Chaotic Dripping Faucet

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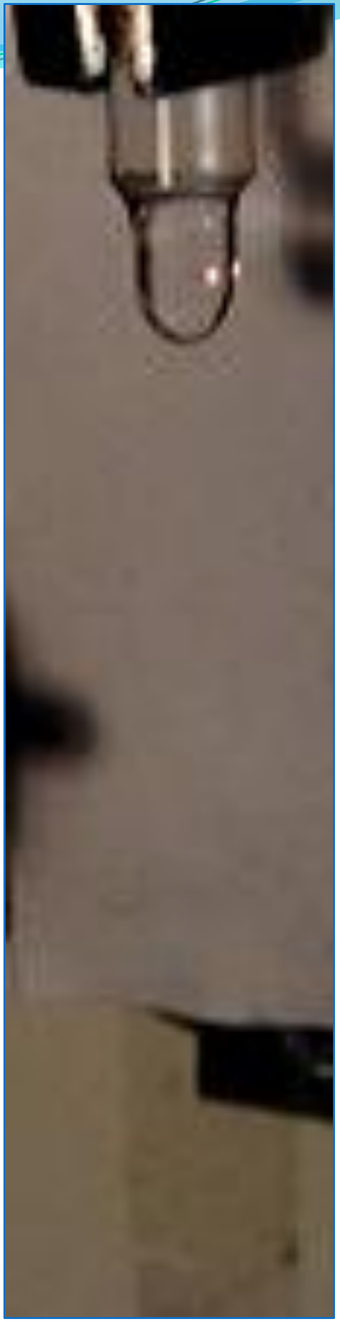
Outline

- Project Description
- Theoretical Background
- Experimental Procedure
- Primary Data
- Model Comparison
- Error Analysis
- Summary

Fig 1: Droplet break off

Project Description

- Route to chaos in droplet formation
- Bifurcation of period as function of flow rate
- Model the water droplet as a harmonic oscillator



Previous Work

- Originated by Shaw, UCSC[1]
- Also investigated by Kiyono and Fuchikami[2], Coullet et al.[6], and others
- Models
  - Damped harmonic oscillator
  - Fluid/hydro-dynamical models

Harmonic Oscillator

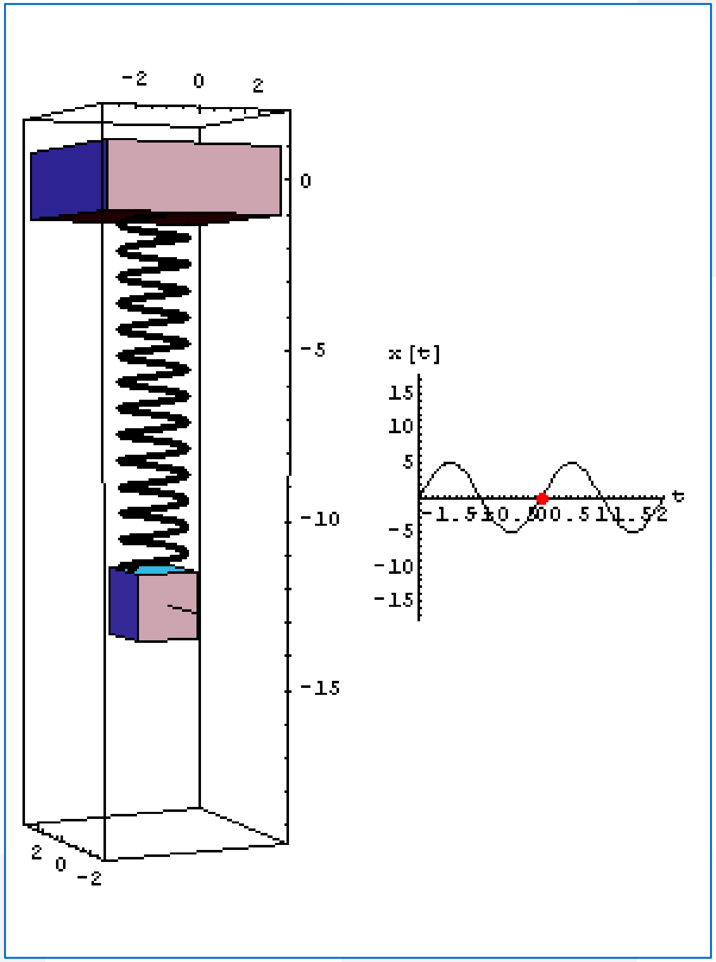
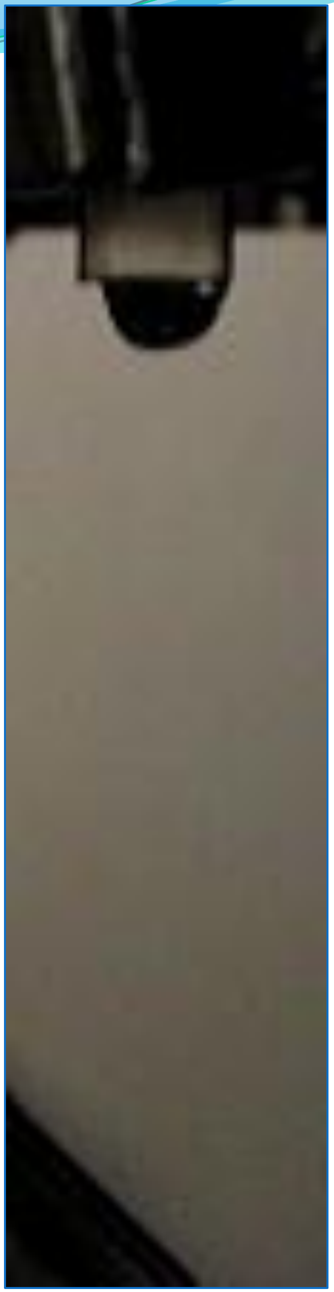


Fig 2: Diagram of harmonic oscillator



Droplet Models

- Shaw's mass-spring model[1]:

$$\frac{d(mv)}{dt} = mg - ky - bv$$

$$\frac{dm}{dt} = flowrate$$

$$v = \frac{dy}{dt}.$$

- Hydrodynamical model[2]
- K&F's improved mass-spring[2]:

$$m\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} + \left(\frac{\mathrm{d}z}{\mathrm{d}t} - v_0\right)\frac{\mathrm{d}m}{\mathrm{d}t} = -kz - \gamma\frac{\mathrm{d}z}{\mathrm{d}t} + mg,$$

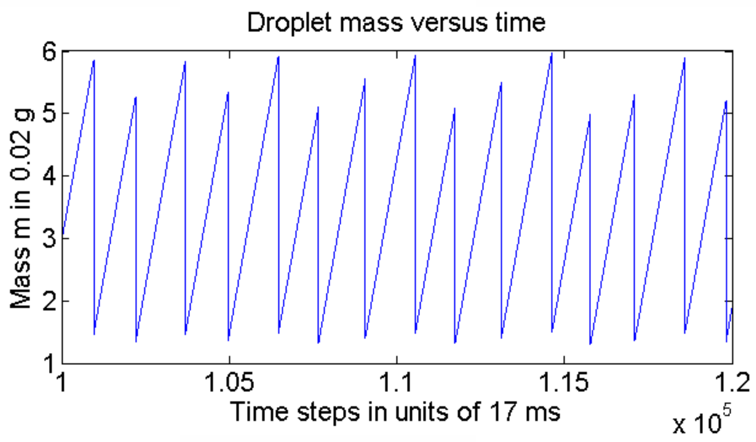
$$k\left(m\right) = \begin{cases} -11.4m + 52.5 & (m < 4.61) \\ 0 & (m \geq 4.61) \end{cases}.$$

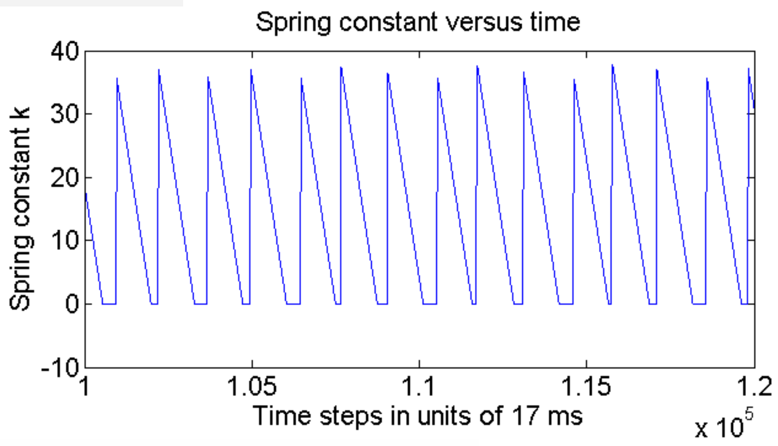
$$\frac{\mathrm{d}m}{\mathrm{d}t} = Q = \pi a^2 v_0.$$

$$m_{\mathrm{r}} = 0.2m + 0.3, \quad \text{when} \quad z = z_{\mathrm{crit}},$$

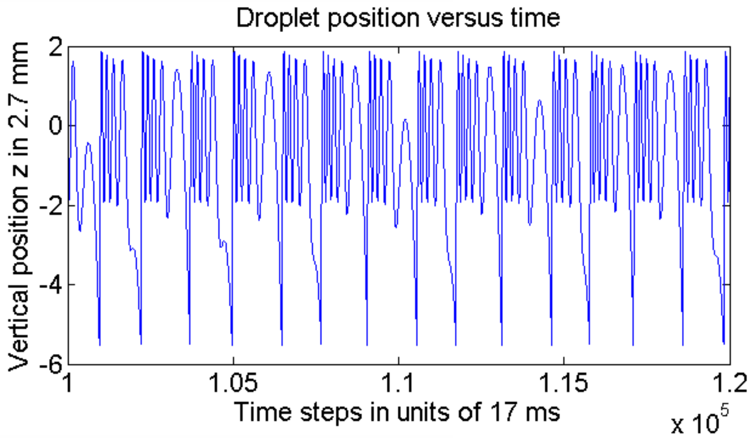
$$\left.\begin{array}{rcl} z & = & z_0, \\ \dot{z} & = & 0, \end{array}\right\} \quad \text{when} \quad z = z_{\mathrm{crit}},$$

Model Analysis

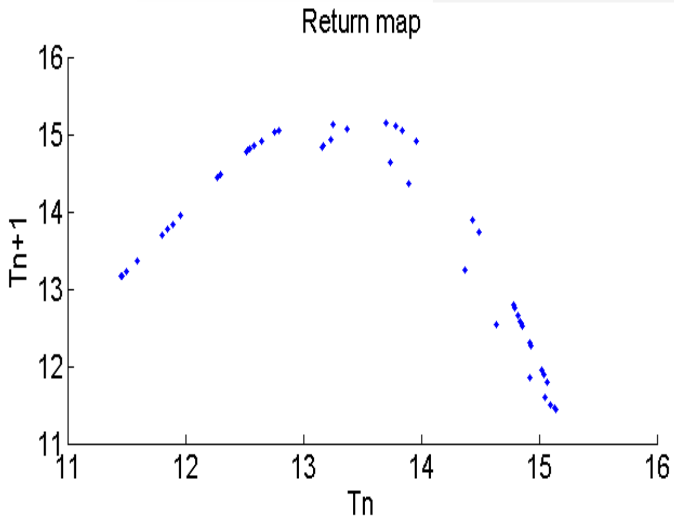


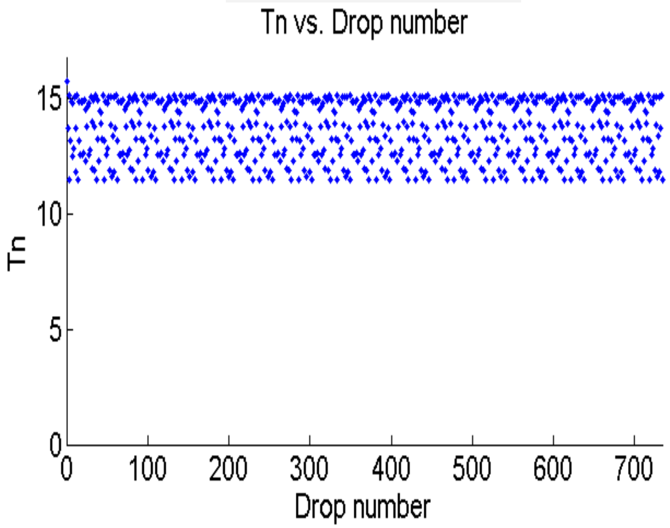


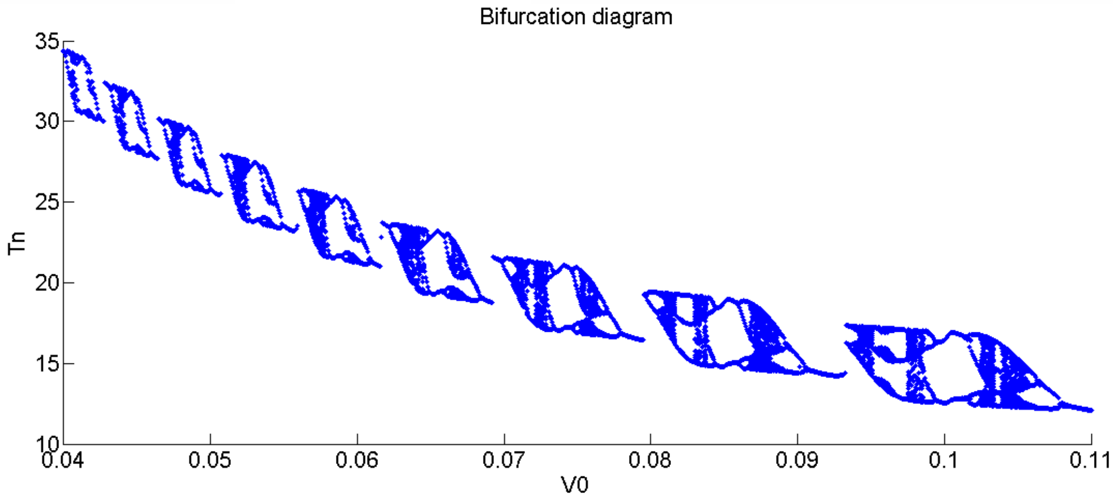
- Linearly increasing mass
- Varying spring constant
- Droplet break off



Model predicted data







Proposed Experimental Procedure

- Feeder tank fills the reservoir tank
- A stopcock controls the flow rate from the reservoir tank
- A laser and photodiode detect falling drops
- The signal is read by an Analog to Digital Converter
- Period of falling drops measured from data
- A high speed camera used to visualize the falling drops

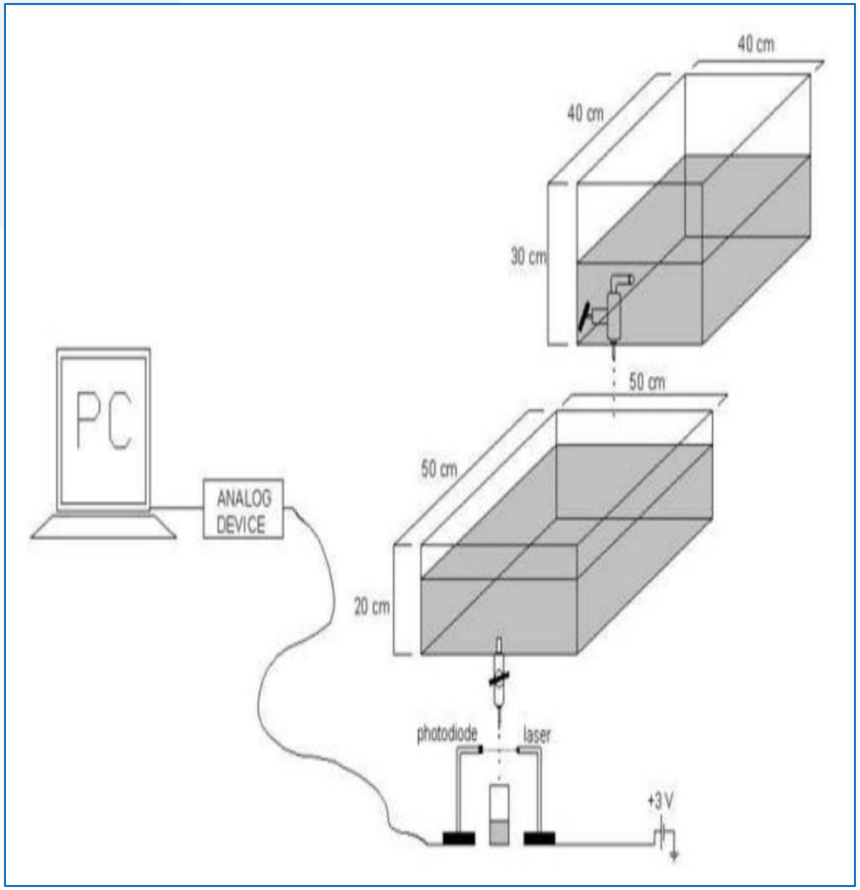


Fig 3: Proposed experimental setup

Initial Attempt

- 3/32” flexible tube
- Bucket with drilled holes

- Problems:
  - Drops not falling straight through laser
  - Difficult to regulate flow rate
  - Difficult to measure flow rate

Fig 4a: Initial experimental setup

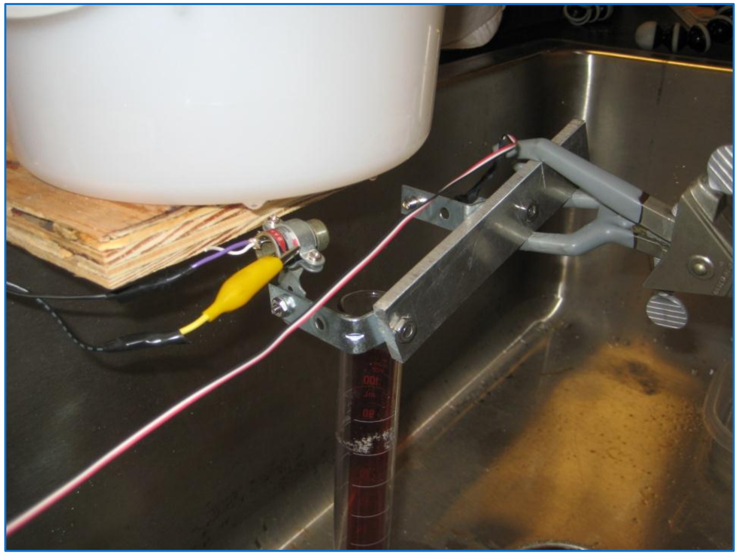


Fig 4b: Photodiode and laser

Attempt 2

- Syringe pump used to dispense fluid at a specified rate
- Problems:
  - Pump possessed undesired cycling
  - Restricted to using small nozzles
  - Limited syringe volume

Fig 5: Syringe pump

Final Setup

1. Photodiode
2. Laser
3. Reservoir
4. Large diameter flexible tubing
5. Flow regulator

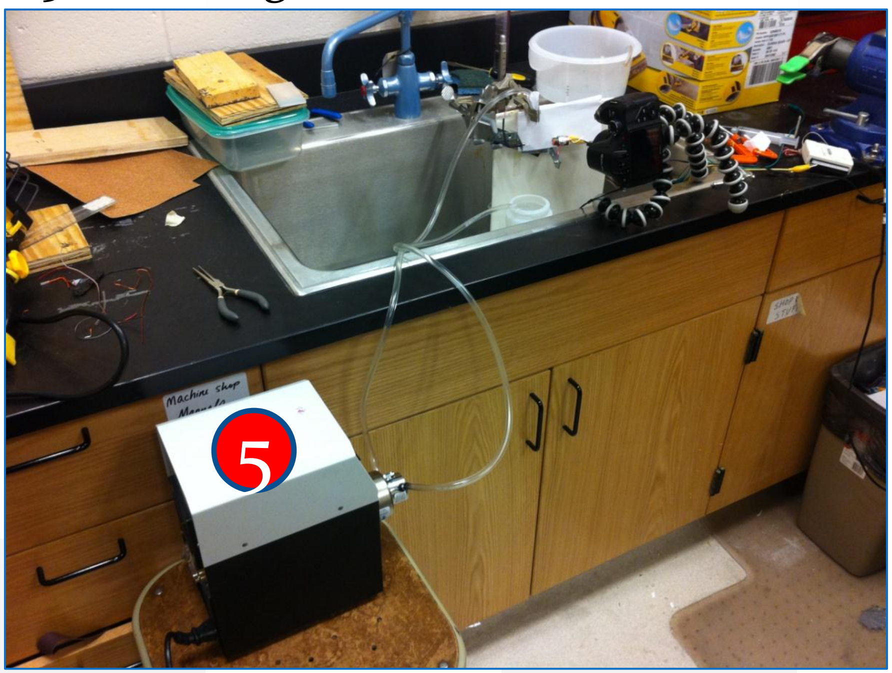


Fig 6a: Final experimental setup

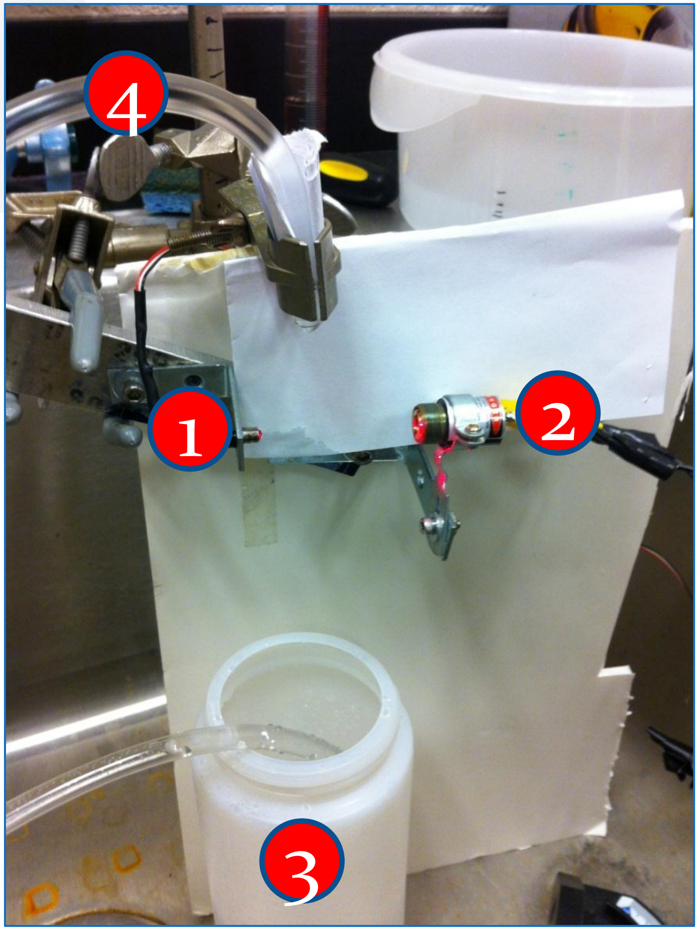


Fig 6b: Final experimental setup

Flow Rate Regulation

- Adjusted flow rate on the pump
- Using linear regression, found conversion factor between FRU and SI units

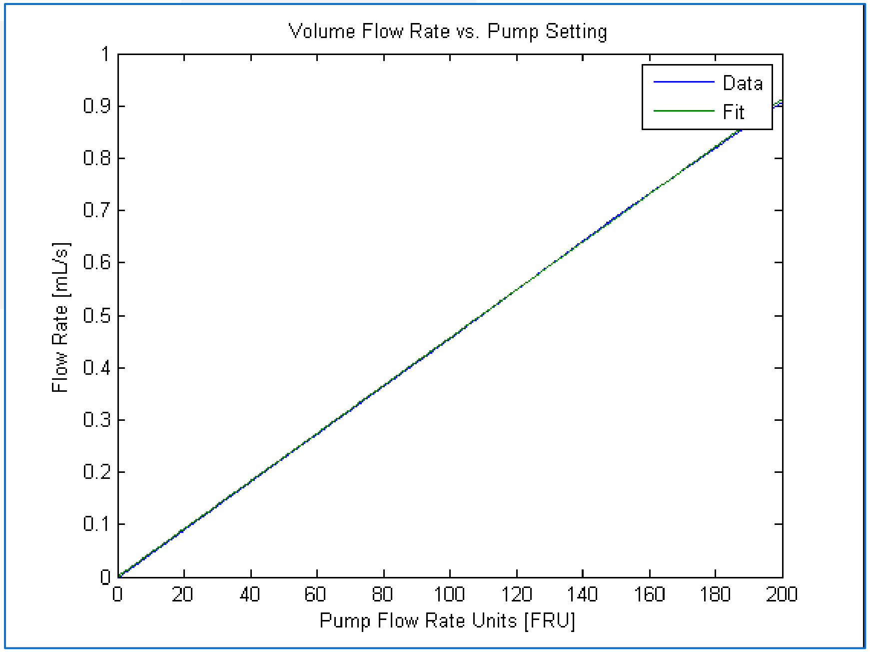


Fig 7: Flow regulator controls

$$\left[\frac{mL}{s}\right] = 0.004563\left[FRU\right]$$

Data Collection

- NI Analog to Digital Converter
- LabVIEW
  - Used VI made by Nick Gravish

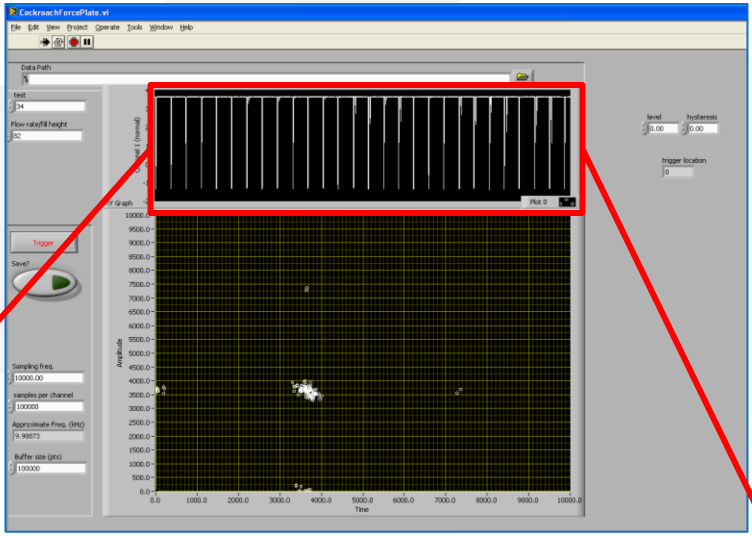


Fig 8: LabVIEW Virtual Instrument

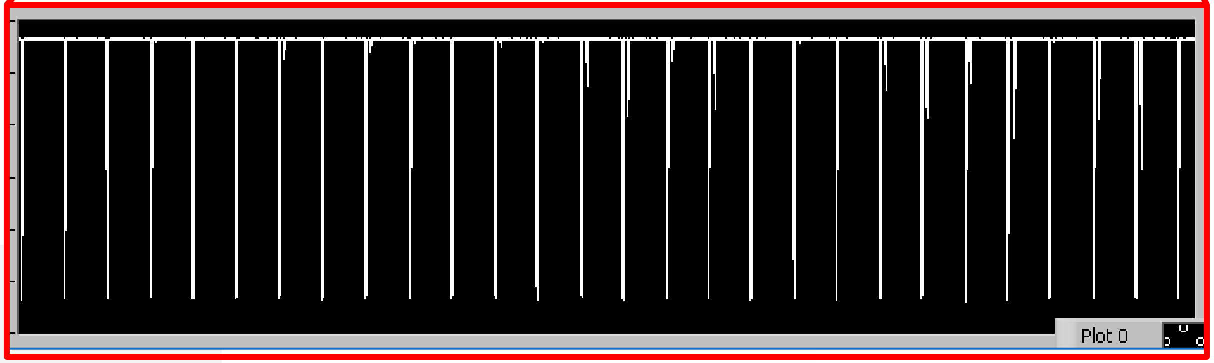


Fig 9: NI ADC

Error Analysis: Nozzle Diameter

- Lateral movement of droplets caused errors in measurement
- Small nozzles magnify imperfections
- After some testing, a larger nozzle diameter produced better results

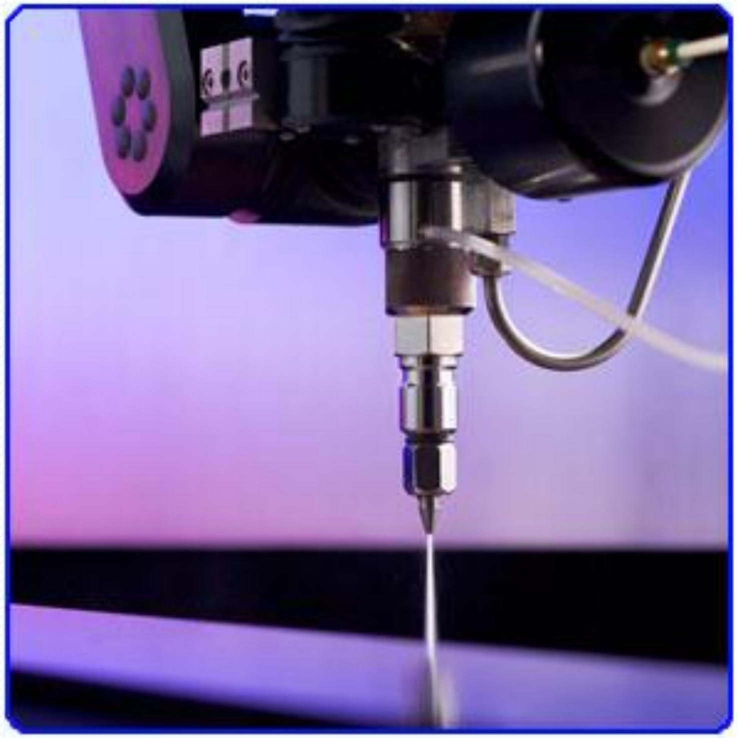
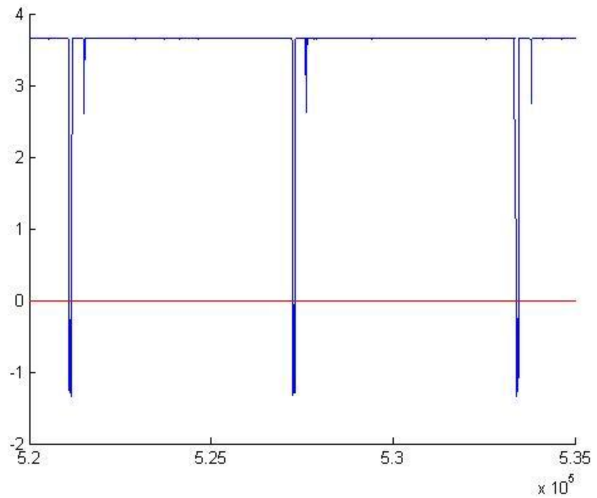
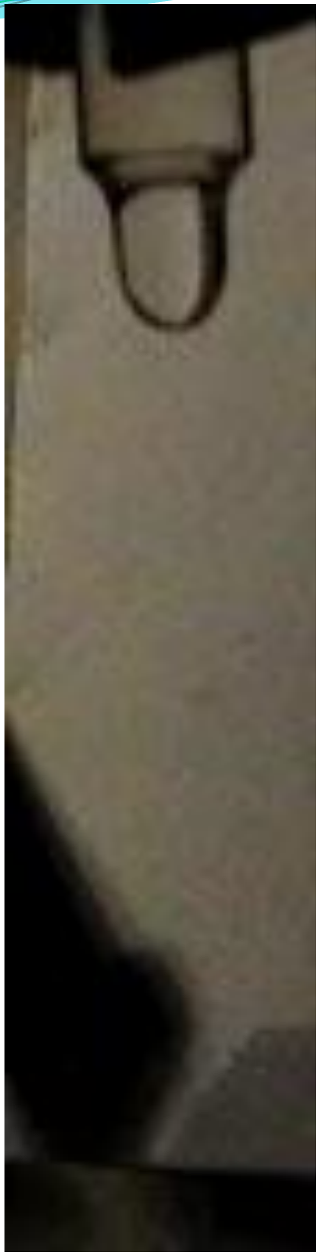


Fig 10: Sensitivity to nozzle diameter

Error Analysis: Satellite Drops

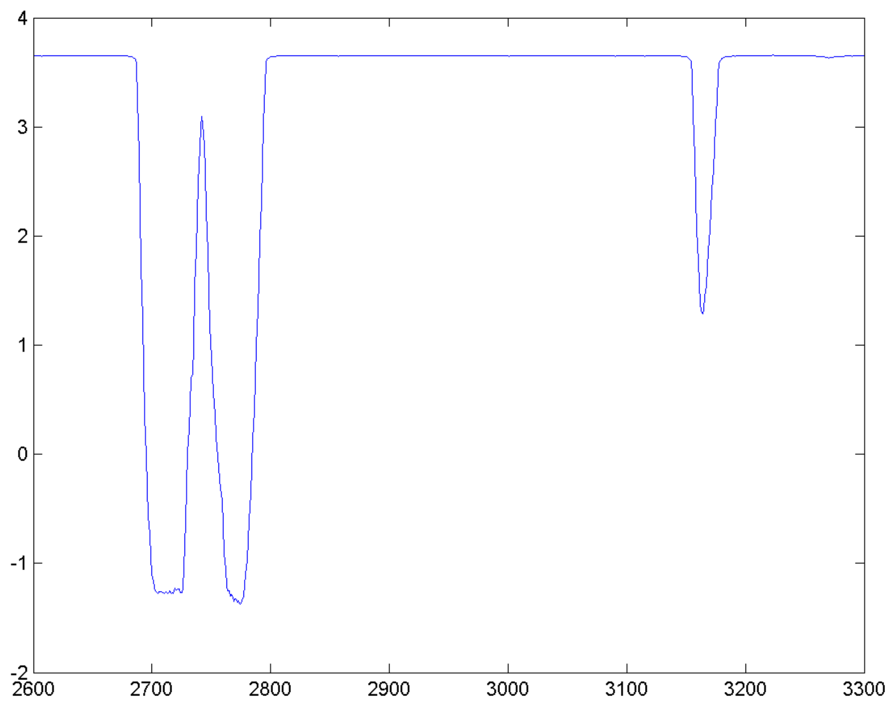
- Missing drops leads to incorrect period measurements
- Satellite drop counting
  - First incorrectly identified satellite drops as the double-period, 4-period, etc data
  - Corrected to skip satellite drops





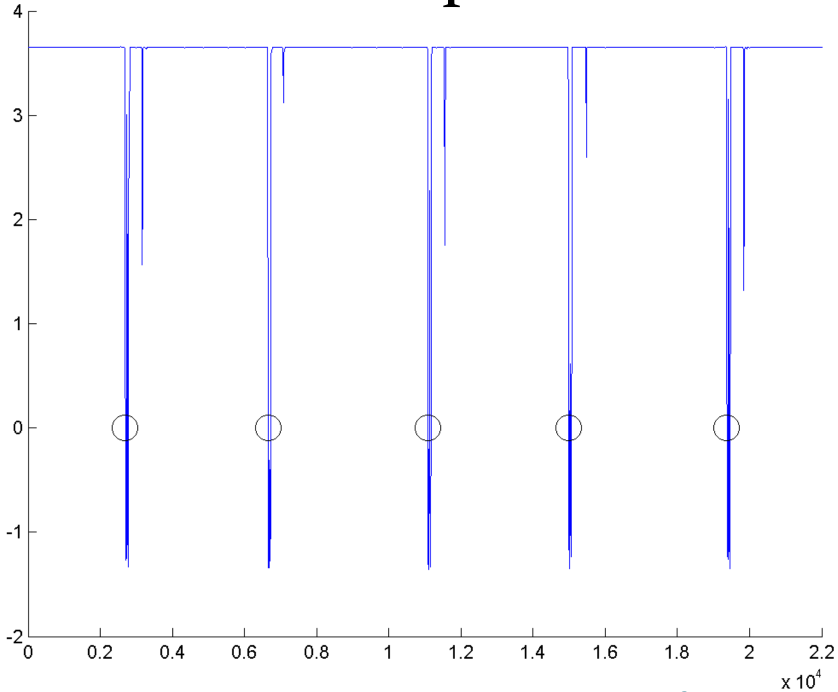
Error Analysis: Debouncing

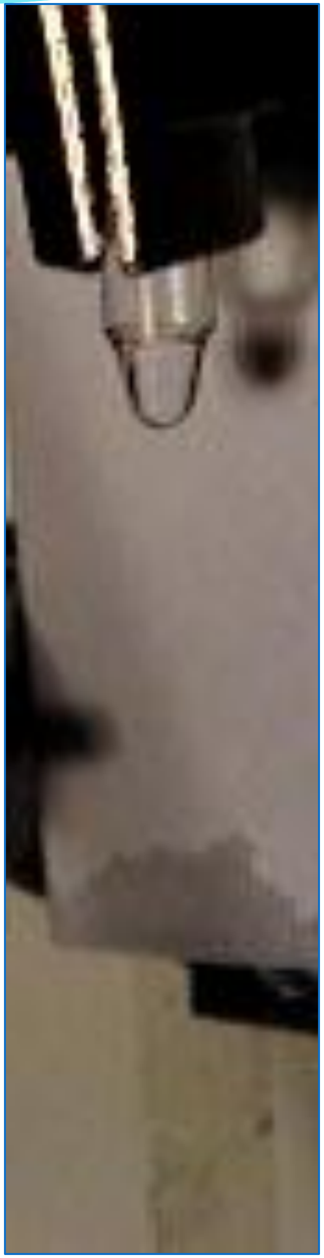
- Double counting top and bottom of drop
- Corrected with measurement refractory period



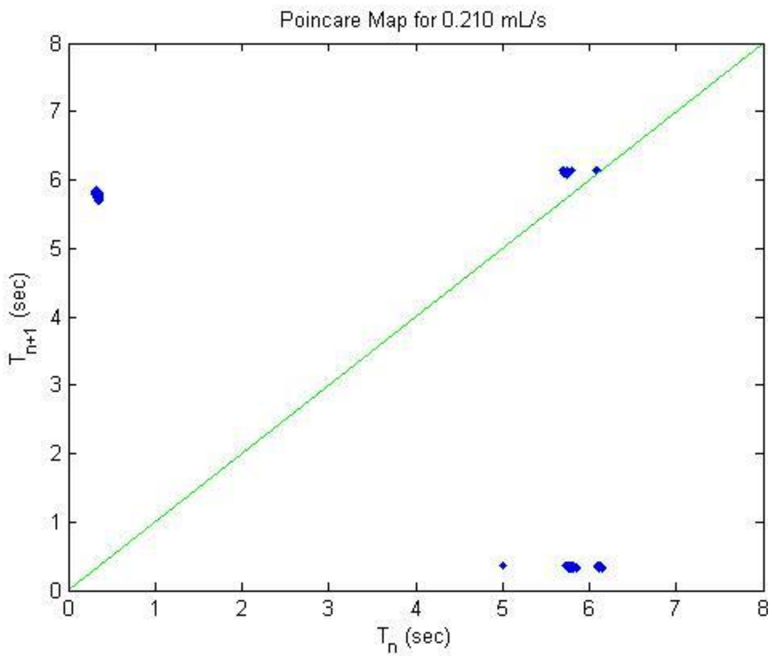
Data Processing Summary

- MATLAB used for post processing
- Set threshold to eliminate satellite drops
- Changed peak counting method to eliminate double peak

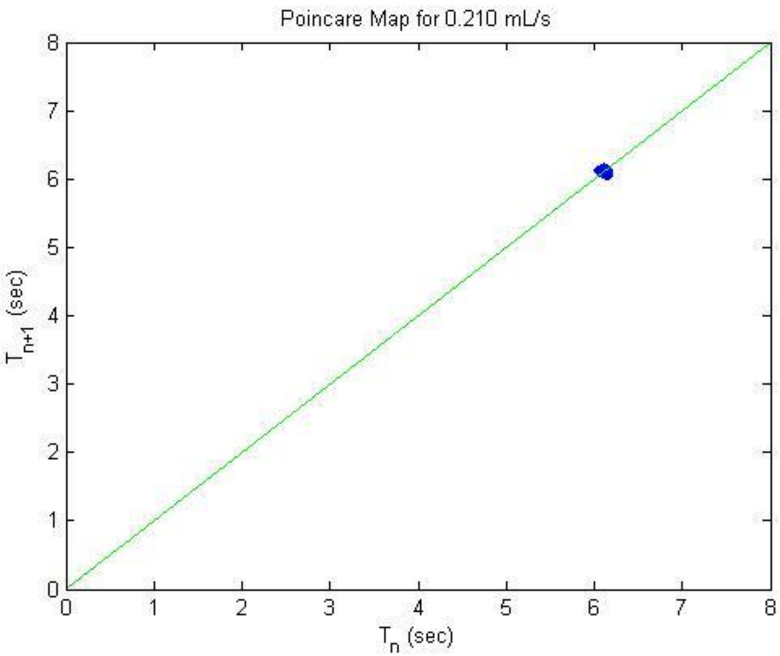




Data Processing Example



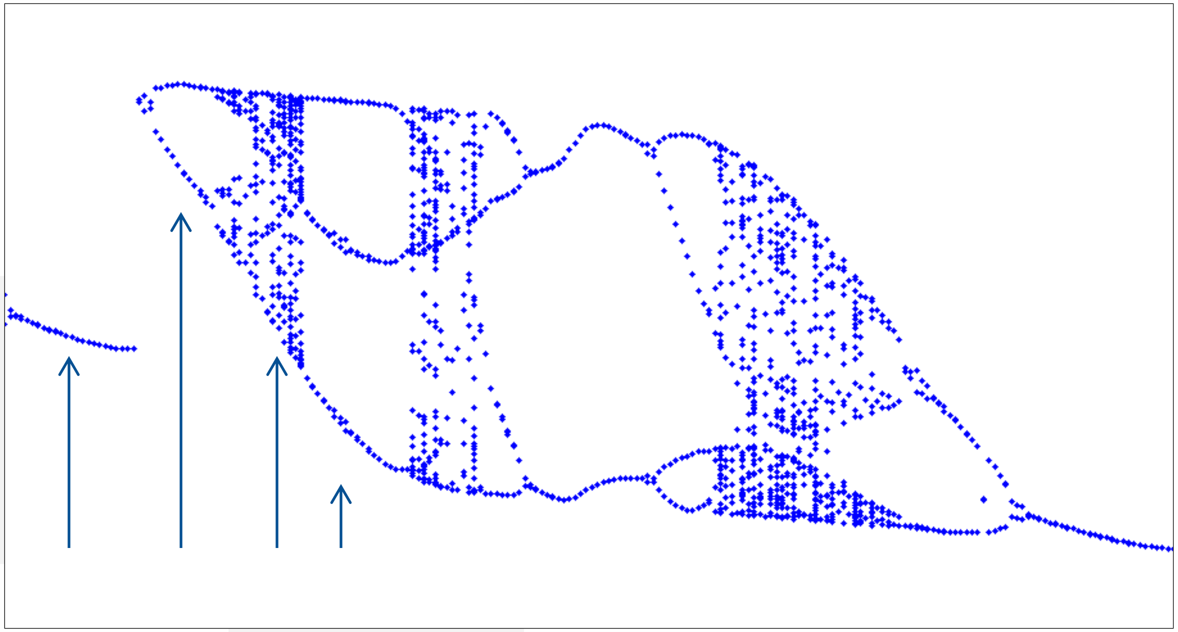
Before



After

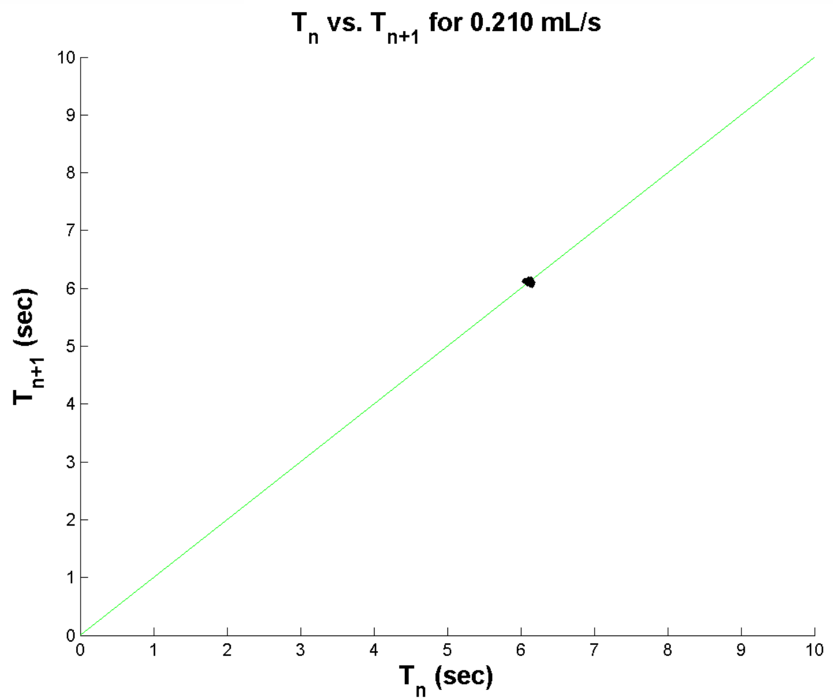
Bifurcation Diagram

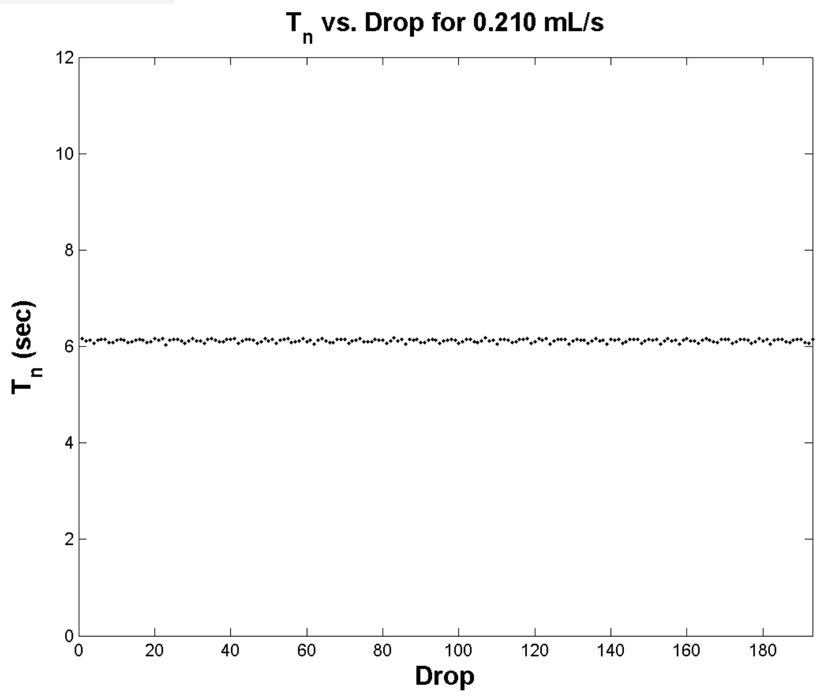
- Period Doubling
- Chaos
- Periodic Windows



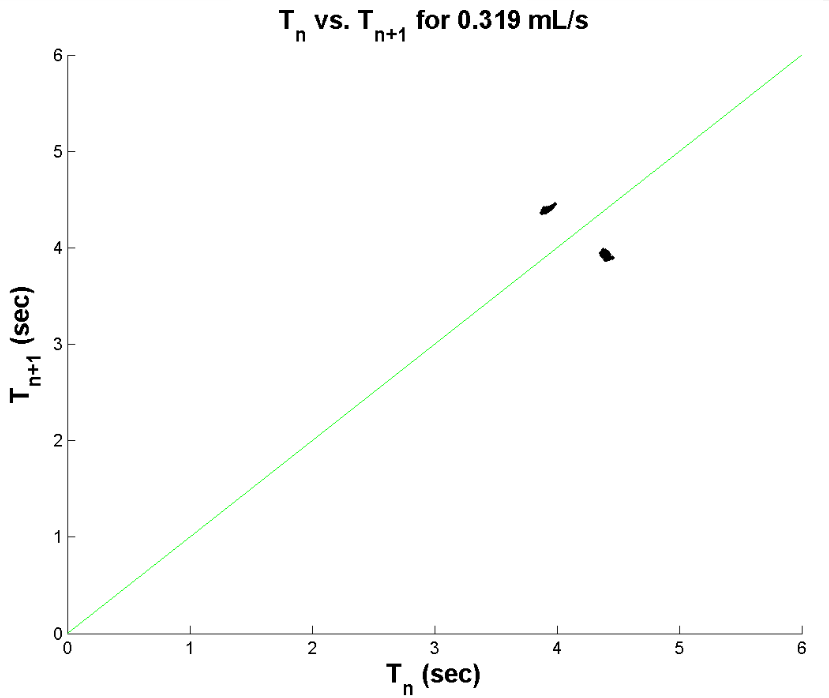
Increasing Flow Rate

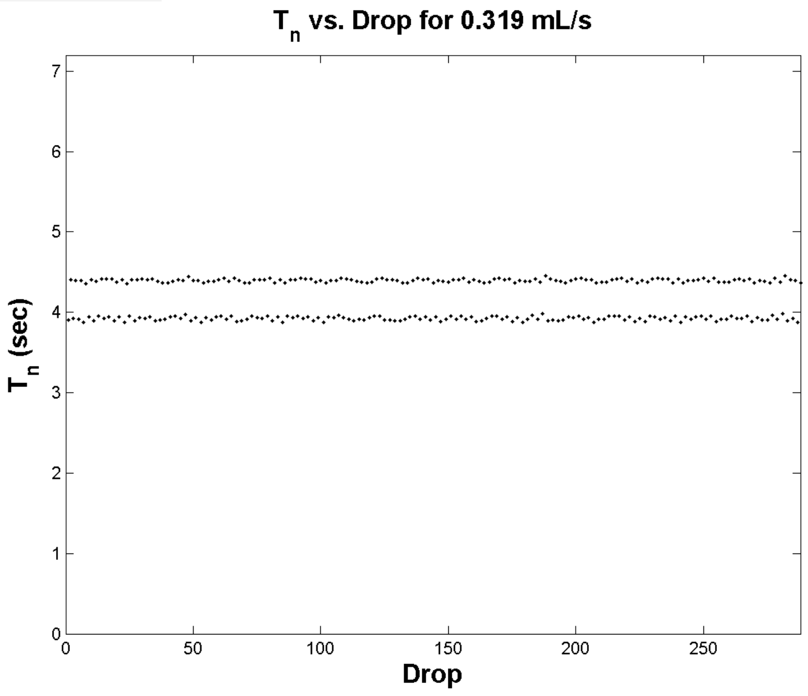
Secondary Data – Period 1



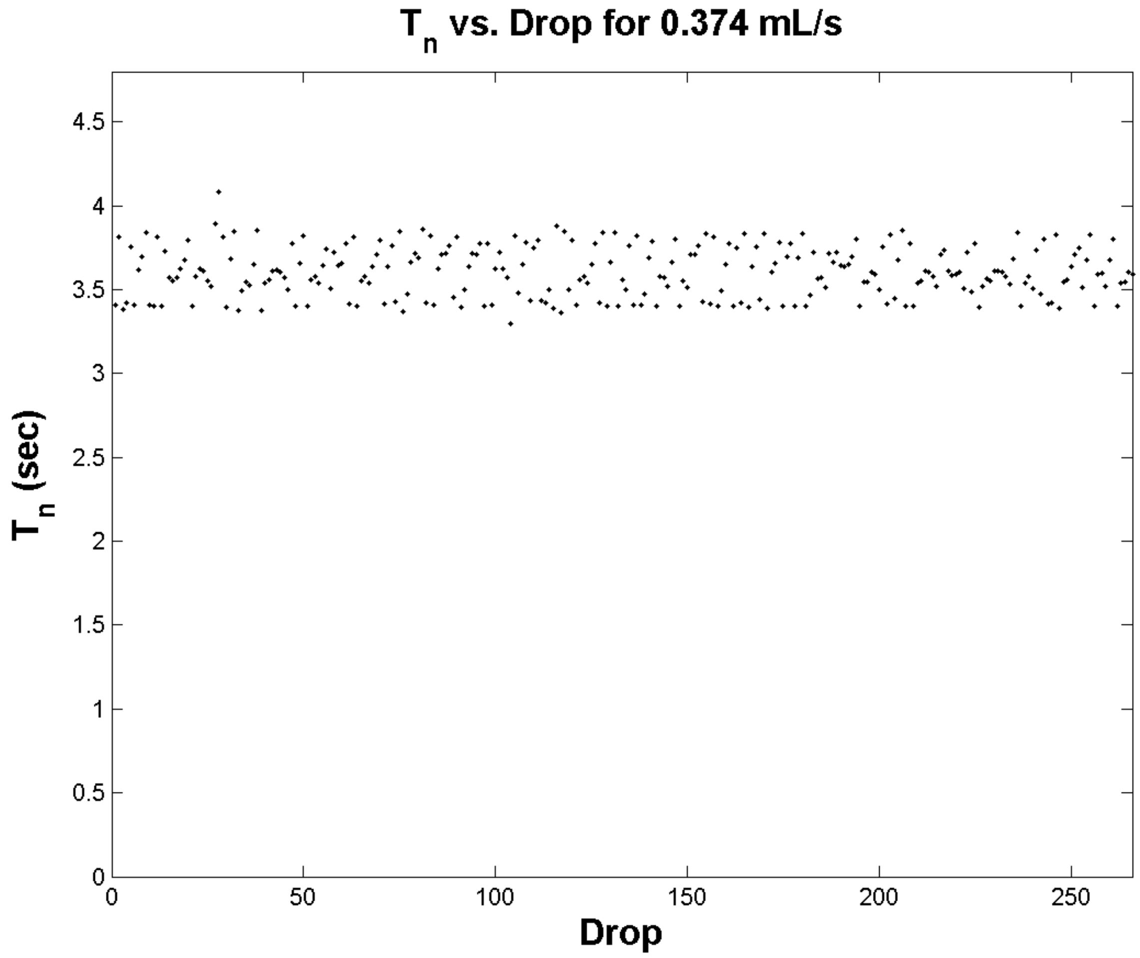


Secondary Data – Period 2

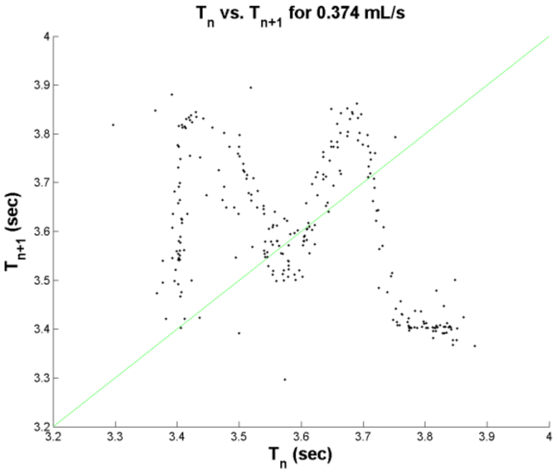


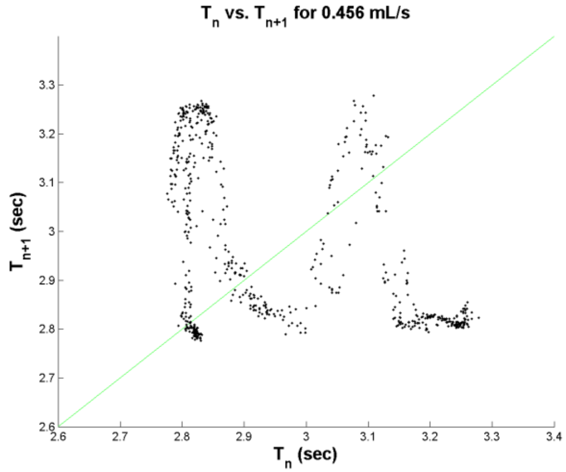


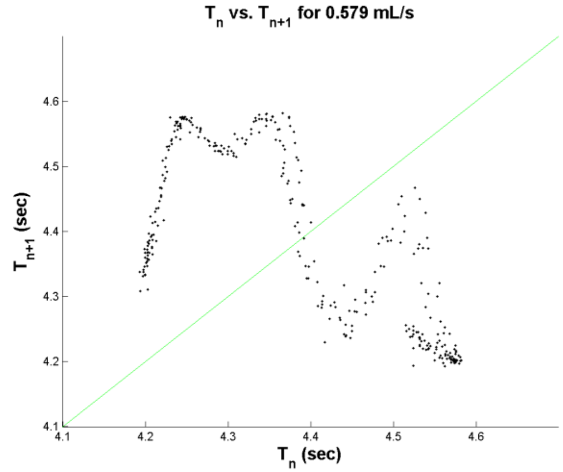
Primary Data – Chaos



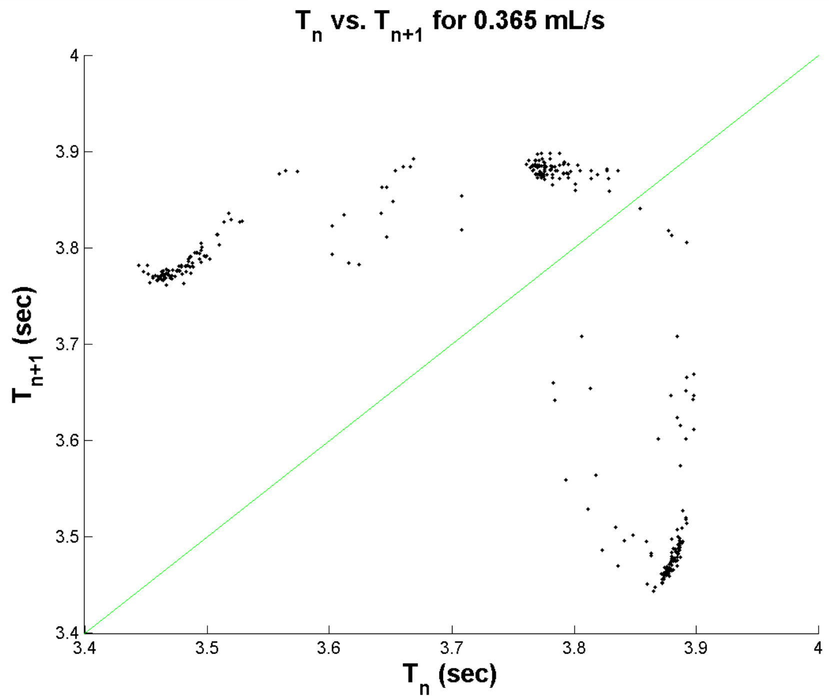
Secondary Data – Chaotic Attractors

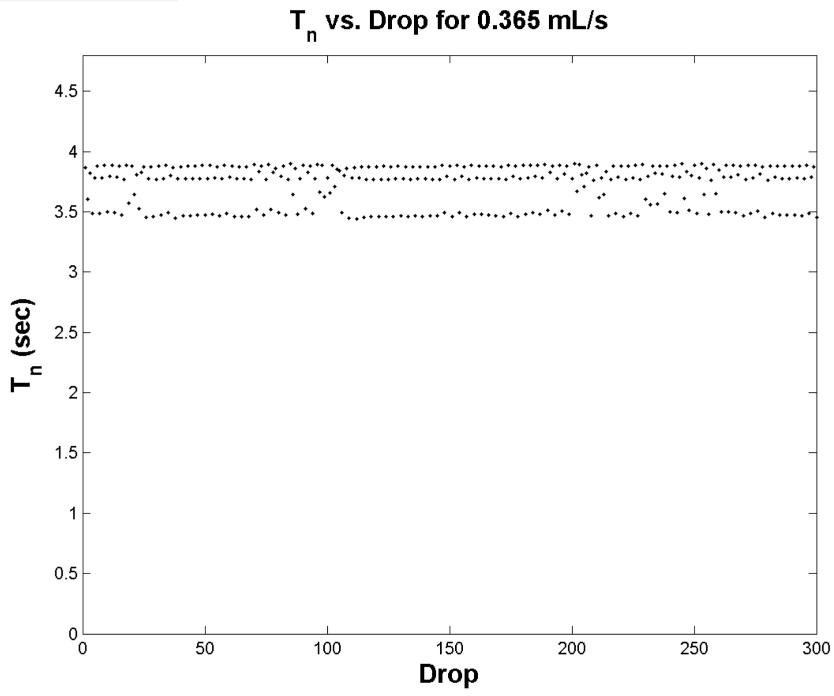






Secondary Data – Period 3





Data Analysis and Comparisons

- Universality
  - Requires bifurcation progression as U-sequence
  - Period doubling, chaos, and periodic windows

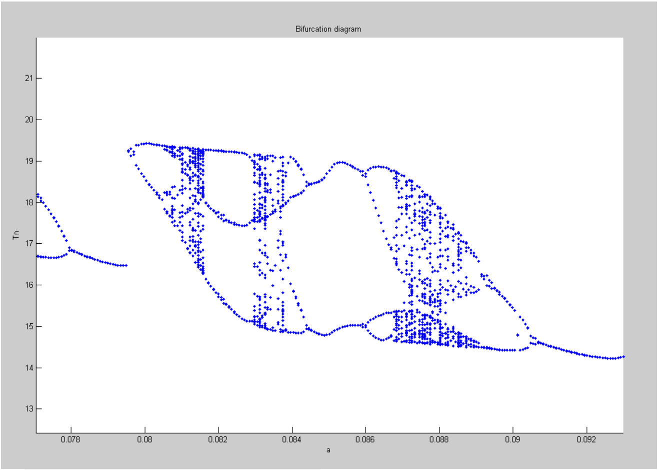


Fig 11: Simulation Bifurcation Diagram

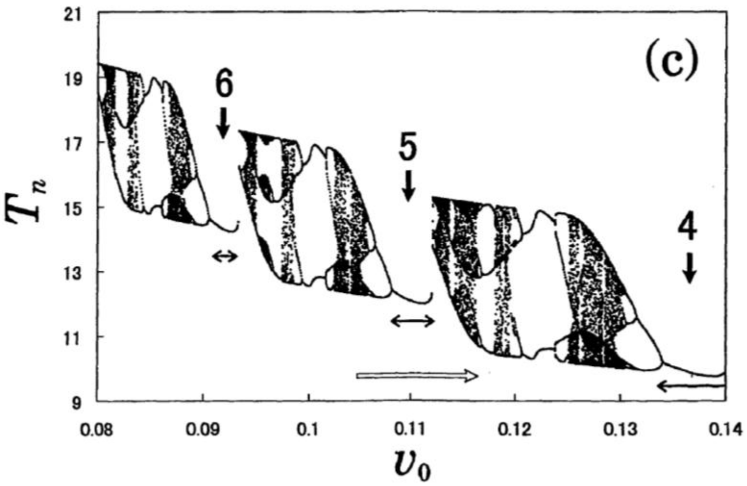


Fig 12: Reference Bifurcation Diagram

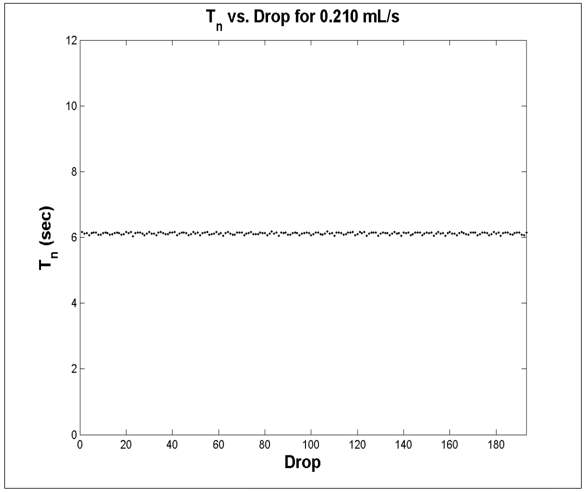
- Though similar, this could suggest the chaotic faucet does not have a unimodal map (current research)[6]

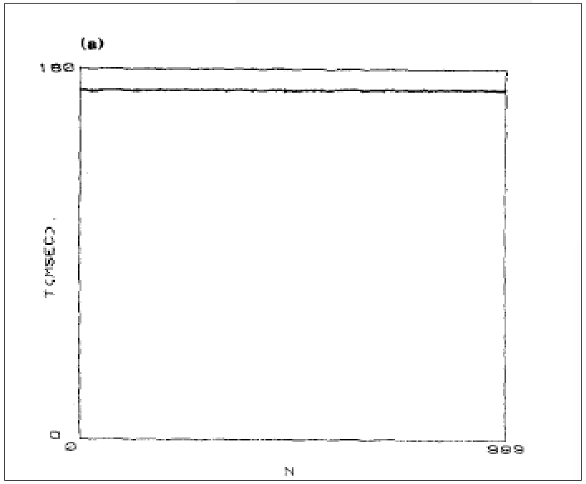
Qualitative Comparison – Single Period

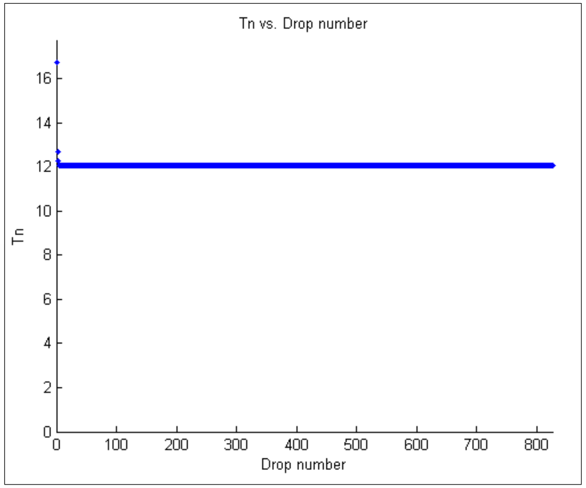
Primary Data

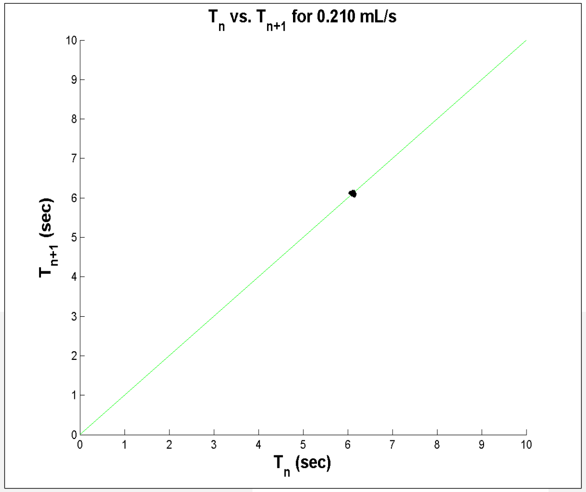
Reference Data[4]

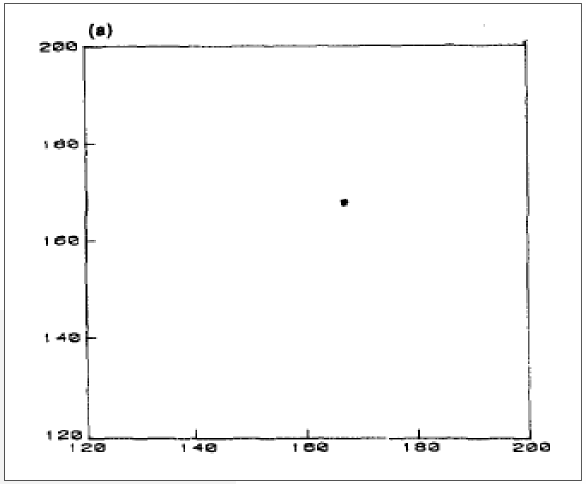
Model Data

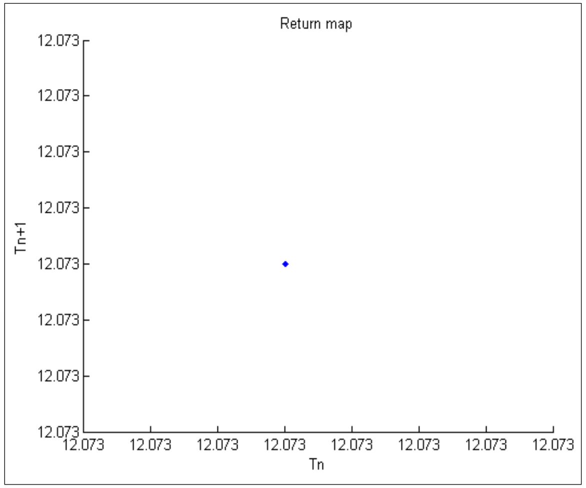






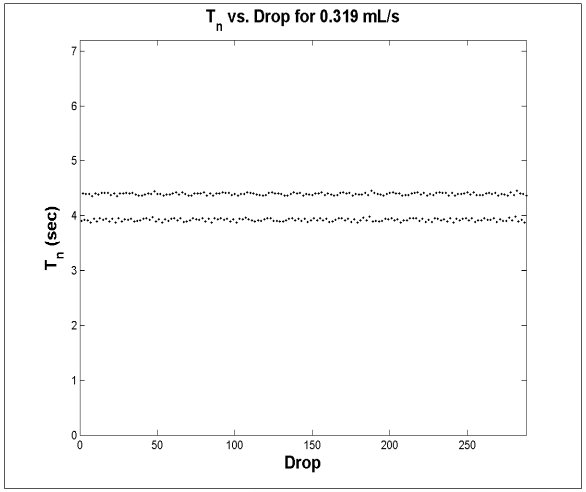


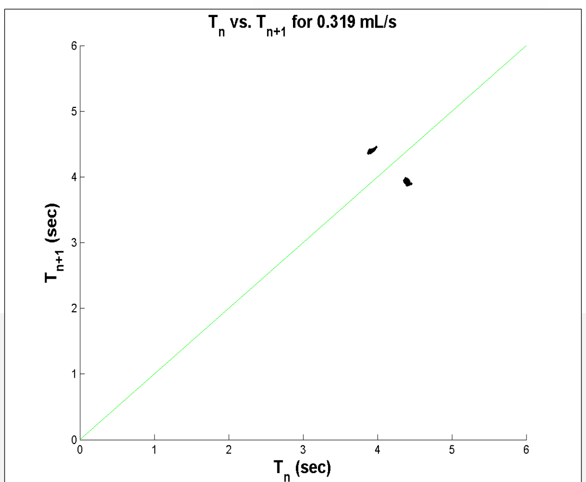




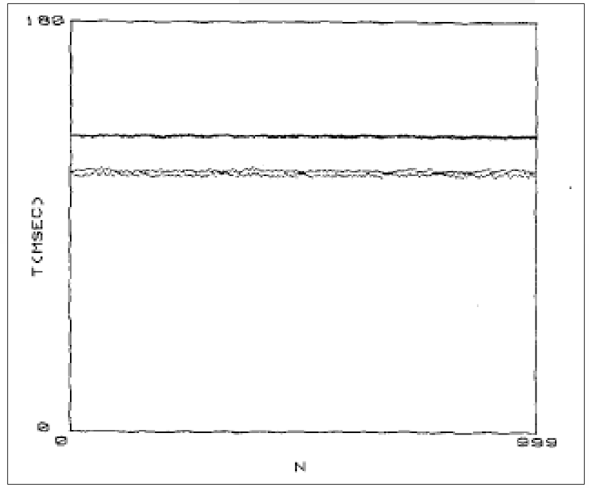
Qualitative Comparison – Two Period

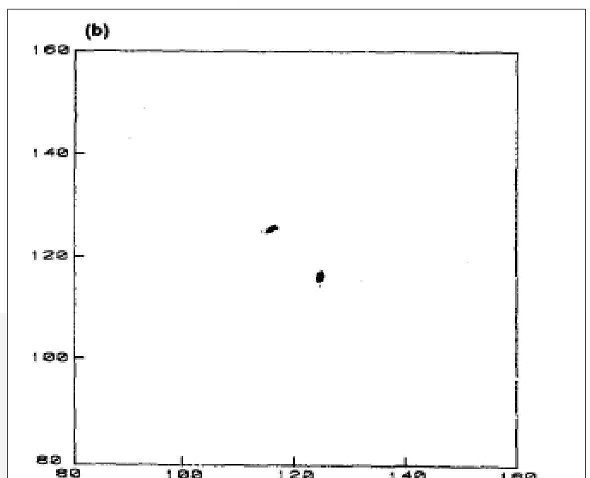
Primary Data



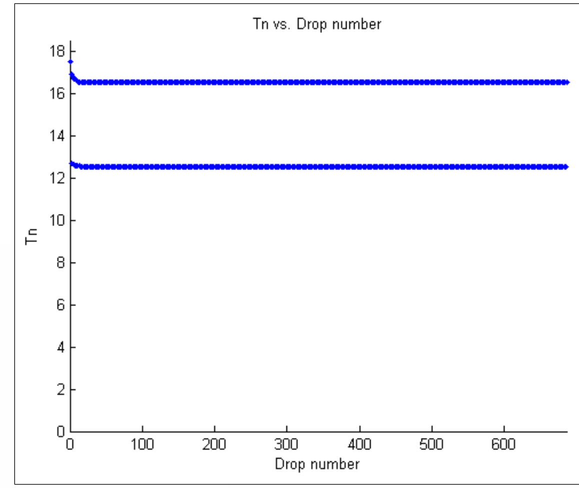


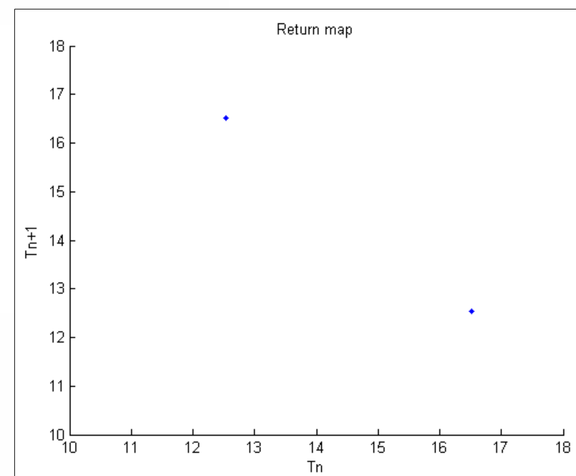
Reference Data[4]





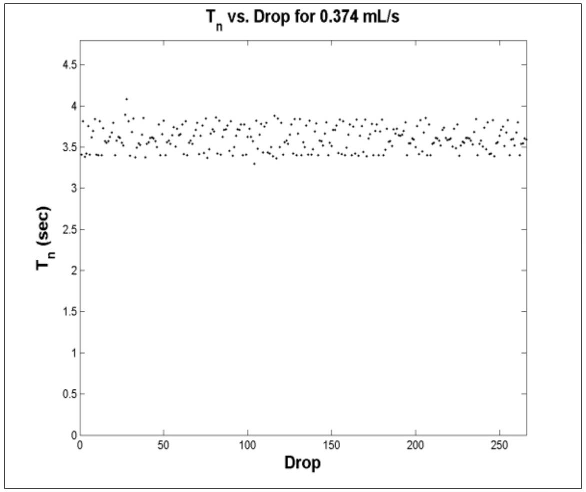
Model Data

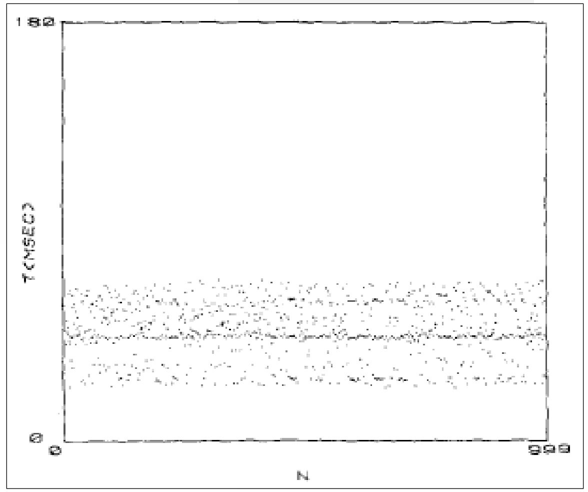


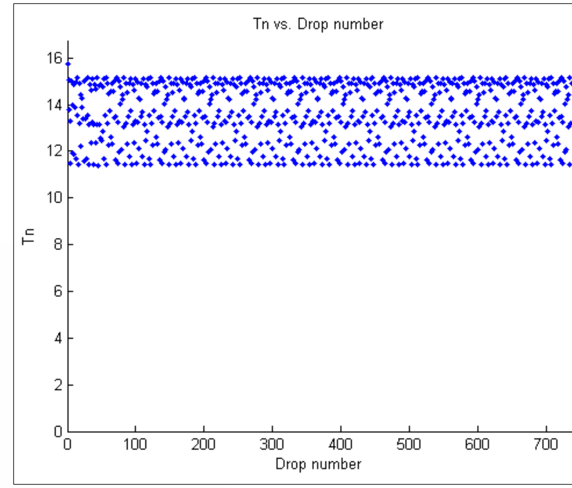


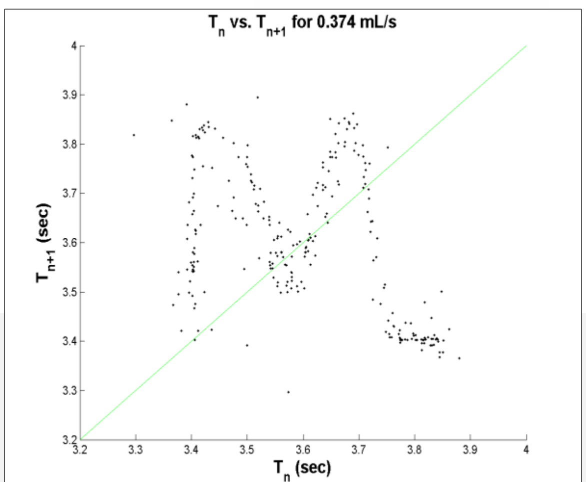
Qualitative Comparison – Chaos

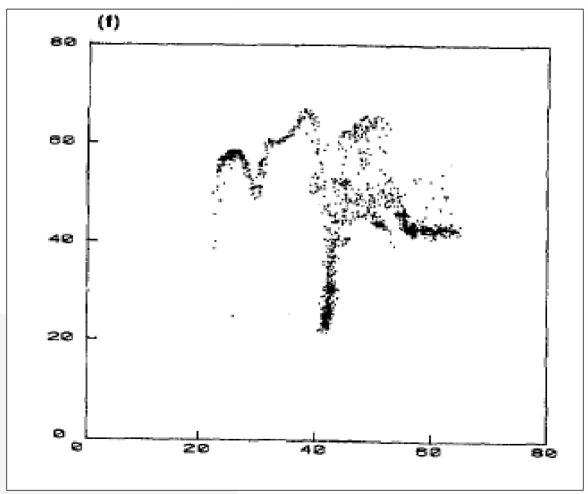
Primary Data | Reference Data[4] | Model Data

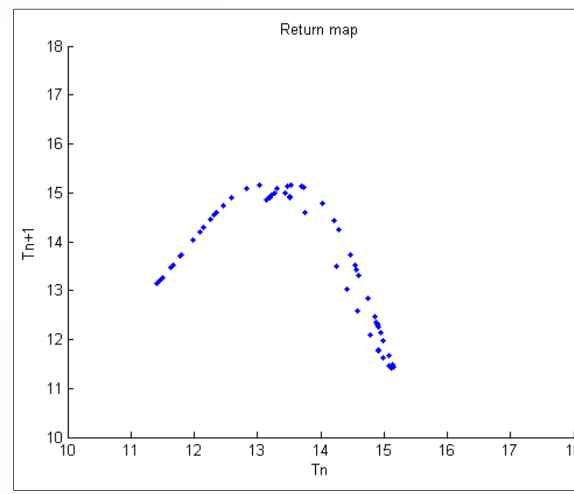








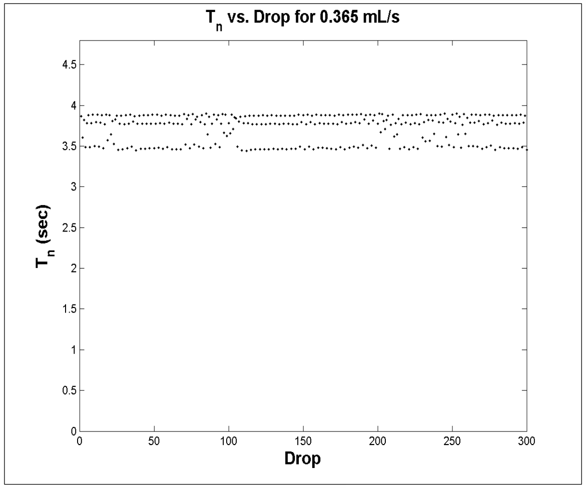


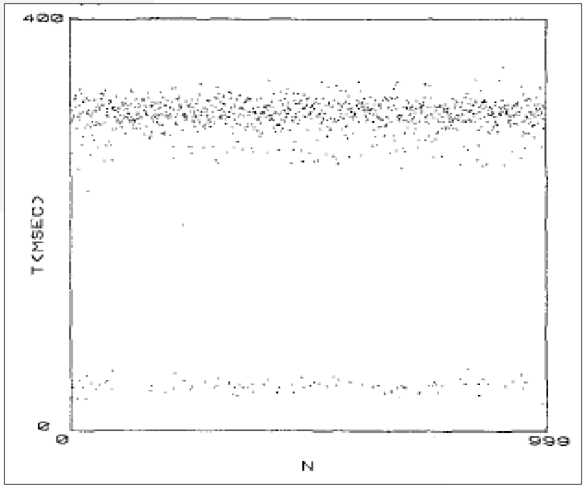


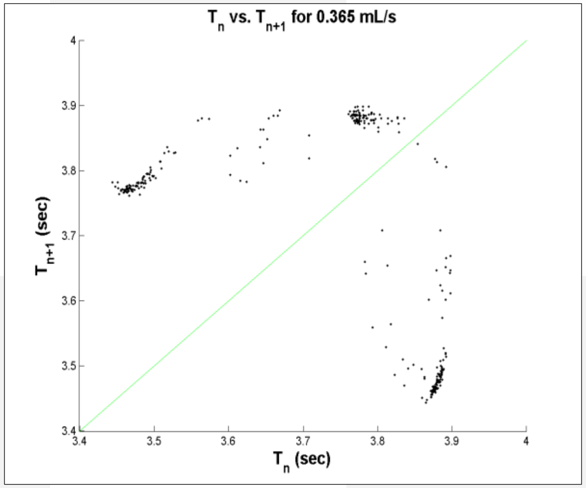
Qualitative Comparison – Periodic Window

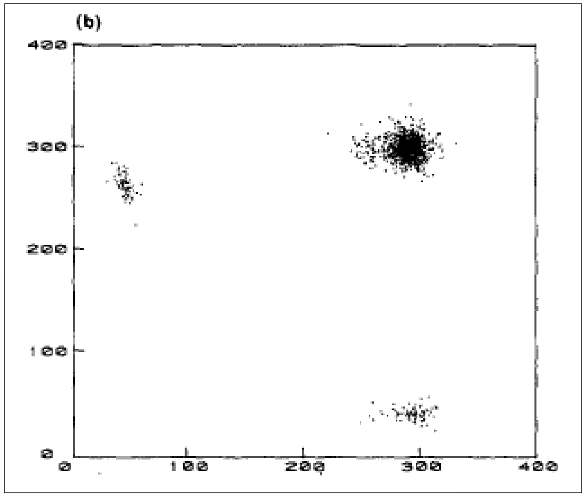
Primary Data

Reference Data[4]





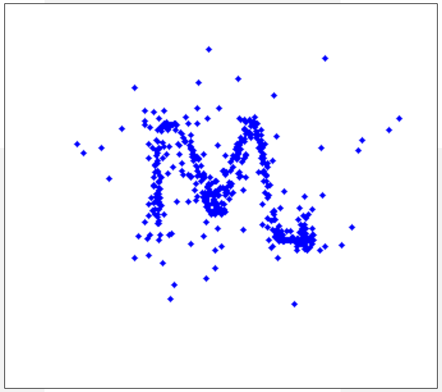




Period Doubling Route to Chaos

- Confirmed period doubling progression
  - Seen in bifurcation diagram
  - Predicted by references and simulation
- Drops from a 5mm nozzle as described by Dreyer, Hickey[4]:

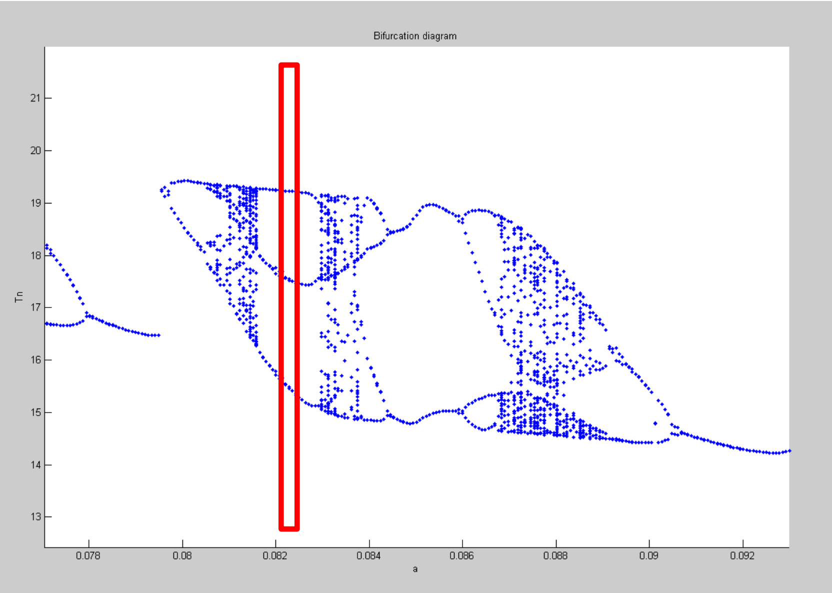
"were seen to follow a bifurcation route to chaos producing period-1 and -2 attractors at lower drip rates and many beautiful examples of strange attractors for higher drip rates with a range of instability between the two regions"



= **A Beautiful Attractor**

Periodic Window Route to Chaos

- Confirmed transient chaos
  - Seen in bifurcation diagram
  - Predicted by references and simulation

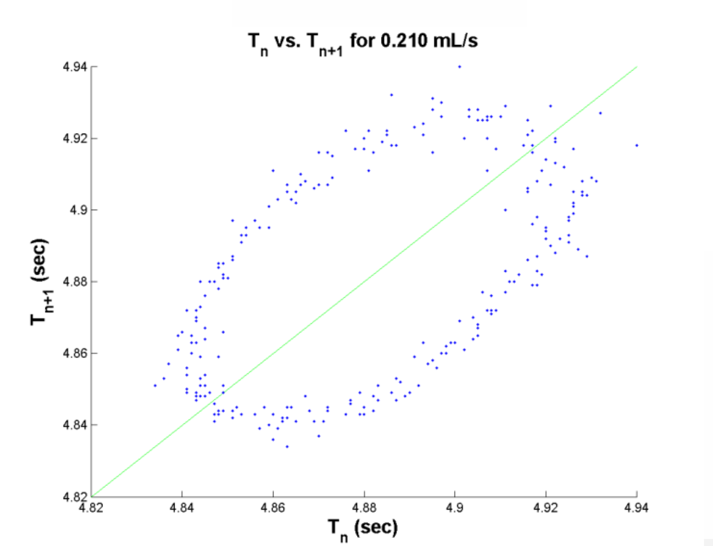


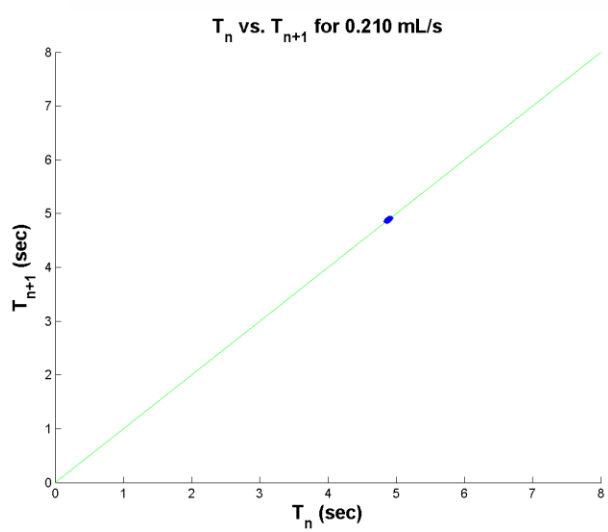
Error Analysis Summary

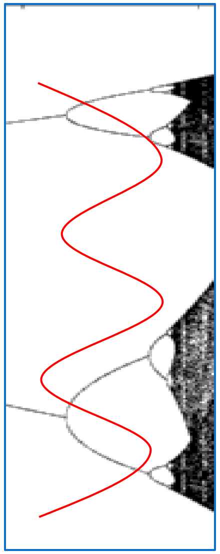
- Debouncing → refractory period
- Satellite drop counting → threshold
- Missed drops dependent on nozzle diameter
- Pump vibrations and mode interactions
  - Visible to naked eye with syringe pump
  - Patterns form before period doubling, indicating external disturbance
  - Possibly mechanically induced

Error Analysis: Periodic Flow Rate

- Sinusoidal disturbance could cause fluctuation over the T-2/T-4/chaotic region
- Single-period circle







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Summary and Conclusions

- Two routes to chaos:
  - Period doubling (period-1 to period-2)
  - Transient/periodic windows
- Experimental setup
  - Accurate flow rate necessary
  - Uniform flow rate necessary
  - Buckets are hard to control flow rate and measure drops
  - Nozzle size is very important (error and dynamics)
- Data analysis
  - Satellite drops should not be included
  - Debouncing of double peaks
- Model matches data and literature qualitatively

Questions?

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