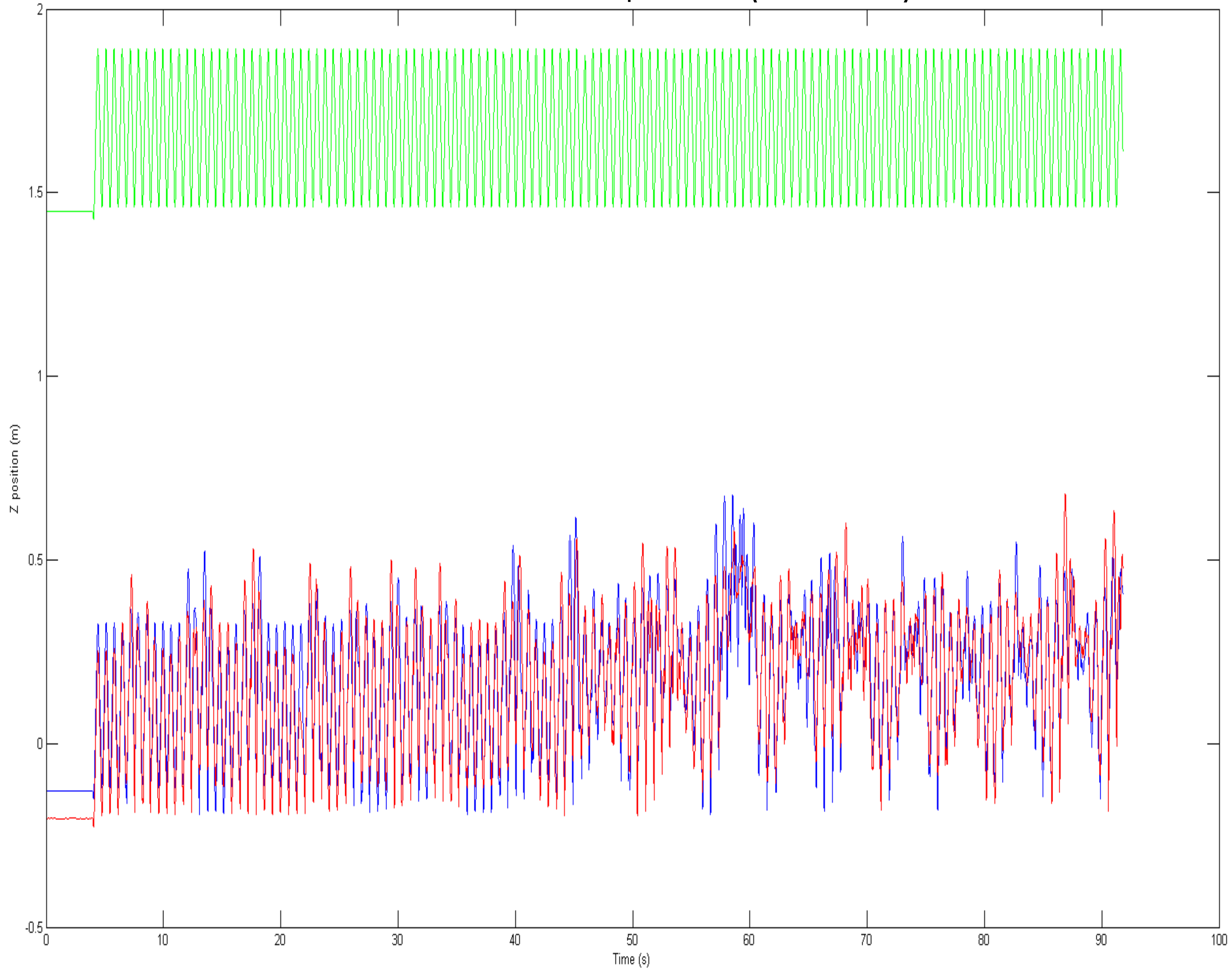
Results (Robotic contd.)

- Splitting occurred more frequently and consistently when forcing was in the x-direction
- What happened most often was the system would begin an orbit but then quickly decay
- We plan to investigate if there is a relationship between the amplitude of the forcing and the decay time

Take 1 dx=265, sp=100 (Robotic)



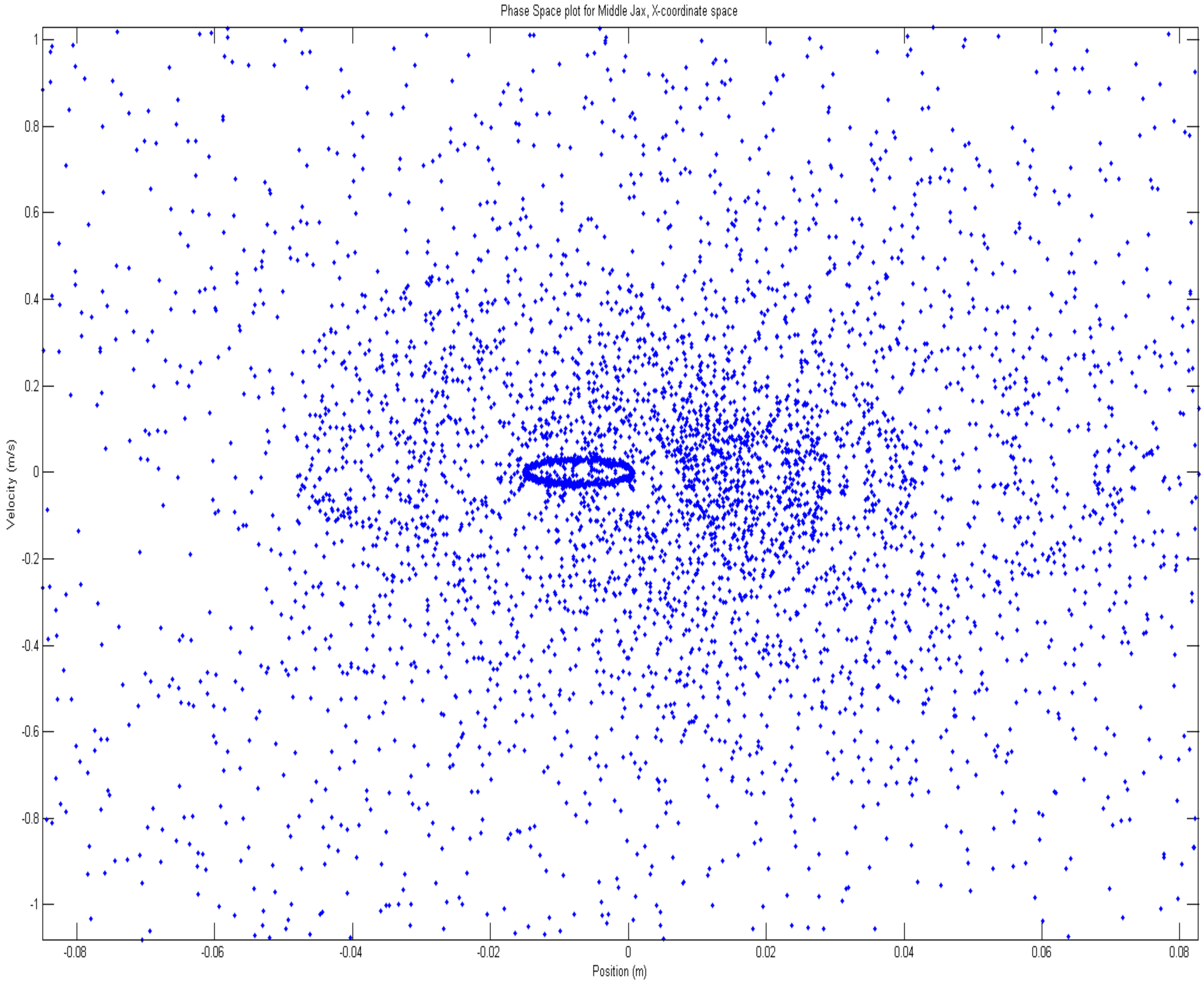
Take 1 dx=265, sp=100 (Robotic)



Phase Space Analysis

- From the OptiTrack data, we can plot position vs. velocity for all three Cartesian coordinates to create phase space plots of each coordinate.
- The following graphs show our resultant phase spaces for the forcing previously shown.

Take 1 dx=265, sp=100 (Robotic)



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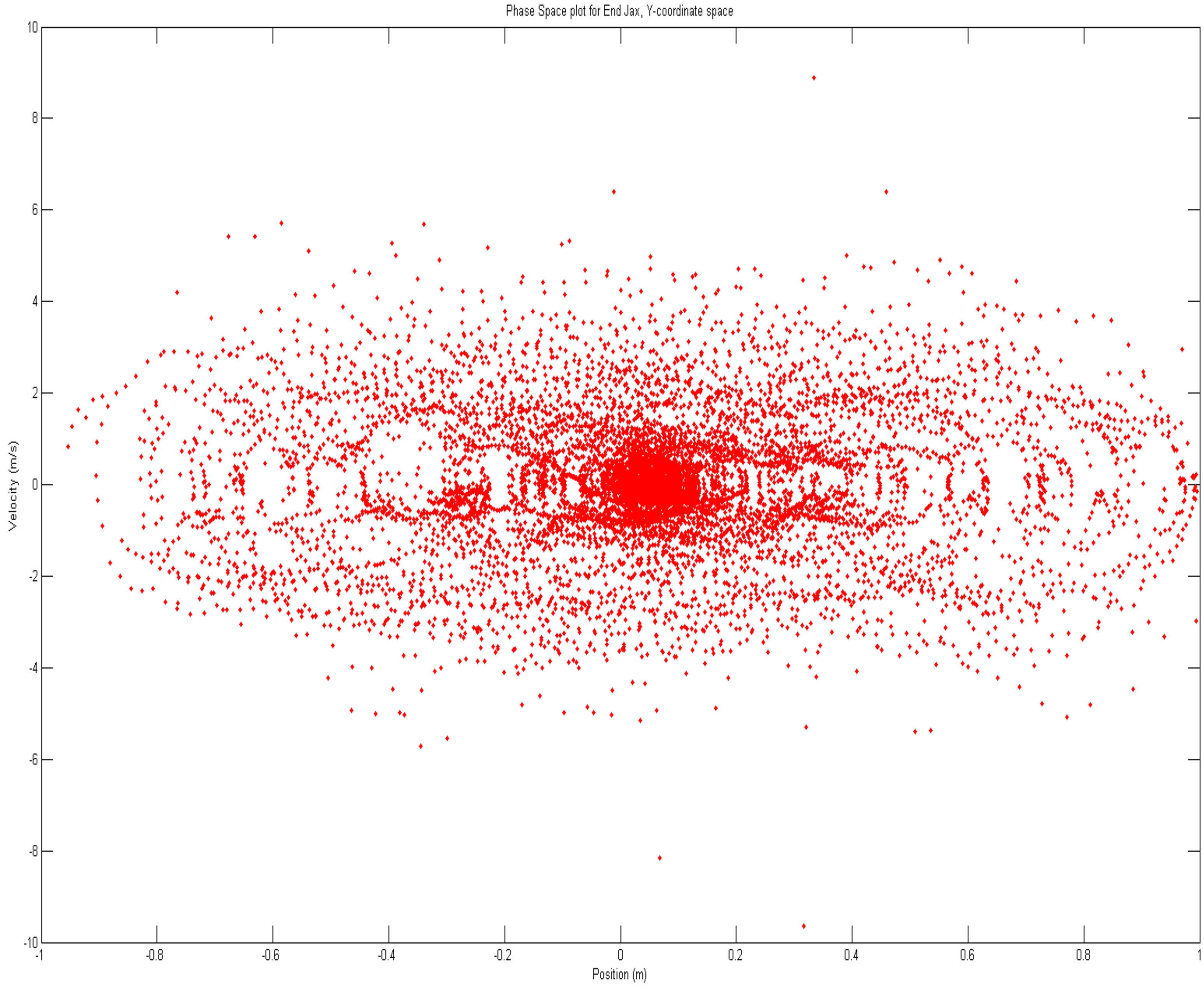

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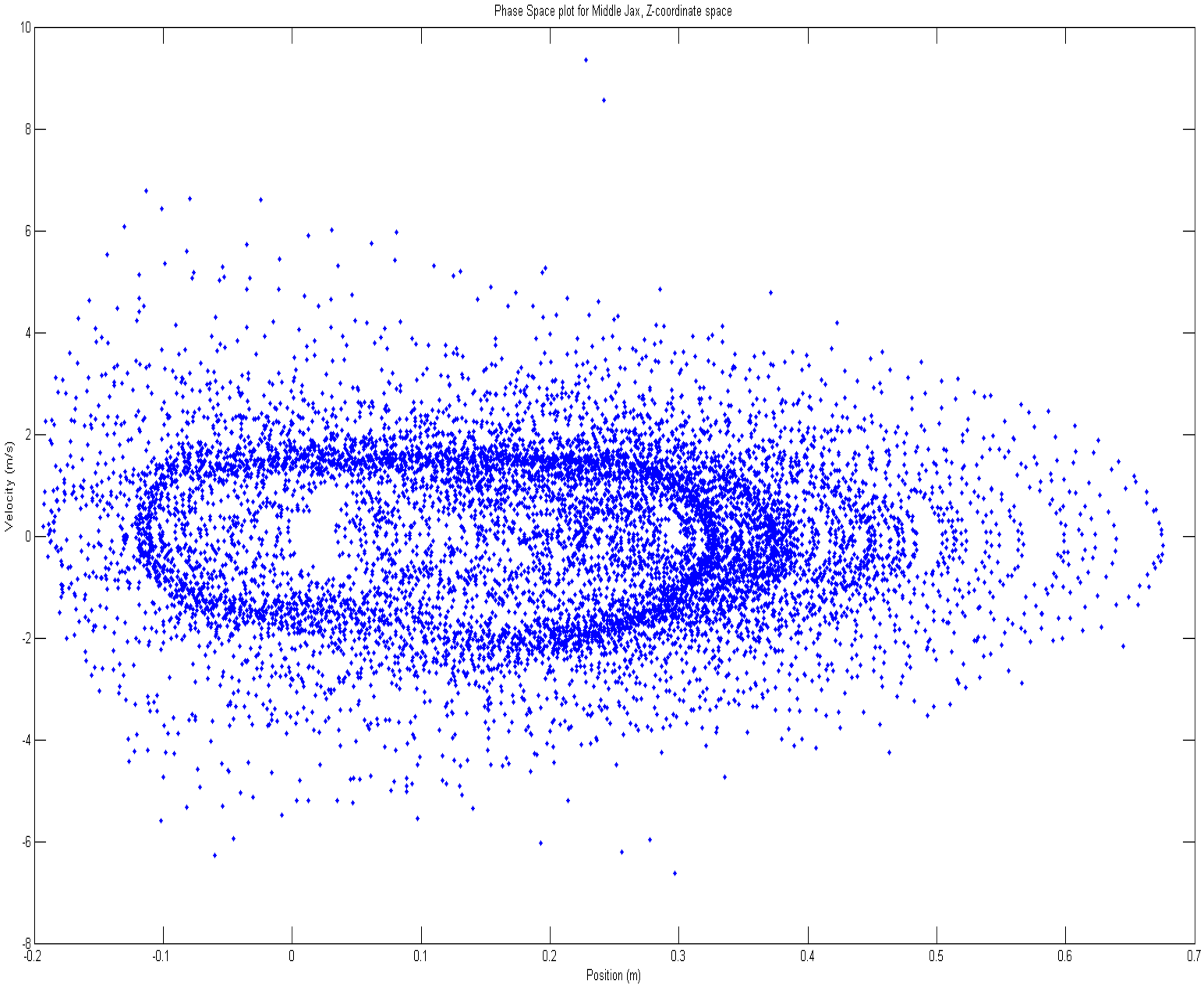
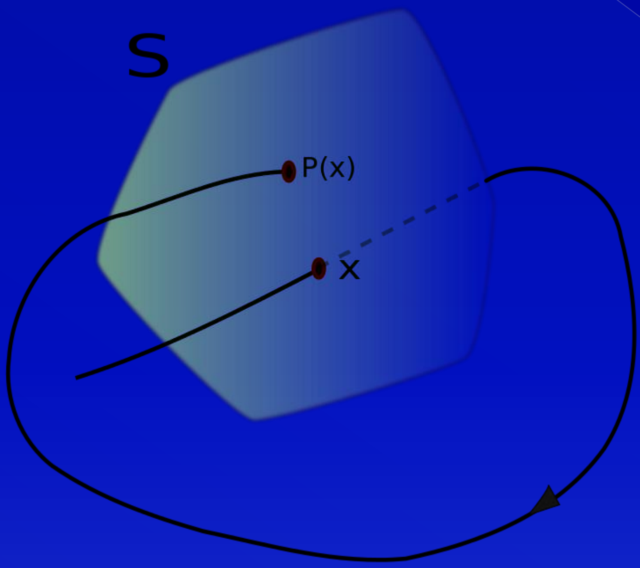
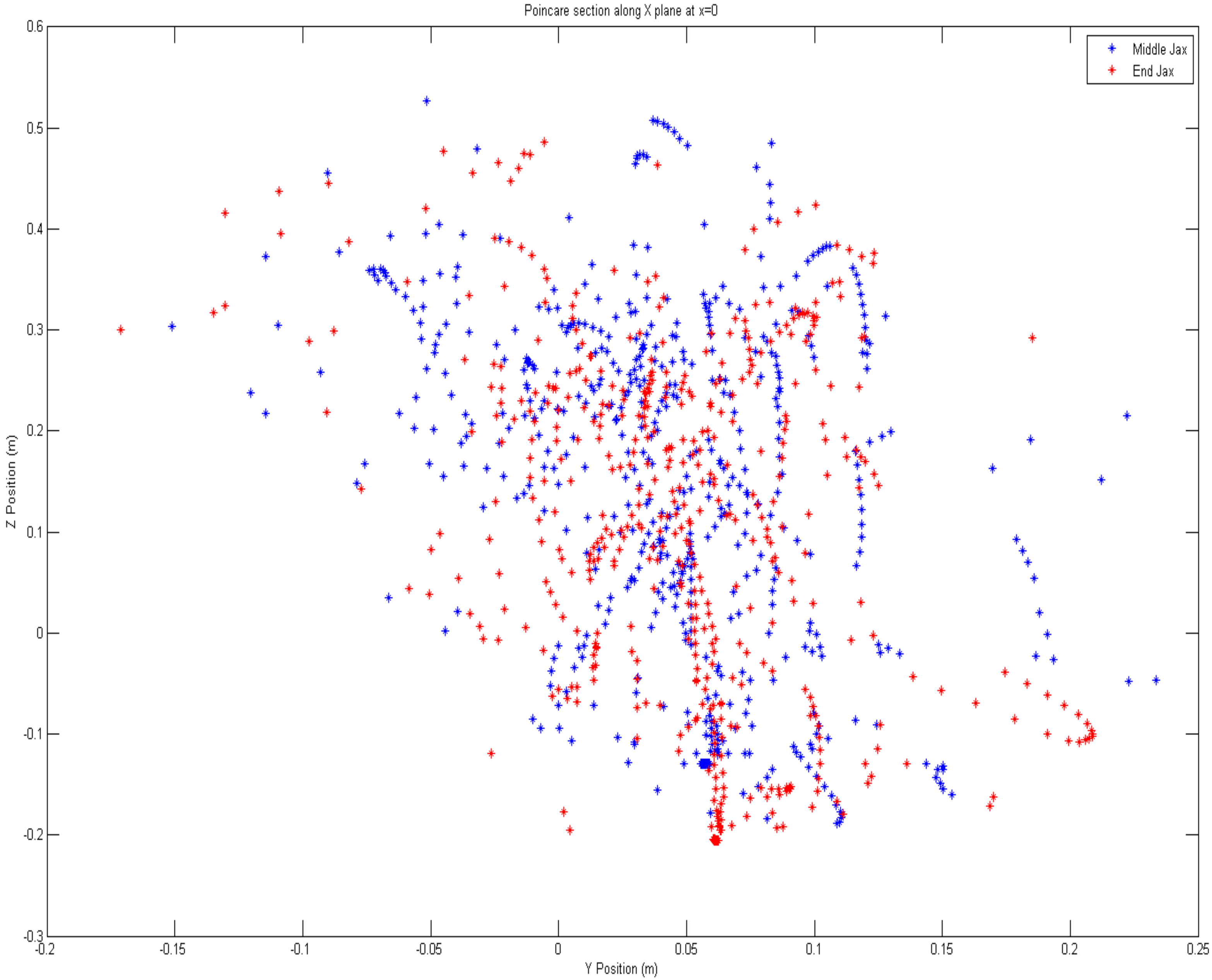


Poincare Section Analysis



- Similarly to how we did the phase space plots, we can take segments of the three Cartesian planes to create Poincare sections that show when each Jax passes through our selected plane in its orbit

Take 1 dx=265, sp=100 (Robotic)



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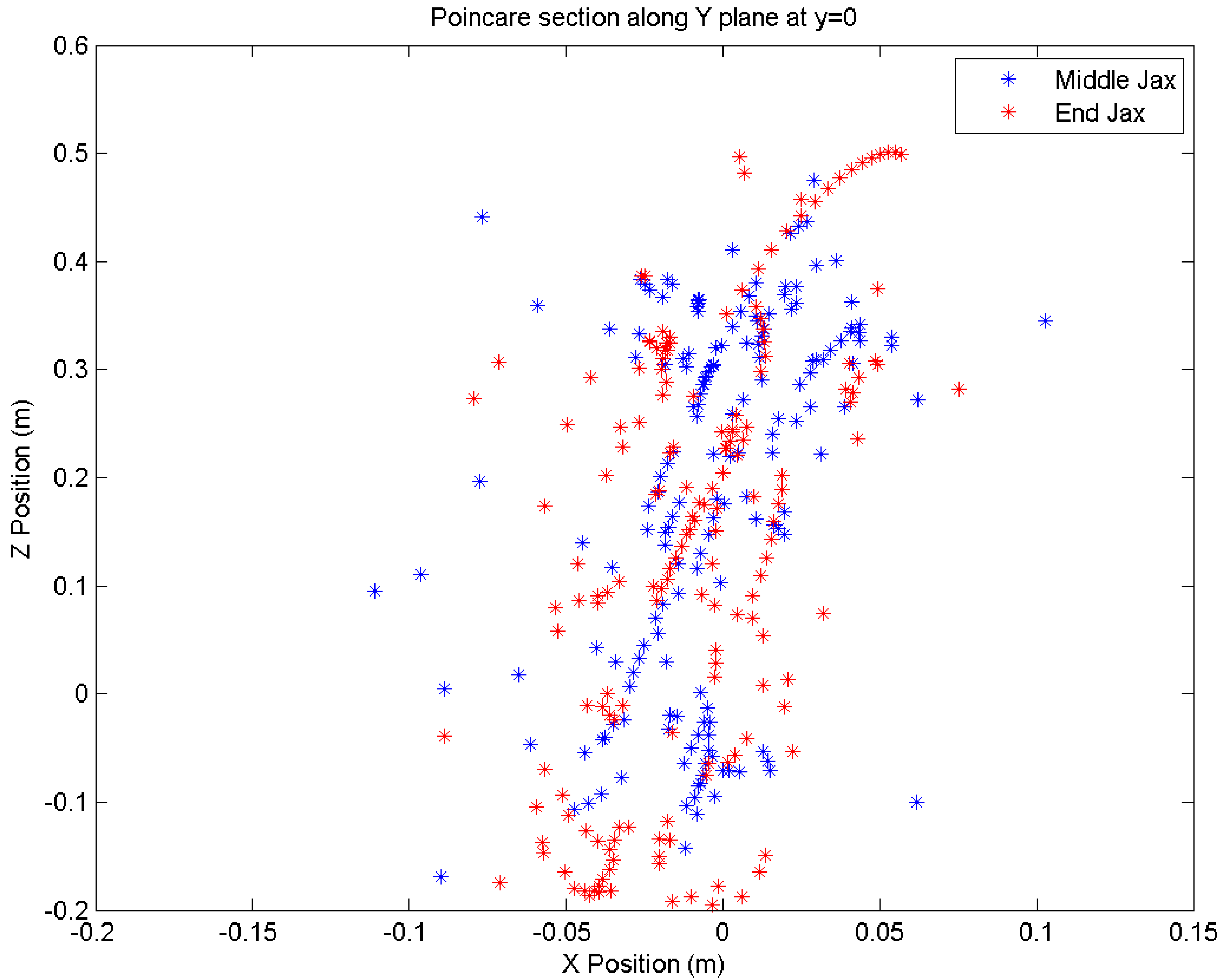

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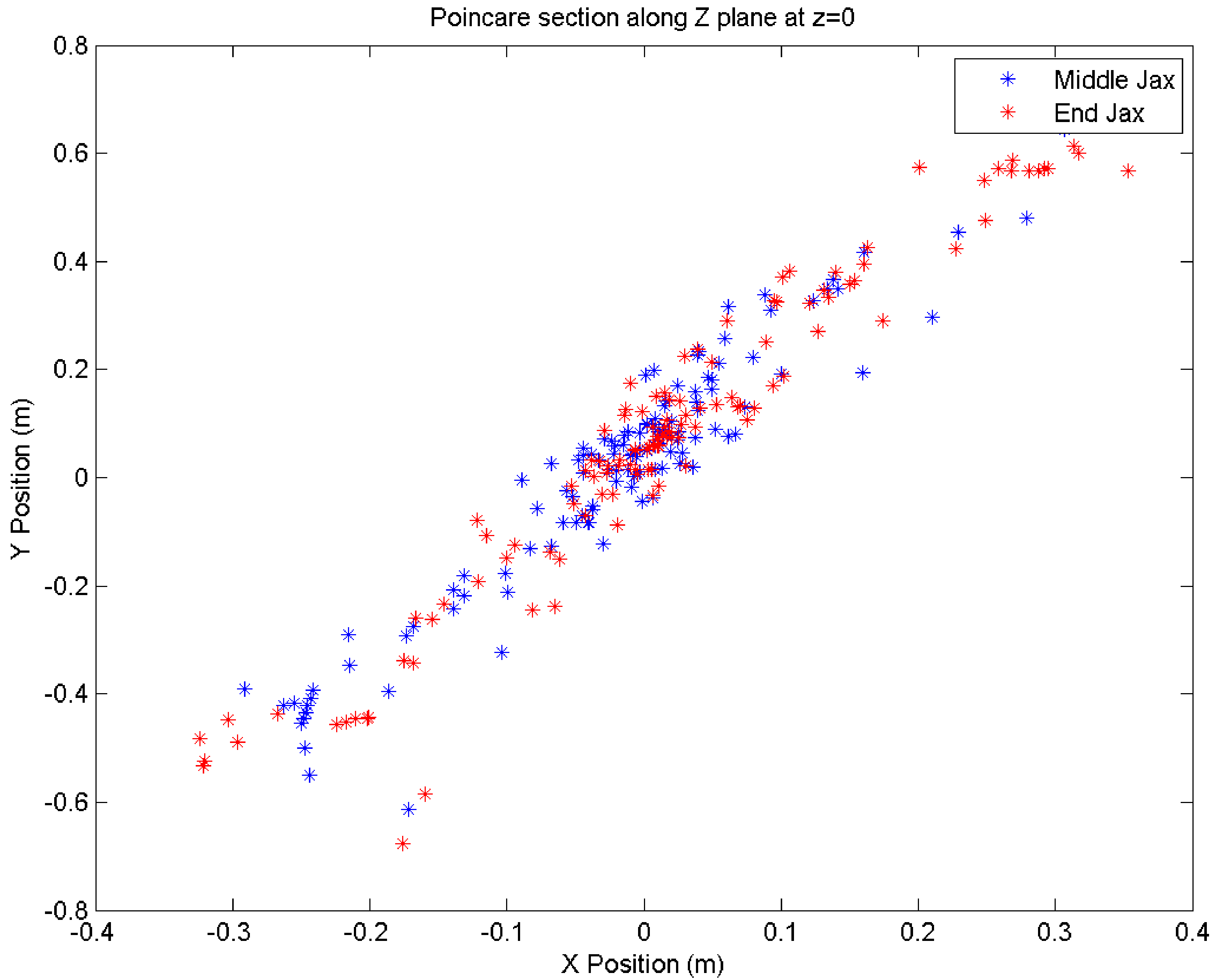
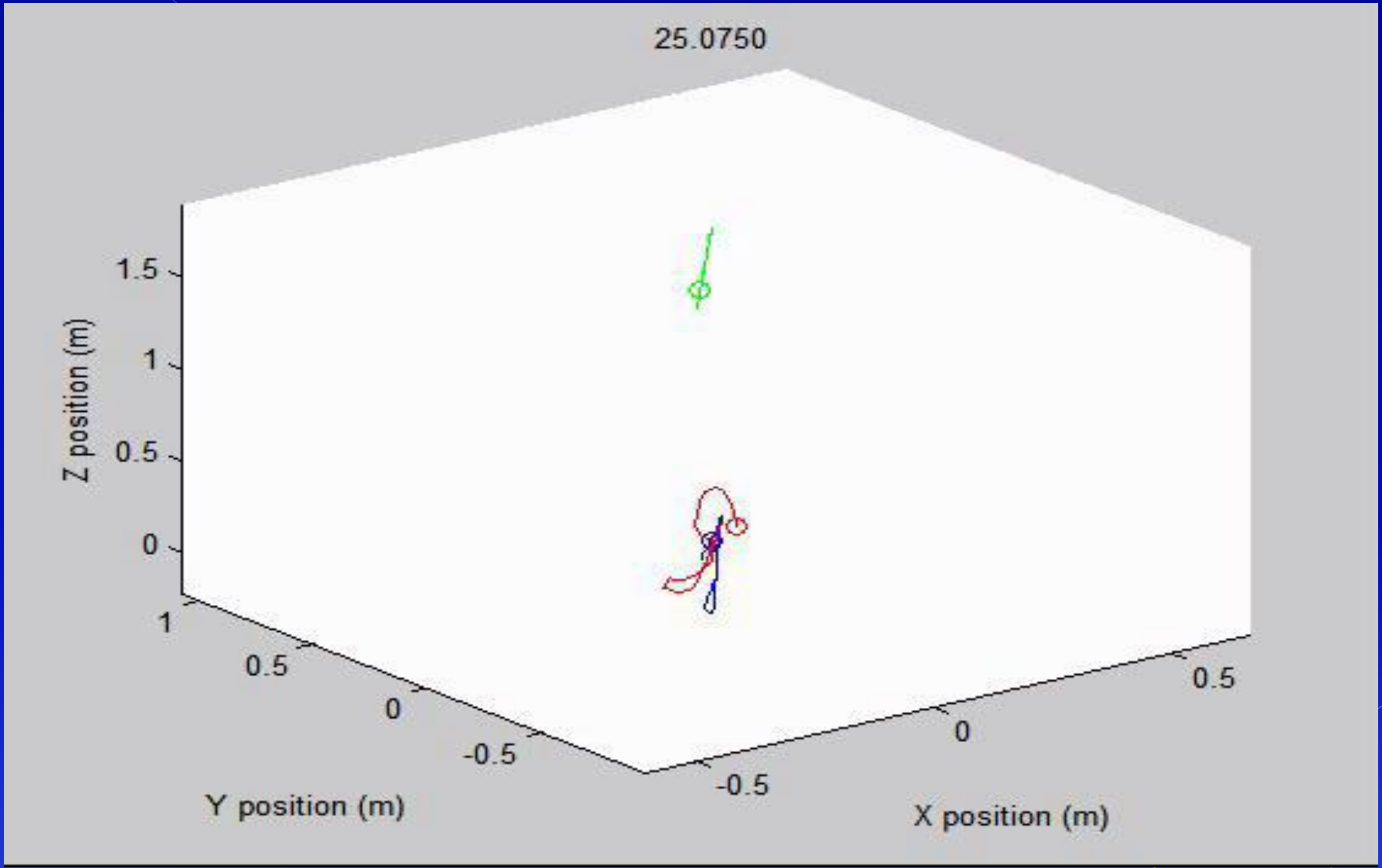


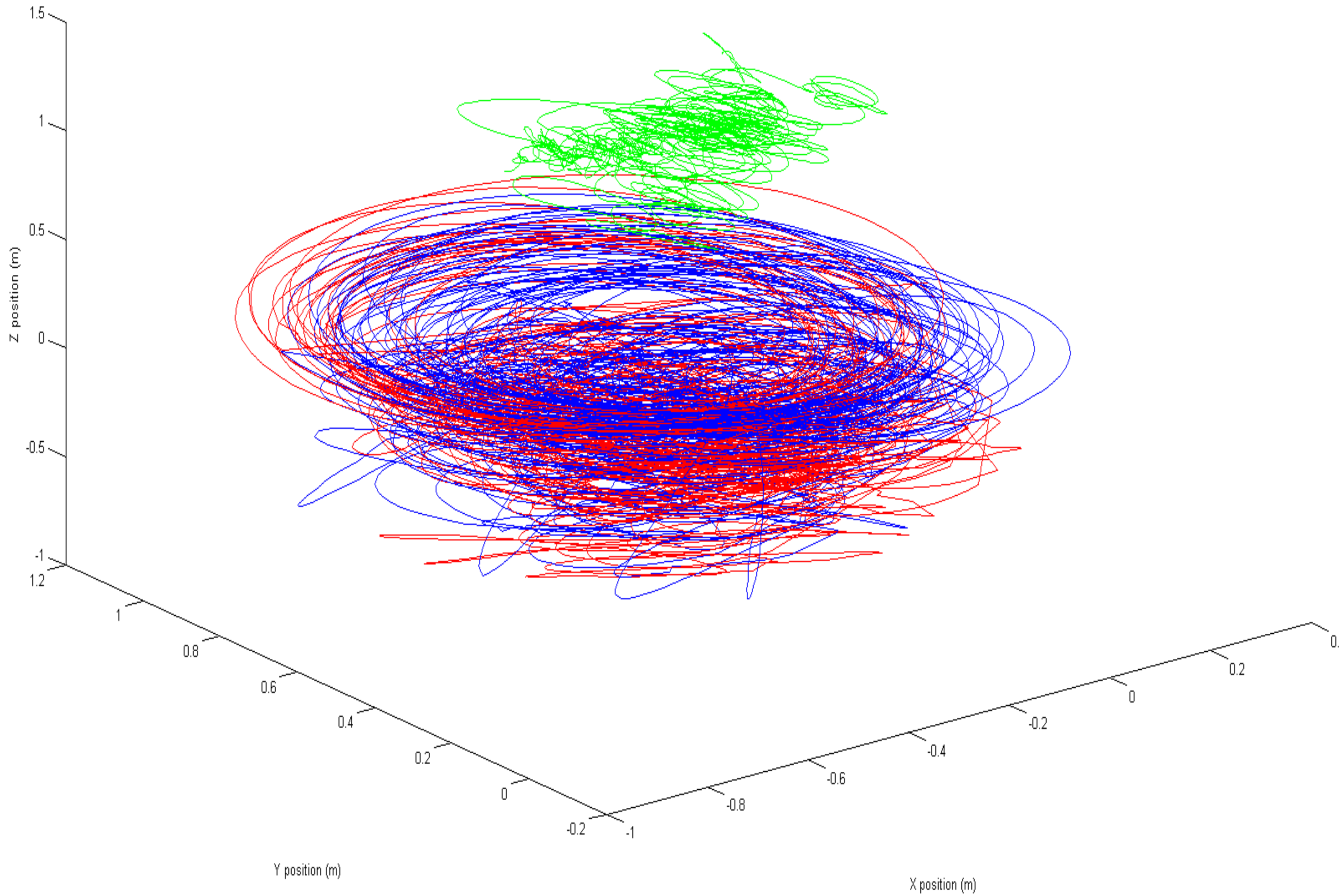
Robotic Forcing Video



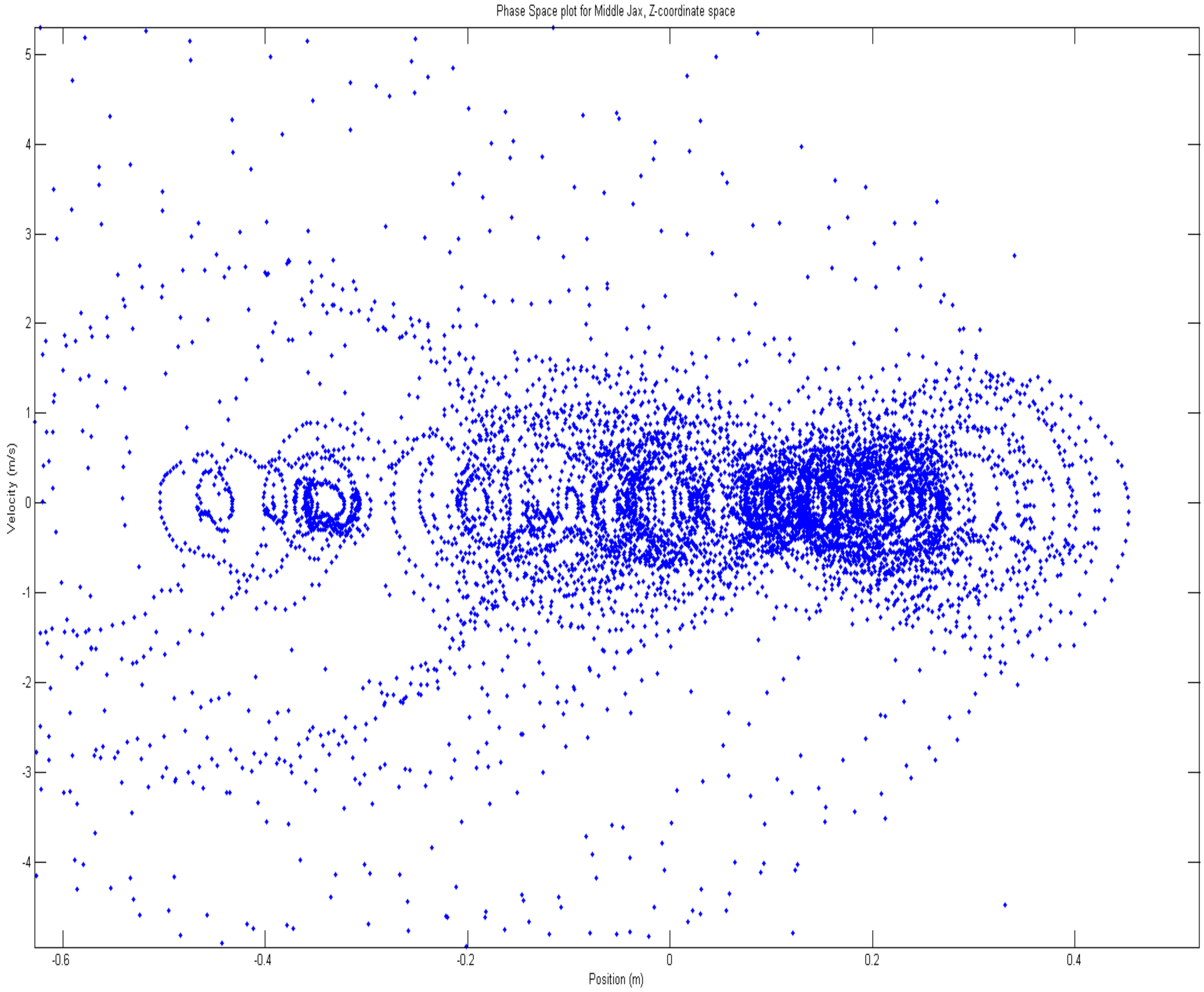
Results (Human)

- Andy was able to obtain orbits for all three forcings
- The starts were a little rough, but once he managed to get it going, there was little to no decay
- We discovered form looking at the OptiTrack footage of Andy's hand that to create a butterfly orbit, he had to move his hand in a figure eight pattern
- The vertical was the result of Andy moving his hand up and down
- The horizontal was the result of Andy moving his hand in an ellipsoid

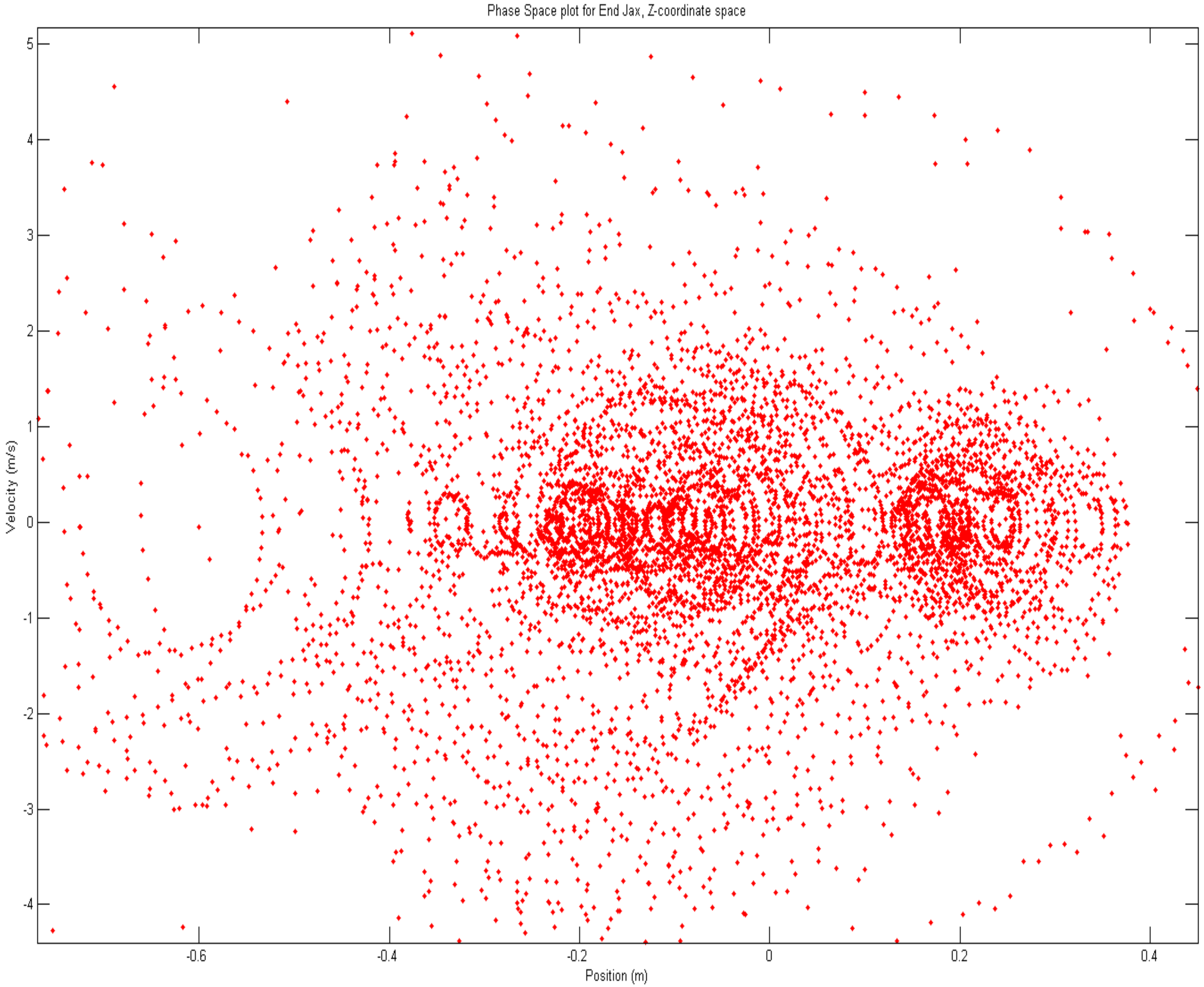
Take 1 Horizontal



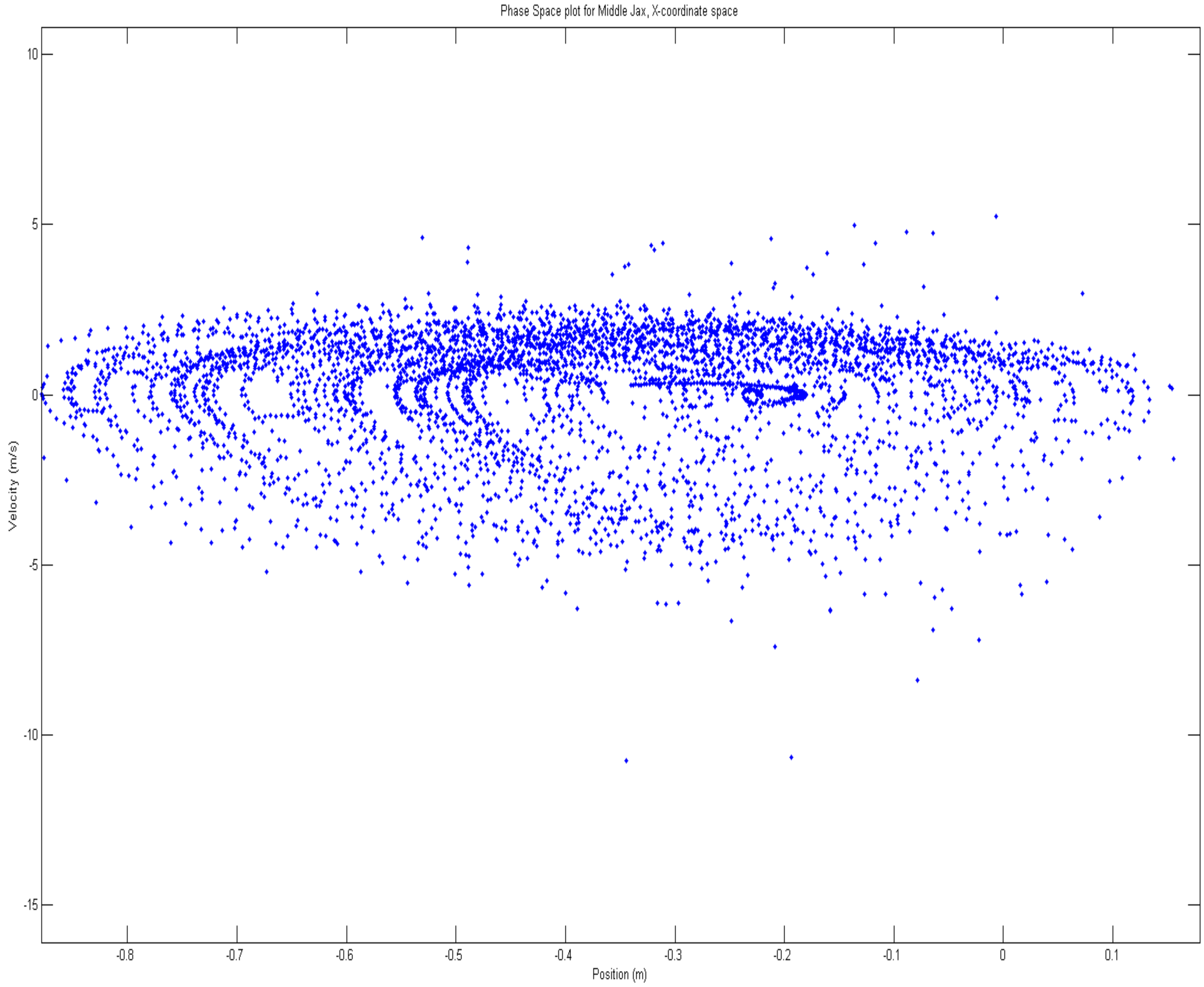
Take 1 Horizontal



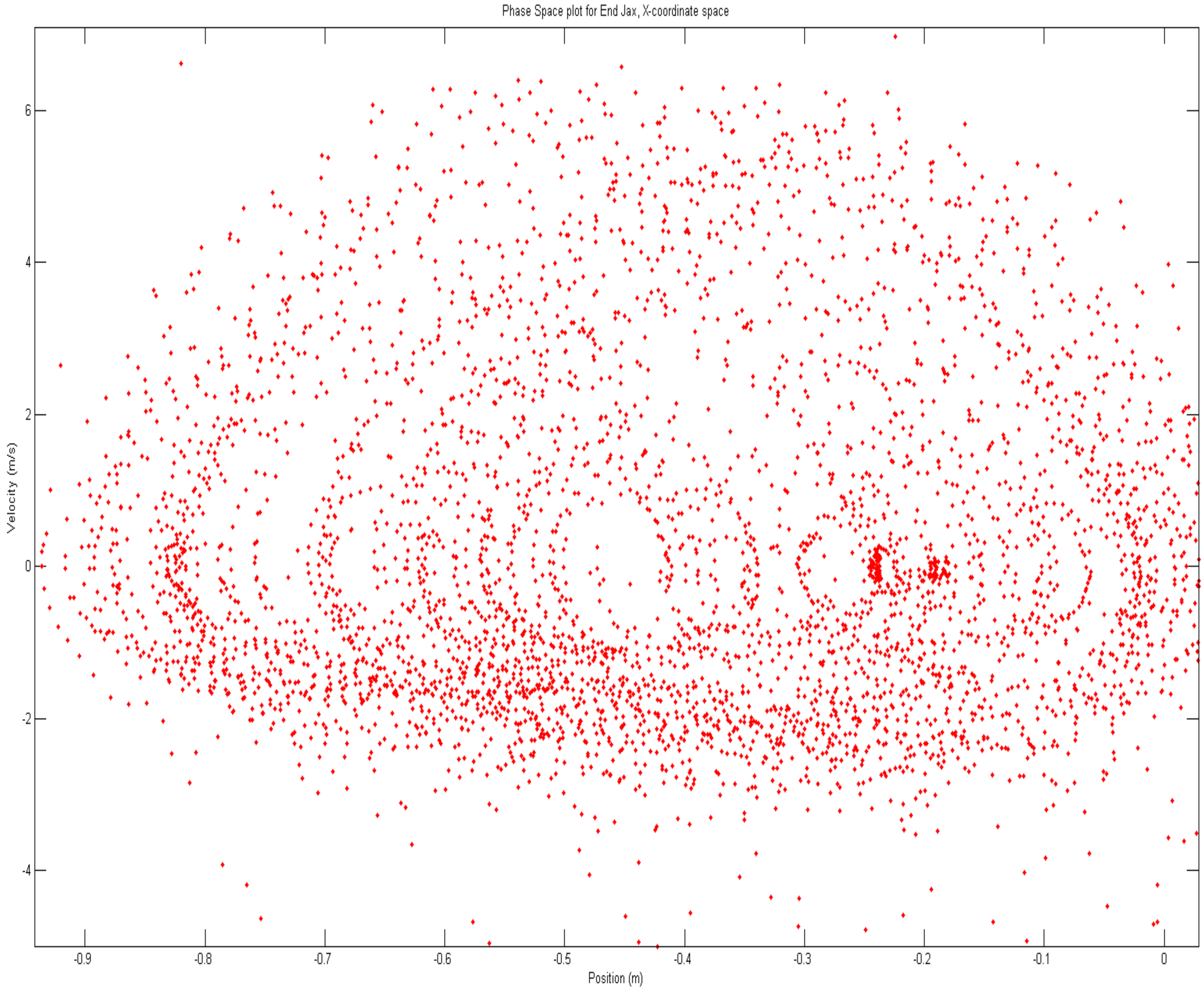
Take 1 Horizontal



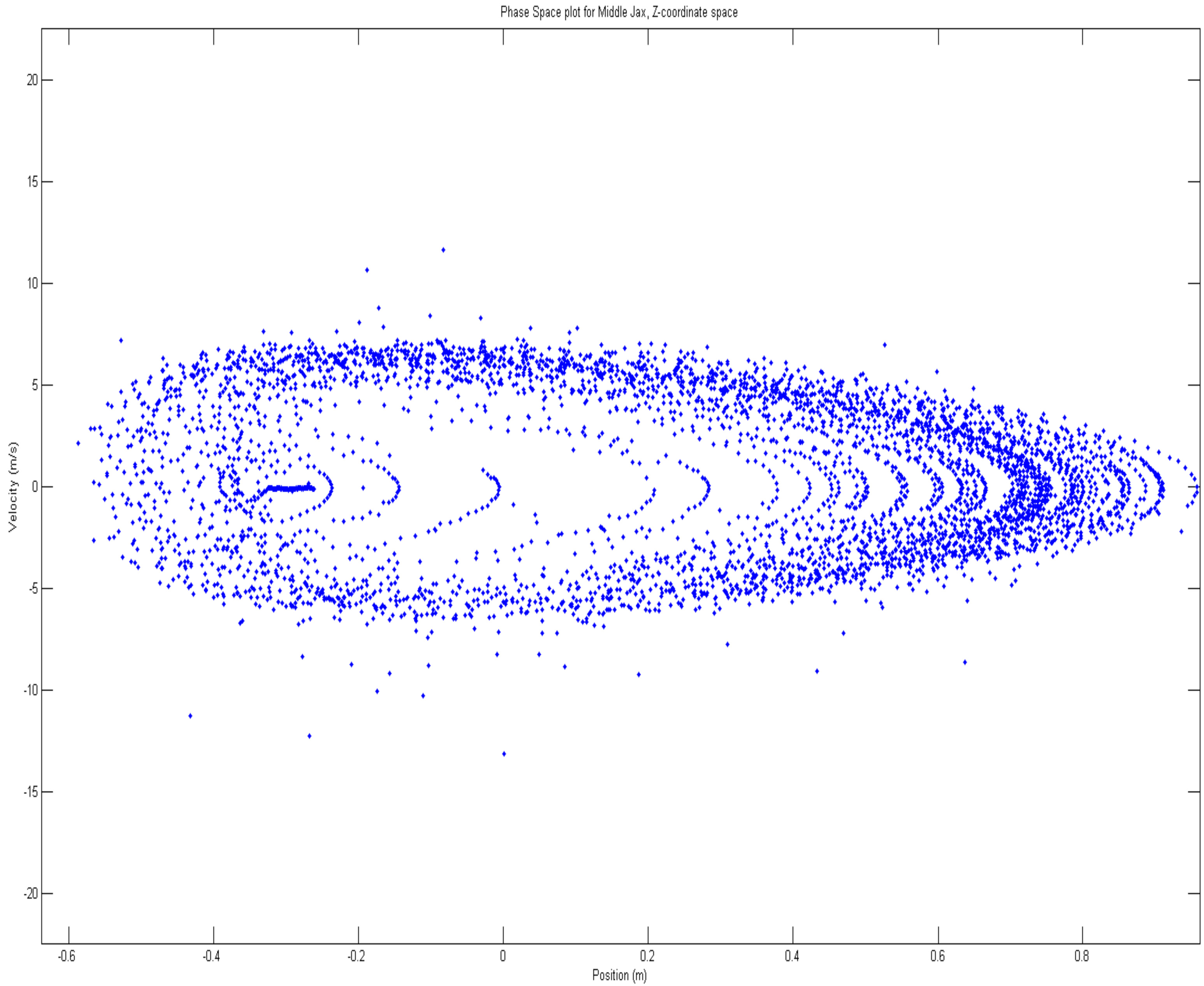
Take 1 Vertical



Take 1 Vertical



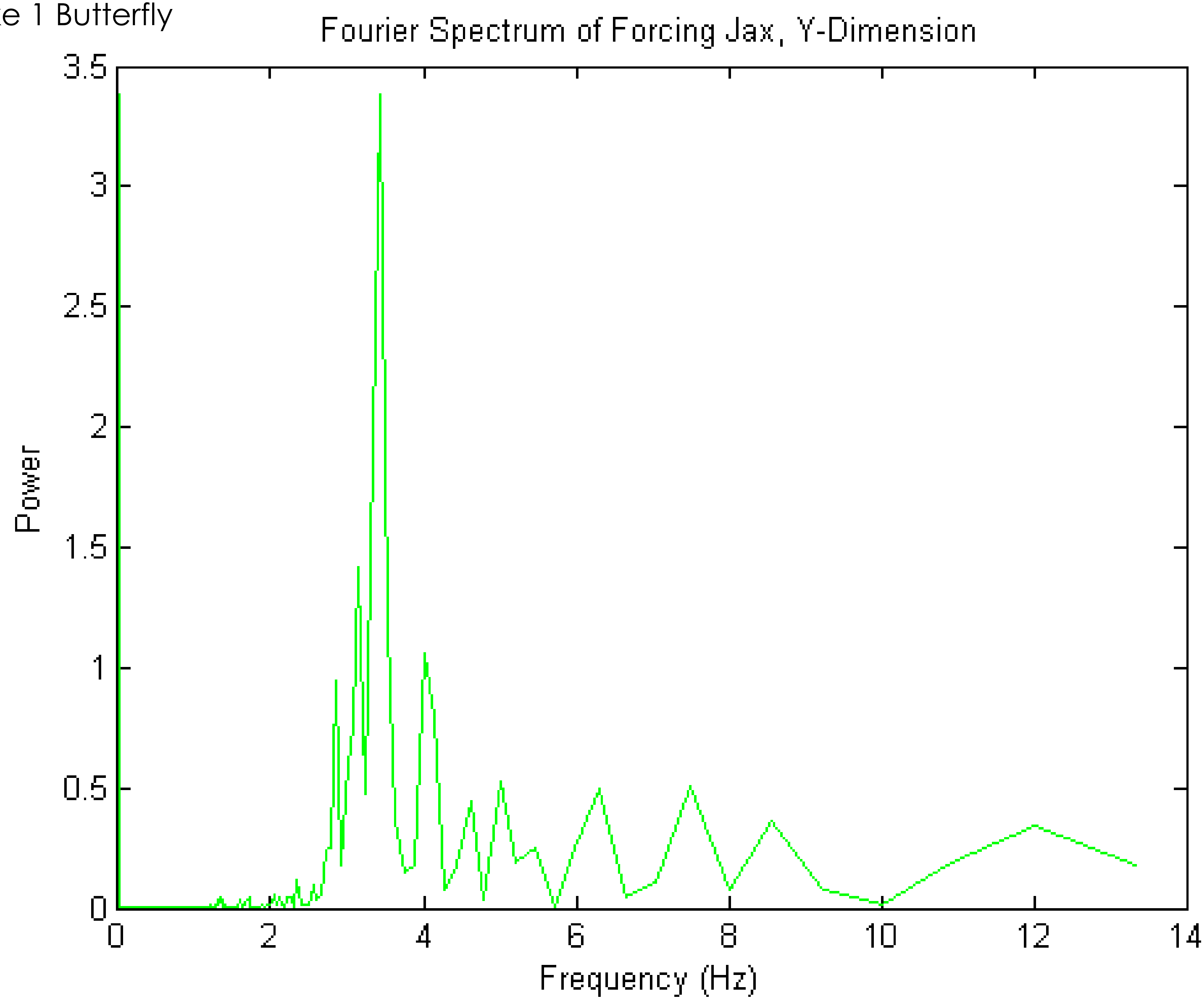
Take 1 Vertical



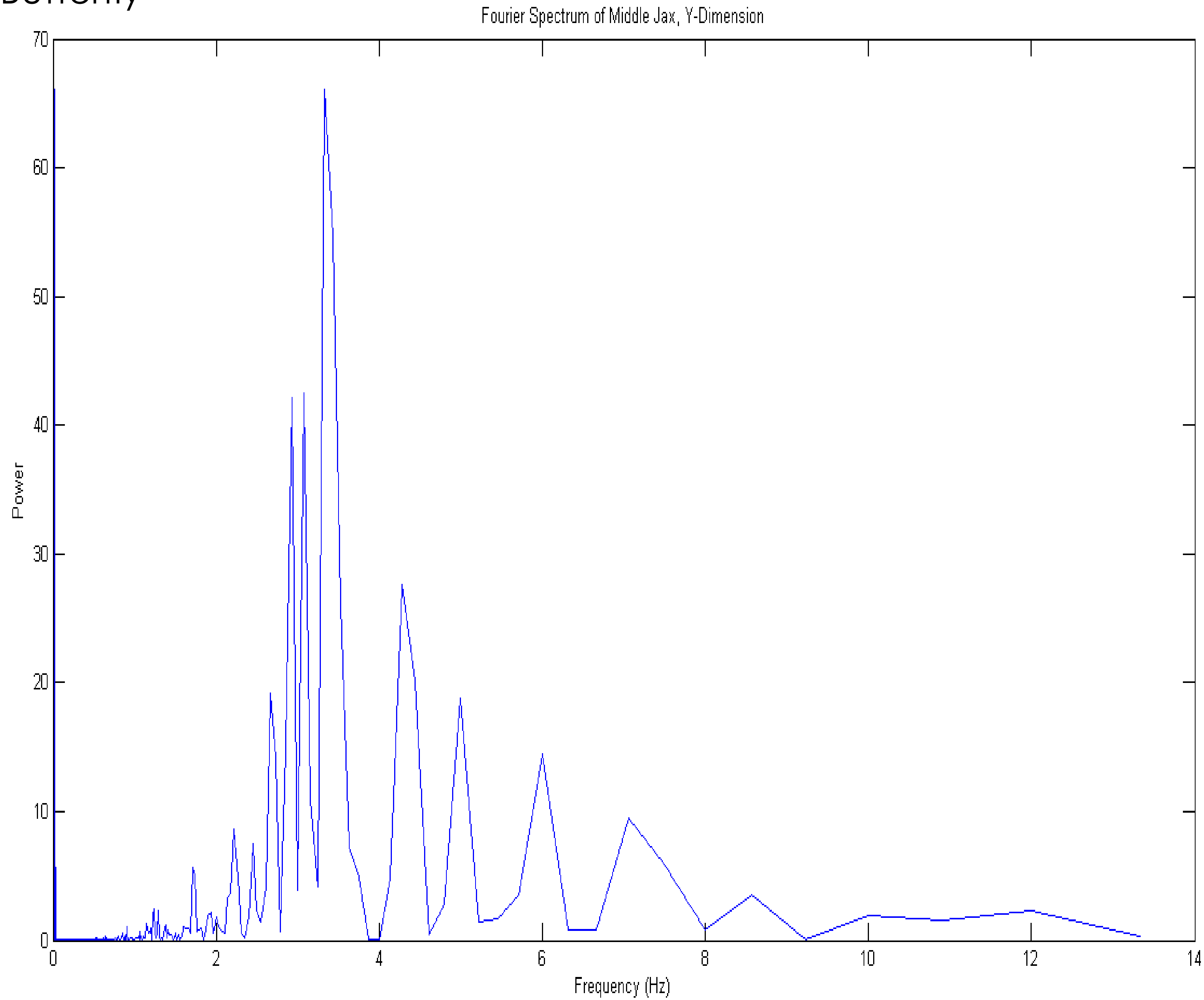
Fourier Spectrum Analysis

- By taking a discrete Fourier transform (DFT) of each Cartesian coordinate in our recorded trajectories, we can see the most dominant harmonic frequency along that coordinate for all tracked markers
- We wish to see what kind of frequencies tend to emerge in this chaotically oscillating system

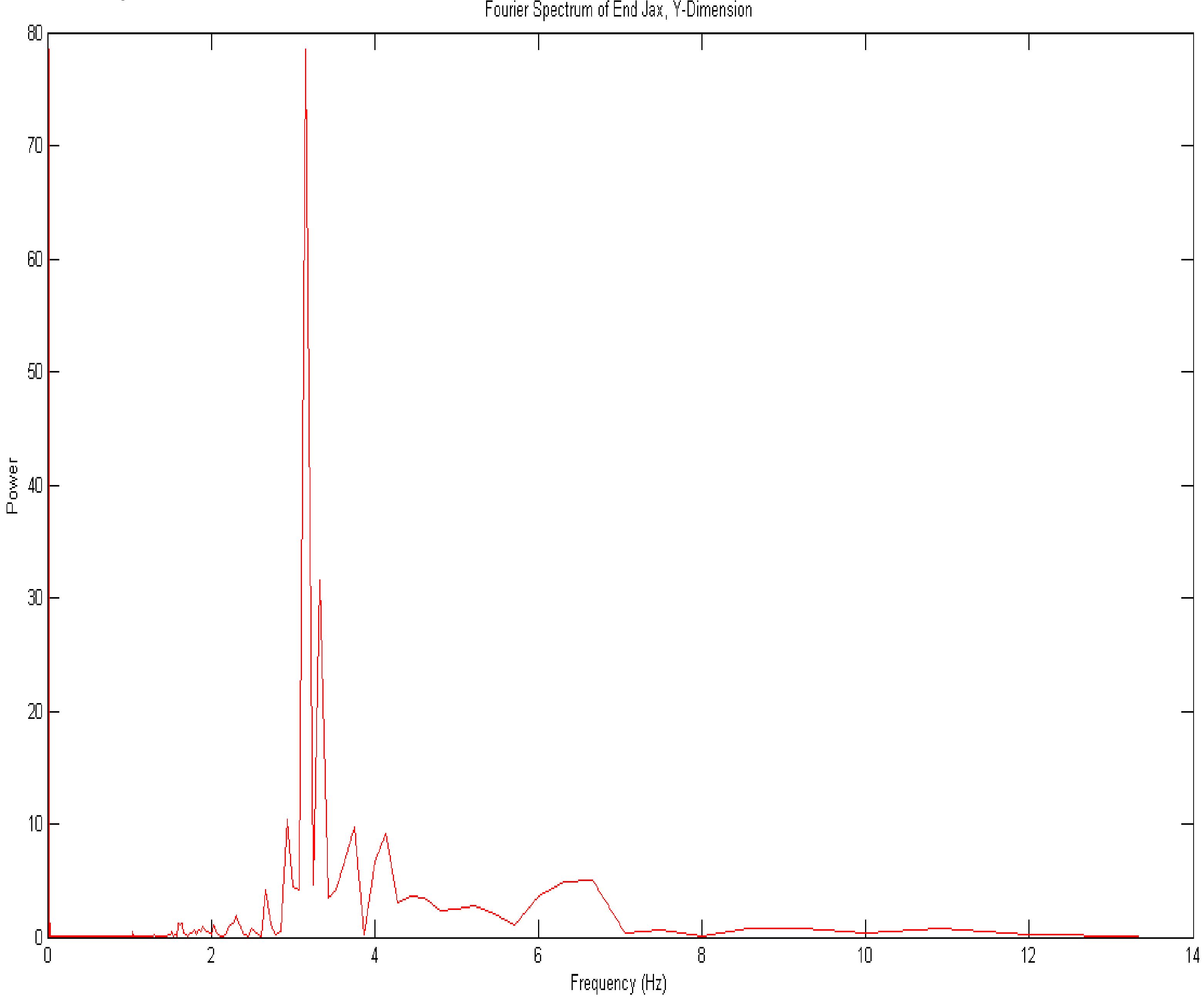
Take 1 Butterfly



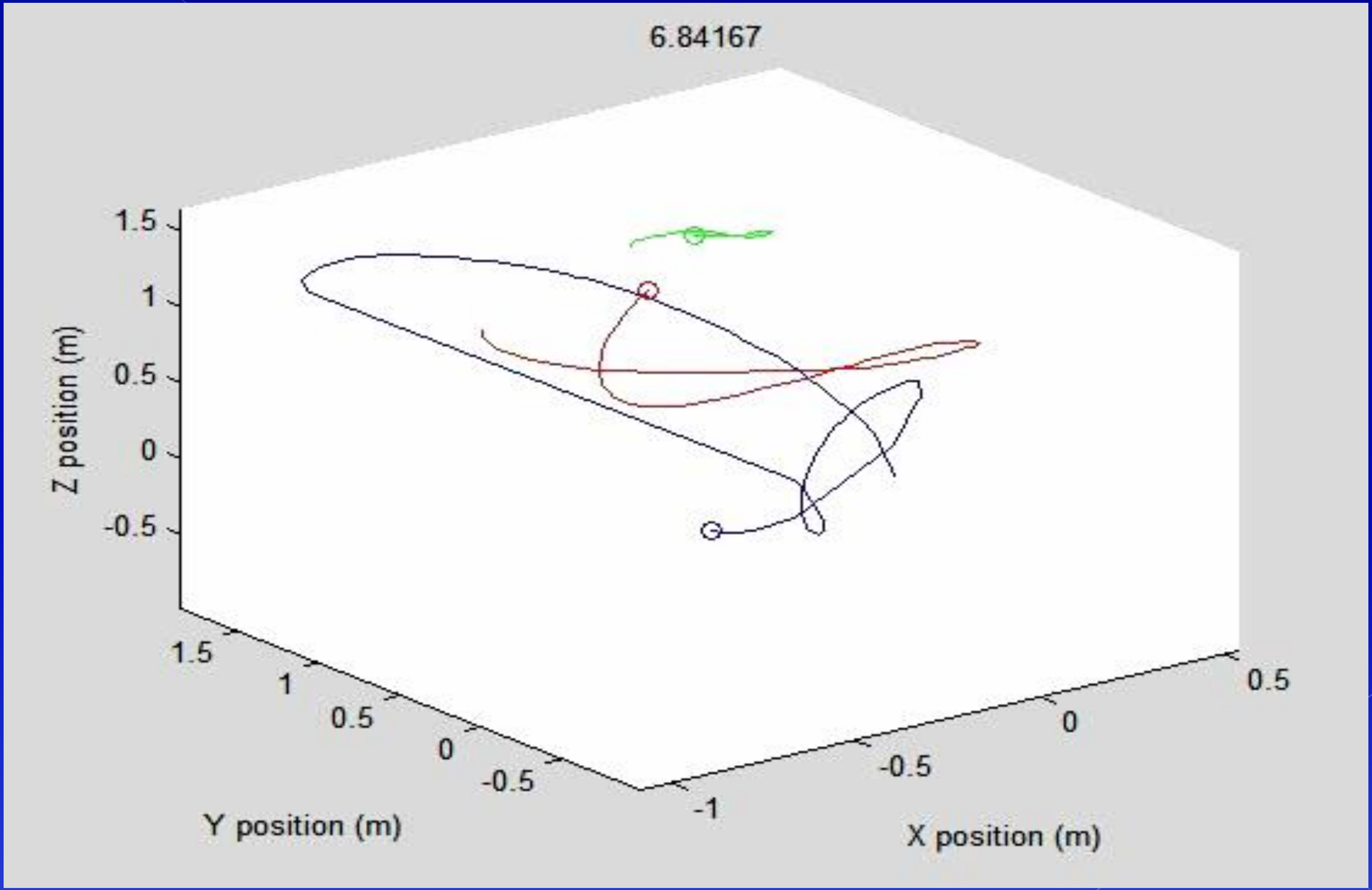
Take 1 Butterfly



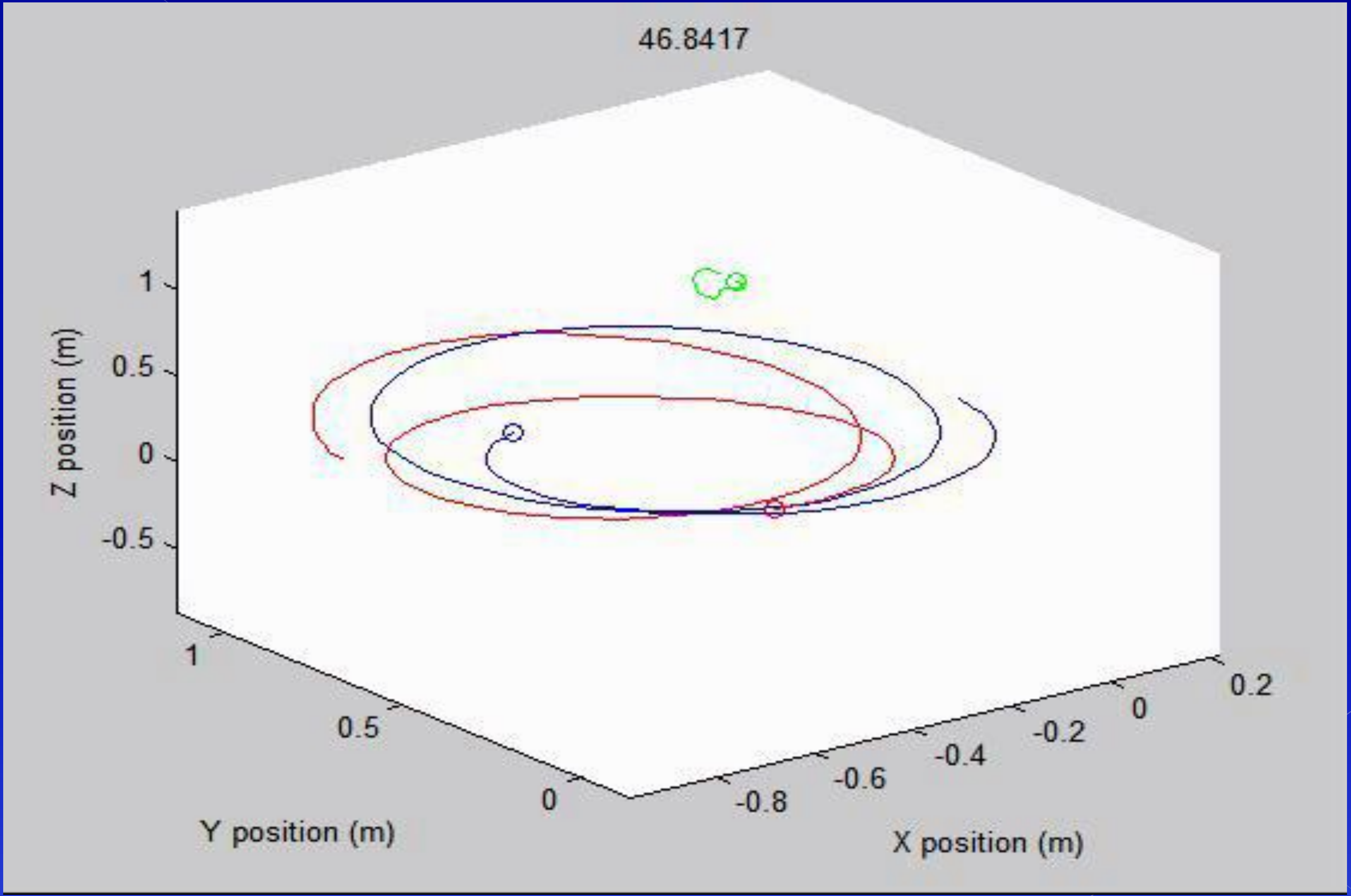
Take 1 Butterfly



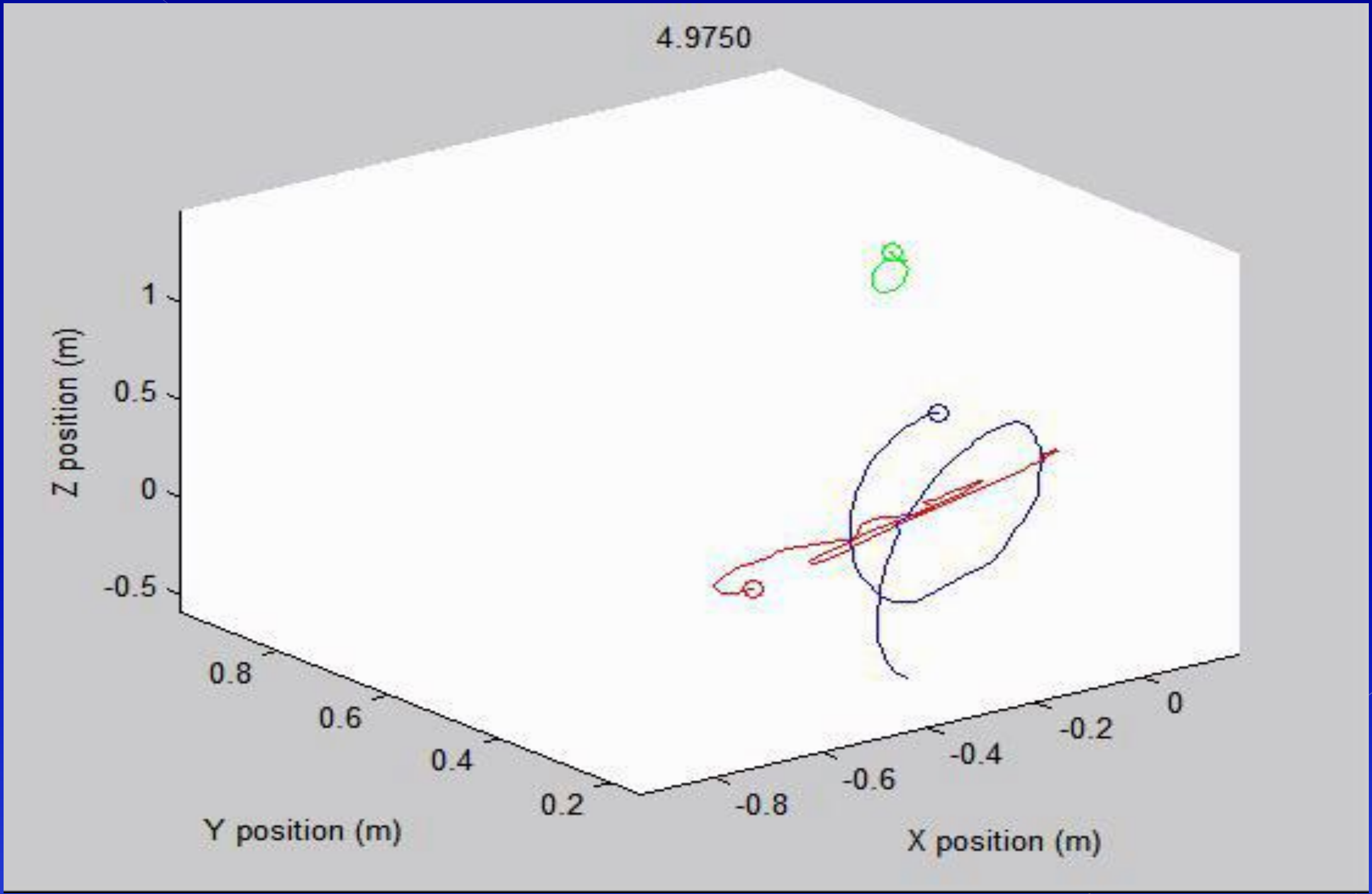
Butterfly Human Forcing



Horizontal Human Forcing



Vertical Human Forcing



Current Conclusions

- Based on our human forcing results, it appears that biofeedback is essential in generating and maintaining stable orbits
- The robot arm is not sufficiently adaptive to create stable orbits with the Astrojax
- There may very well be no stable orbits for precise, periodic forcing

Current Conclusions (contd.)

- From the Poincare sections of robotic forcing, we see emerging recurrence paths of the Astrojax that appear to decay or diverge from their respective attractors
- From the phase space plots, we can see concentric rings in all three dimensions that represent correlations between velocity and position while in an orbit
- From the Fourier spectrum we can easily see the most dominant harmonic frequency of the different bobs

References and Acknowledgments

- Du Toit, Philip 2005. *The Astrojax Pendulum and the N-Body Problem on the Sphere: A study in reduction, variational integration, and pattern evocation*

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