Chaotic Dripping Faucet

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Outline

- Project Description
- Theoretical Background
- Experimental Procedure
- Primary Data
- Model Comparison
- Error Analysis
- Summary

Fig 1: Droplet break off
Project Description

- Route to chaos in droplet formation
- Bifurcation of period as function of flow rate
- Model the water droplet as a harmonic oscillator
Previous Work

- Originated by Shaw, UCSC\(^1\)
- Also investigated by Kiyono and Fuchikami\(^2\), Coullet et al.\(^6\), and others

Models
- Damped harmonic oscillator
- Fluid/hydro-dynamical models
Harmonic Oscillator

Fig 2: Diagram of harmonic oscillator
Droplet Models

- Shaw’s mass-spring model\(^1\): 
  \[
  \frac{d(mv)}{dt} = mg - ky - bv \\
  \frac{dm}{dt} = \text{flowrate} \\
  v = \frac{dy}{dt}
  \]

- Hydrodynamical model\(^2\)

- K&F’s improved mass-spring\(^2\):
  \[
  m \frac{d^2z}{dt^2} + \left( \frac{dz}{dt} - v_0 \right) \frac{dm}{dt} = -kz - \gamma \frac{dz}{dt} + mg, \\
  k(m) = \begin{cases} 
  -11.4m + 52.5 & (m < 4.61) \\
  0 & (m \geq 4.61)
  \end{cases} \\
  m_r = 0.2m + 0.3, \quad \text{when} \quad z = z_{\text{crit}}, \\
  z = z_0, \quad \dot{z} = 0, \quad \text{when} \quad z = z_{\text{crit}} \]

\[Q = \pi a^2 v_0.\]
Model Analysis

- Linearly increasing mass
- Varying spring constant
- Droplet break off
Model predicted data
Proposed Experimental Procedure

- Feeder tank fills the reservoir tank
- A stopcock controls the flow rate from the reservoir tank
- A laser and photodiode detect falling drops
- The signal is read by an Analog to Digital Converter
- Period of falling drops measured from data
- A high speed camera used to visualize the falling drops

Fig 3: Proposed experimental setup
Initial Attempt

- 3/32” flexible tube
- Bucket with drilled holes

Problems:
- Drops not falling straight through laser
- Difficult to regulate flow rate
- Difficult to measure flow rate

Fig 4a: Initial experimental setup
Fig 4b: Photodiode and laser
Attempt 2

- Syringe pump used to dispense fluid at a specified rate

Problems:
  - Pump possessed undesired cycling
  - Restricted to using small nozzles
  - Limited syringe volume

Fig 5: Syringe pump
Final Setup

1. Photodiode
2. Laser
3. Reservoir
4. Large diameter flexible tubing
5. Flow regulator

Fig 6a: Final experimental setup

Fig 6b: Final experimental setup
Flow Rate Regulation

- Adjusted flow rate on the pump
- Using linear regression, found conversion factor between FRU and SI units

\[
\left[ \frac{mL}{s} \right] = 0.004563 [\text{FRU}]
\]

Fig 7: Flow regulator controls
Data Collection

- NI Analog to Digital Converter
- LabVIEW
  - Used VI made by Nick Gravish

Fig 8: LabVIEW Virtual Instrument
Fig 9: NI ADC
Error Analysis: Nozzle Diameter

- Lateral movement of droplets caused errors in measurement
- Small nozzles magnify imperfections
- After some testing, a larger nozzle diameter produced better results

Fig 10: Sensitivity to nozzle diameter
Error Analysis: Satellite Drops

- Missing drops leads to incorrect period measurements
- Satellite drop counting
  - First incorrectly identified satellite drops as the double-period, 4-period, etc data
  - Corrected to skip satellite drops
Error Analysis: Debouncing

- Double counting top and bottom of drop
- Corrected with measurement refractory period
Data Processing Summary

- MATLAB used for post processing
- Set threshold to eliminate satellite drops
- Changed peak counting method to eliminate double peak
Data Processing Example

Before

After

Poincare Map for 0.210 mL/s

Poincare Map for 0.210 mL/s
Bifurcation Diagram

- Period Doubling
- Chaos
- Periodic Windows

Increasing Flow Rate
Secondary Data – Period 1

$T_n$ vs. $T_{n+1}$ for 0.210 mL/s

$T_n$ vs. Drop for 0.210 mL/s
Secondary Data – Period 2

$T_n$ vs. $T_{n+1}$ for 0.319 mL/s

$T_n$ vs. Drop for 0.319 mL/s
Primary Data – Chaos

$T_n$ vs. Drop for 0.374 mL/s
Secondary Data – Chaotic Attractors

\( T_n \) vs. \( T_{n+1} \) for 0.374 mL/s

\( T_n \) vs. \( T_{n+1} \) for 0.486 mL/s

\( T_n \) vs. \( T_{n+1} \) for 0.579 mL/s
Secondary Data – Period 3

$T_n$ vs. $T_{n+1}$ for 0.365 mL/s

$T_n$ vs. Drop for 0.365 mL/s
Data Analysis and Comparisons

- Universality
  - Requires bifurcation progression as U-sequence
  - Period doubling, chaos, and periodic windows

- Though similar, this could suggest the chaotic faucet does not have a unimodal map (current research)\(^6\)
Qualitative Comparison – Single Period

Primary Data

Reference Data

Model Data

$T_n$ vs. Drop for 0.210 mL/s

$T_{n+1}$ vs. $T_n$ for 0.210 mL/s

Return map

1/12/2012
Qualitative Comparison – Two Period

Primary Data

Reference Data

Model Data

1/12/2012
Qualitative Comparison – Chaos

Primary Data

Reference Data

Model Data

Tₙ vs. Drop for 0.374 mL/s

Tₙ vs. Drop number

Tₙ vs. Tₙ+1 for 0.374 mL/s

Return map
Qualitative Comparison – Periodic Window

Primary Data

Reference Data

1/12/2012
PeriodDoublingRoute to Chaos

- Confirmed period doubling progression
  - Seen in bifurcation diagram
  - Predicted by references and simulation
- Drops from a 5mm nozzle as described by Dreyer, Hickey\textsuperscript{4}:

  “were seen to follow a bifurcation route to chaos producing period-1 and -2 attractors at lower drip rates and many beautiful examples of strange attractors for higher drip rates with a range of instability between the two regions”

= A Beautiful Attractor
Periodic Window Route to Chaos

- Confirmed transient chaos
  - Seen in bifurcation diagram
  - Predicted by references and simulation
Error Analysis Summary

- Debouncing $\rightarrow$ refractory period
- Satellite drop counting $\rightarrow$ threshold
- Missed drops dependent on nozzle diameter
- Pump vibrations and mode interactions
  - Visible to naked eye with syringe pump
  - Patterns form before period doubling, indicating external disturbance
  - Possibly mechanically induced
Error Analysis: Periodic Flow Rate

- Sinusoidal disturbance could cause fluctuation over the T-2/T-4/chaotic region
- Single-period circle

\[ T_n \text{ vs. } T_{n+1} \text{ for 0.210 mL/s} \]
Summary and Conclusions

- Two routes to chaos:
  - Period doubling (period-1 to period-2)
  - Transient/periodic windows

- Experimental setup
  - Accurate flow rate necessary
  - Uniform flow rate necessary
  - Buckets are hard to control flow rate and measure drops
  - Nozzle size is very important (error and dynamics)

- Data analysis
  - Satellite drops should not be included
  - Debouncing of double peaks

- Model matches data and literature qualitatively
Questions?
References


List of Figures

3. See reference 3
4. Photo credits Caleb Royer
5. Photo credits Nick Pritchard
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7. Photo credits Ricky Patel
8. Photo credits Josh Job
9. NI.com
11. Photo credits Josh Job
12. See reference 2