Physics 6268, Group 5: Chaotic Dripping Faucet

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The dripping faucet experiment seeks to provide a relatively simple visualization of a system that exhibits seemingly predictable dynamics that quickly transitions to an entirely unpredictable, chaotic behavior under certain conditions. This behavior is first modeled through equations determine from previosu research and papers in order to understand the general phenomenon and established a baseline to comapre results against. Through a straightforward experimental setup, the time between drips of a faucet is measured using a photodiode and a laser. The number of drips are dictated through precise control of flow rate through a flow regulator. The photodiode records this drop as the water drop temporarily diffracts the laser path to the diode. The photodiode is interfaced to a computer through an analog to digital converter. The data is easily analyzed and visualized through the computer. After conducting error analysis, the processed data was examined in MATLAB and analyzed.The lessons learned through this research should include fine attention to detail regarding the experimental setup. It is imperative to have accurate flow control and uniform flow rate. It is suggested that shunting the flow to allow finer resolution of the water passing through the nozzle, while still using the crude units of the pump, be performed in order to increase the flow rate resolution. This would allow finer investigation of the 70-90 flow-rate unit (FRU) region of the pump, where the period doubling and chaotic behavior seemed to be most apparent. Furthermore, large nozzle size is important to minimize imperfections and allow harmonic oscillator dynamics. For data collection, care must be taken to exclude satellite drops and debounce the laser-photodiode to remove the double crossings generated by falling drops. The results show two main regimes: a period doubling regime, and a chaotic regime. The system gradually transitions from a single period to double period, followed by four periods. Once the flow rate is increased past these period doubling regimes, the time between drips becomes apparently unpredictable. Plotting the current

period against the next period, the outwardly chaotic behavior begins to show signs of an attraction basin. This represents a classic example of chaotic behavior of a system that seems to contain easily understandable system dynamics and physical phenomena. Additionally, the seemingly random behavior after the initial period doubling also shows the basins of attractions that seem to exist for the chaotic regime. Overall, the model and secondary data match the literature and simulation qualitatively.

I. PROJECT DESCRIPTION

The behavior of the dripping faucet was investigated. Within the field of chaos, it is a well known canonical example of chaotic behavior. The project concept originates from the dripping behavior seen from faucets that are not fully stopped. These drops fall from the faucet head at a given period based on their flow rate. As the flow rate increases, the dropping behavior begins to show period doubling. The single period drop transitions to double period, then four period until it soon becomes chaotic, and finally a solid stream of water. This process not only exhibits a classic example of chaos, but also undergoes periods of non-chaotic behavior in between chaotic regions, known as periodic windows. The project goals were to find bifurcations in the periods of drops and their relationship to flow rate, as well as observing the transition to chaos through period doubling.

II. BACKGROUND[2,3,4]

Research into the chaotic dripping faucet began in 1984 with Robert Shaw, a paper titled "The Dripping Faucet as a Model Chaotic System". The paper described the experimental results, as well as a straightforward simple model of a damped harmonic oscillator. The mass-spring model was a damped, harmonic oscillator with linearly increasing mass that was driven by a forcing function. Through this model, the droplet is viewed as a mass that increases as water continues to flow into the droplet, pulling the droplet downward due to gravity. The spring is the surface tension of the droplet itself. While this model provides results that are qualitatively similar to the actual phenomena, it does not take into account any of the fluid dynamics. The equations of the Shaw model are as follows:

$$\frac{d(mv)}{dt} = mg - ky - bv, \frac{dm}{dt} = flowrate, v = \frac{dy}{dt}$$

Over time, it has become a prime example of chaos due to its simplicity and well established knowledge of the system involved. Since Robert Shaw's paper, it was been investigated by numerous scientists experimentally as well as theoretically, such as Kiyono and Fuchikami et al, Coullet et al, and others. The Kiyono and Fuchikami paper published in 1999 provided a much more complex hydrodynamical model, providing the equations of motion gained from solving an integral that involved the Lagrangian through the estimation of kinetic energy, gravitational potential energy, and the surface tension energy for the droplet. The results of this analysis provided a qualitatively far superior system. The team also published another paper in 1999 that detailed the more advanced mass-spring model. The new model takes into account the spring constant dependence on mass, and includes the pinch-off of the droplet. This differs substantially from Shaw's model, which did not take into account the pinch-off dynamics. For the Kiyono and Fuchikami model, the model is based on an analysis of their previous hydrodynamical model. This model also uses a special unit system, where distances are measured in units of 2.7 mm, time in 17 ms, and mass in .2 g. This makes g = 1 (gravity) and simplifies the system overall. For a faucet with 5mm diameter, the model predicts a z0 of 2.0 and zcrit of 5.5, in units of 2.7 mm. An advantage of this system over Shaws is the parameters are explicitly stated for the reset values of z, zdot, and m are. This makes it difficult to make any sort of predictions due to the drastic changes in behavior for small variations. The equations for the Kiyono and Fuchikami advance model are:

$$m\frac{d^2z}{dt^2} + (\frac{dz}{dt} - v_0)\frac{dm}{dt} = -kz - \gamma\frac{dz}{dt} + mg$$

$$\frac{dm}{dt} = Q = \pi.a^2.v_0$$

$$k(m) = -11.4m{+}52.5[m < 4.61]; 0[m \geq 4.61]$$

$$m = 0.2m + 0.3 when z = zcrit$$

$$z = z_0$$

Because of the time intensive nature of the simulation, a full fluid dynamics simulation was not undertaken. However, some figures using MATLAB were created to illustrate the concept of the simulation results. The mass on increases linearly in time until the drop-off, it is then reset to a value dependent on its mass prior to pinch-off. The spring constant is linearly dependent on the mass, until it reaches a critical value at which point it is simply zero. The behavior is meant to simulate the pinching off of the droplet. After creating these models, the experimental data was collected in order to determine the veracity of these models.

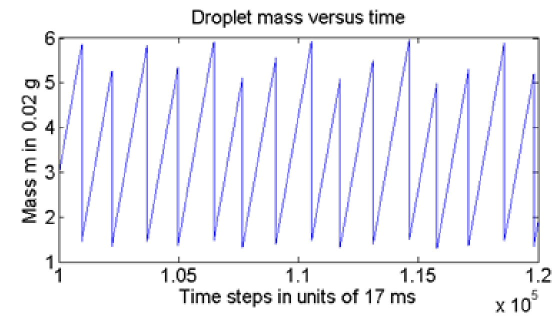

FIG. 1. Simulation data showing droplet mass variation versus time

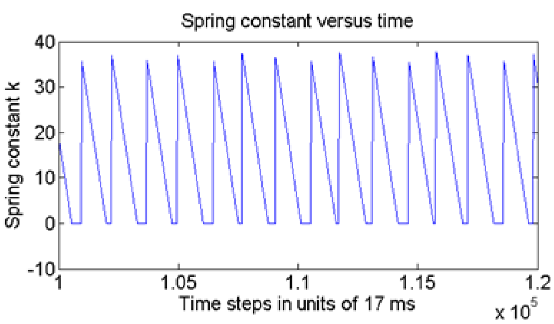

FIG. 2. Simulation data showing the change in spring constant over time

III. PROCEDURE

The following is the proposed experimental setup for measuring the period between drops:

1. Feeder tank fills the reservoir tank
2. A stopcock controls the flow rate from the reservoir tank
3. A laser and photodiode detect falling drops
4. The signal is read by an Analog to Digital Converter
5. Period of falling drops measured from data
6. A high speed camera used to visualize the falling drops

In reality, the experimental setup varied considerably from the actual setup. For the first attempt, a 3/32" flexible tube was used along with a bucket with drilled holes. The setup was quite simple but had a host of issues. One issue was the drop did not fall directly through the laser, causing drops to be missed. Secondly, it was quite difficult to regulate the flow rate, as the field height of the bucket waterline changed considerable. Finally, the bucket flow rate could not be accurately measured. These drawbacks required the development of a new experimental setup.

For the second iteration, a syringe pump was used to dispense the fluid at a specified rate. Ideally, this would allow the flow rate to be accurately set. In reality, the syringe pump also suffered from a host of failures. The pump possessed undesired cycling, which created an unsuitable flow rate. In addition, a large nozzle would dispense fluid at a totally unusable rate. In order to remove transience from the system, the syringe pump would have to run for an initial period of time. This would cause issues as the syringe was limited in volume and could not provide enough fluid for both data measurement and removing transience from the system.

The final setup consisted of a photodiode, laser, reservoir, large diameter flexible tubing, and flow regulator. The flow regulator allows the liquid to be dispensed at a set rate, measured in flow rate units (FRU). Using linear regression, a conversion factor between FRU and SI units was found to be:

$$\frac{mL}{s} = 0.004563[FRU]$$

IV. MODEL[1,5]

Using the previously discussed models by Kiyono and Fuchikami, simulation data was generated in order to replicate their results. Using MATLAB, the results were plotted to show the results they discovered, which were used as a benchmark for the actual project results. The first plot is the resulting simulation for a one period band versus drop number in Figure 3. This result is quite similar to the obtained data. The Poincare map is the same one period simulation data is shown in Figure 4. Increasing the number of periods to two period, the band and Poincare map are plotted (Figures 5, 6:

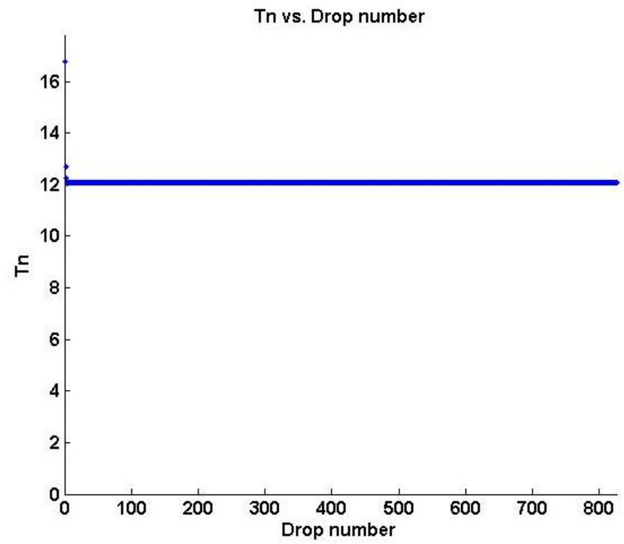

FIG. 3. period-1 T_n vs Drop

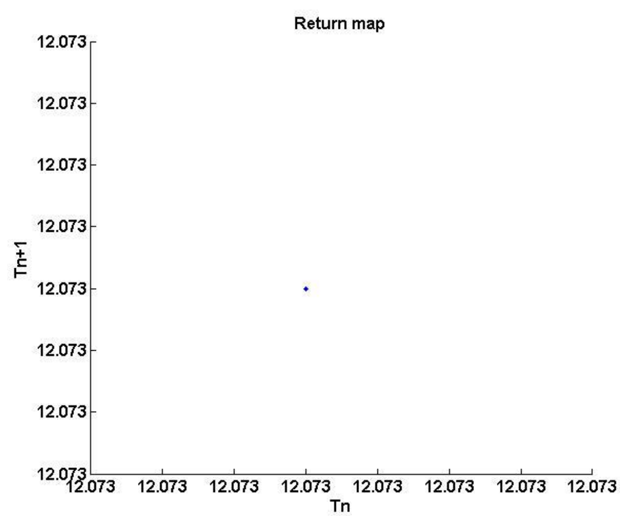

FIG. 4. period-1 T_{n+1} vs T_n

Finally, the simulation model was used to plot the chaotic band and map plots. The results of the chaotic map show some semblance of a non-unimodal result, while the chaotic band plot is expected (Figures 7, 8)

From these results, it is clear that the models give expected results. In order to determine whether the data correlates with the modeled data, the data was first analyzed for error to determine their inaccuracies and issues to ensure proper data collection and processing.

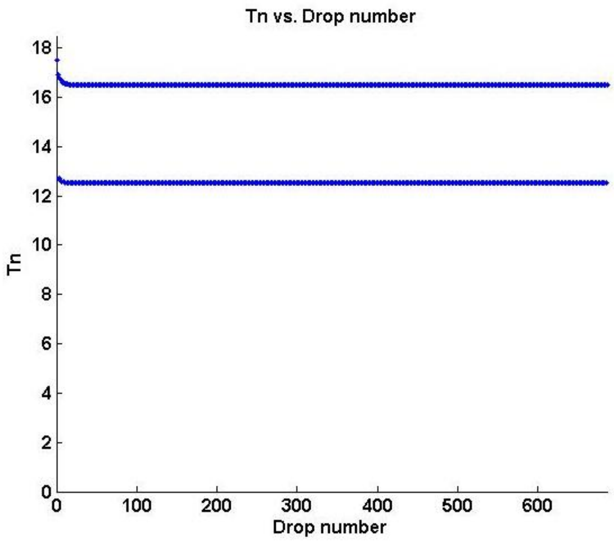

FIG. 5. period-1 T_n vs Drop

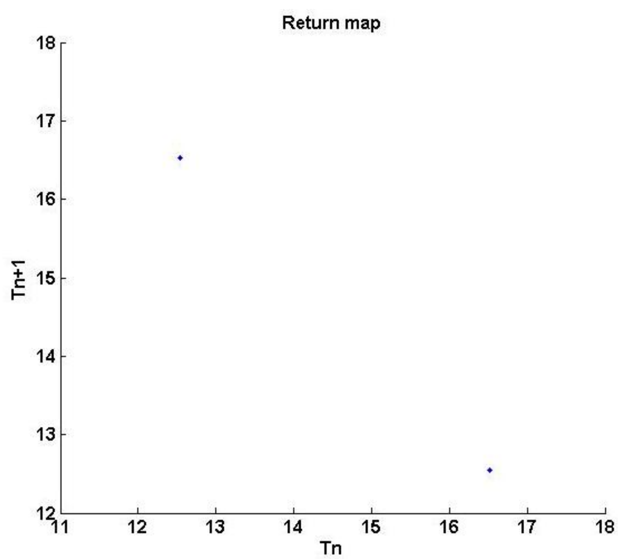

FIG. 6. period-1 T_{n+1} vs T_n

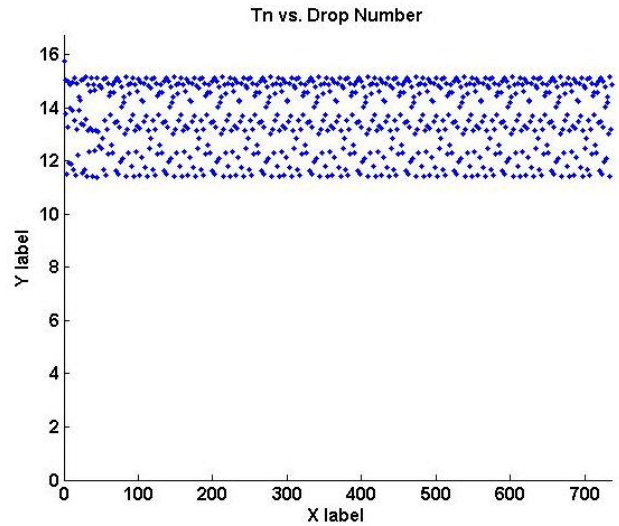

FIG. 7. period-1 T_n vs Drop

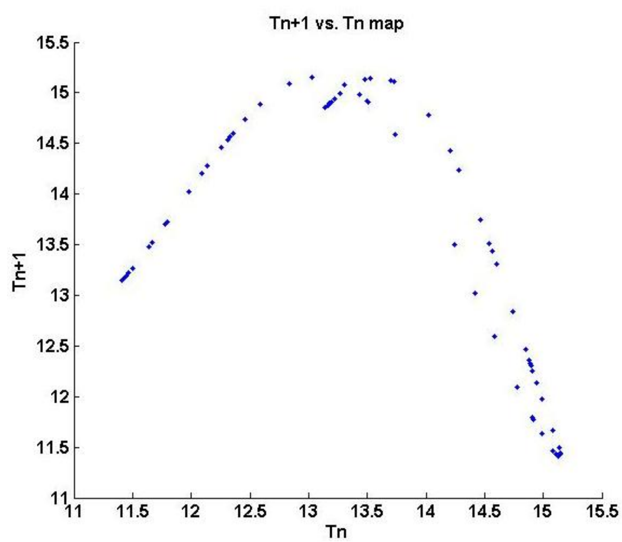

FIG. 8. period-1 T_{n+1} vs T_n

V. ERROR ANALYSIS

When analyzing the data, major issues were uncovered during data collection. One such issue was the nozzle diameter. The lateral movement of droplets caused errors in the measurement of the drops. The large lateral movement would cause the water droplets to fall out of the range of the laser, causing the laser to report either abnormally long or short period measurements due to 'missed' drops. In addition, small nozzles magnified the imperfections of the tubing itself, causing the flow exiting the tube to be turbulent, jittery, or generally not uniform and controlled. After extensive testing, a correlation was found between larger nozzle diameters and better results. In conclusion, it was determined that larger nozzle diameters led to better results and were therefore chosen for the experiment.

Another source of error was the presence is satellite drops. As the main drop fell off, it produced several 'satellite' drops that came off the main falling drop. This presented several issues with data collection, as the satellite drop would trigger the period measurement and incorrectly attribute the satellite drop as another main drop, leading to far smaller periods. This lead to incorrectly identifying satellite drops as the double period, four period. In order to remove the satellite drop data, the data was post processed by setting a threshold to remove all points beneath the main drop.

In addition to the the satellite drops, an issue with the actual laser measurement is contact bouncing. Contact bouncing occurs when the laser double counts the top and bottom of the top as crossings. This means the periods recorded were extremely close, as they occurred when the top and bottom of the drop crossed the laser. In order to account for this, the data was debounced during post processing to correct the double counting by use of a refractory period, where the program was instructed to ignore the measurement that occurs immediately after a laser crossing. This change was coupled with the threshold for satellite drops and large nozzle diameters to provide a method for combating experimental errors through post-processing with the MATLAB scripting environment.

VI. DATA

The data for the experiment was collected using a National Instruments Analog to Digital Converter (ADC) that was interfaced through LabVIEW using a virtual instrument (VI) file made by Nick Gravish, the TA for PHYS6268 for the Fall 2011.

An example of period-1 behavior occured at a flow rate of 0.210 mL/s. The plots show the data after it the errors and issues were removed during post-processing. The Poincare map of next period (Tn+1) versus current period (Tn) is plotted in Figure 9, with the line of symmetry drawn in green. The black dots, representing the data obtained, are all clustered in one location at about six seconds. Since there is only one cluster, and this cluster lies on the line of symmetry, a period of six seconds is a fixed point for the flow rate of 0.210 mL/s. This is shown in another way in the Figure 10. Here period (Tn) is plotted versus drop count. The single horizontal line indicates that the period remains constant with each drop.

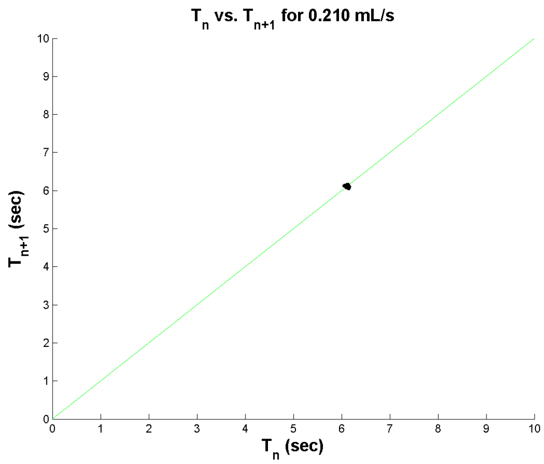

FIG. 9. period-1 T_{n+1} vs T_n

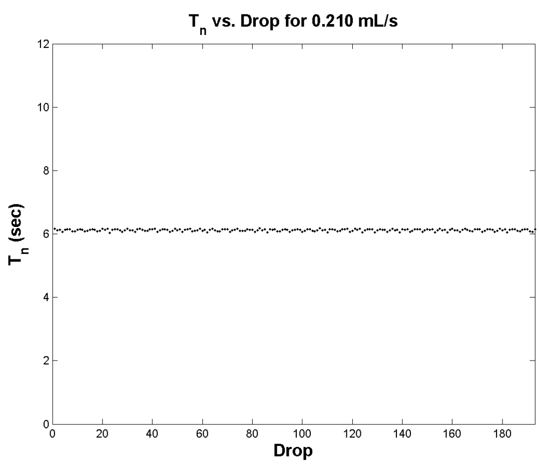

FIG. 10. period-1 T_n vs Drop

Next, an example of period-2 behavior occured at a flow rate of 0.319 mL/s. Like before, there are two plots below showing the post-processed data: the left figure shows a Poincare map and the right shows a time series of the same data. The Poincare map shows two distinct clusters, one slightly under four seconds and the other a little under four and a half seconds in Figure 11. These two period regions shows that the period switches in between the two values. For example, if a pair of drops falls with a period of four seconds, the corresponding following period (Tn+1) shows that the next period will be four and a half seconds. Using this value as the current period (Tn) value, the point falls in the lower right cluster of the map. The corresponding next period (Tn+1) shows that the next period will be four seconds which returns us to our initial period. Figure 12 shows two horizontal, relatively flat lines which represent the he two alternating values of period.

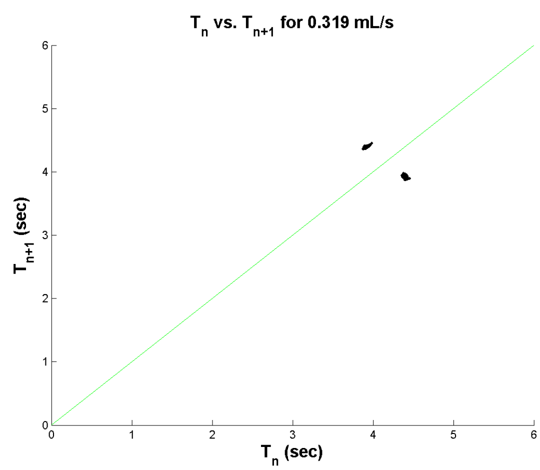

FIG. 11. period-1 T_{n+1} vs T_n

Next, chaos was found in a range of flow rates. The plots below show period versus drop count at a flow rate of 0.374 mL/s. The behavior of the drops seems to be random in Figure 13. Although the range of periods

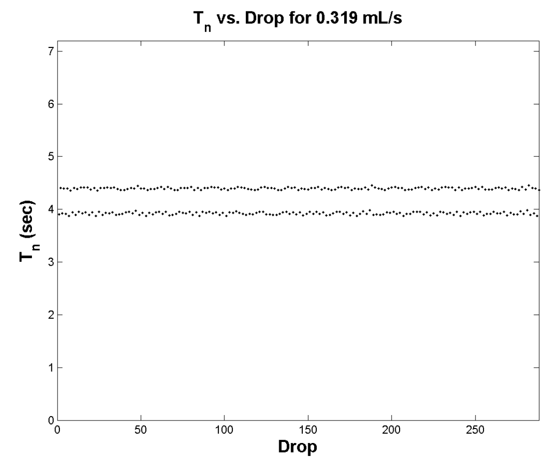

FIG. 12. period-1 T_n vs Drop

appears to be bounded between about three and a half and four seconds, there does not seem to be any predictable pattern. This is known as chaotic behavior, as no correlation is clearly evident in the plot.

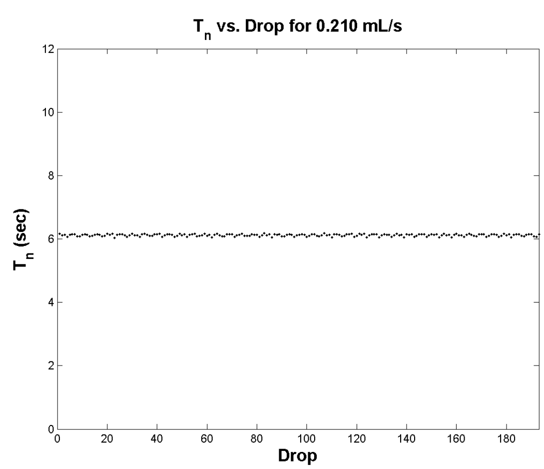

FIG. 13. period-1 T_n vs Drop

When the Poincare map is plotted in Figures 14, 15, 16, clear attractors are visible. Shown are the three Poincare maps for three distinct flow rates. Even though the flow rates differ, the attractors shapes remain very similar, producing a shape similar to the letter 'M'. The plot showing a dissimilar shape was actually obtained from using a different nozzle diameter, explaining its slight difference in shape.

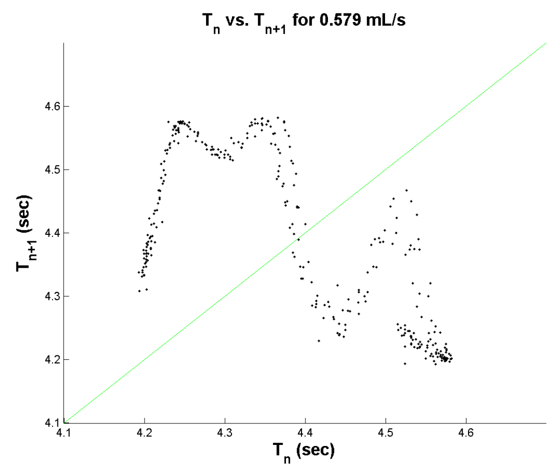

FIG. 14. period-1 T_{n+1} vs T_n

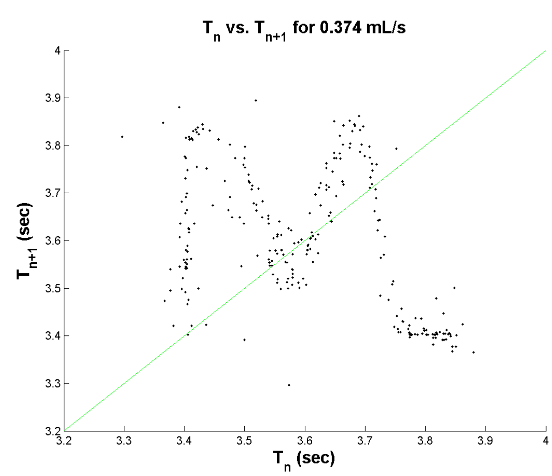

FIG. 15. period-1 T_{n+1} vs T_n

Finally, an example of period-3 behavior was found at a flow rate of 0.365 mL/s. Once again there are two plots below showing the post-processed data: a Poincare map, and a

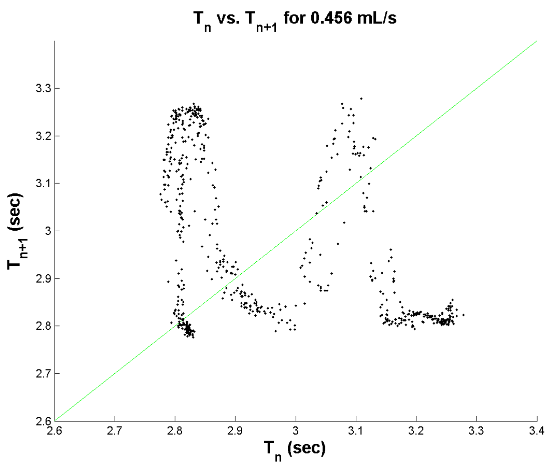

FIG. 16. period-1 T_{n+1} vs T_n

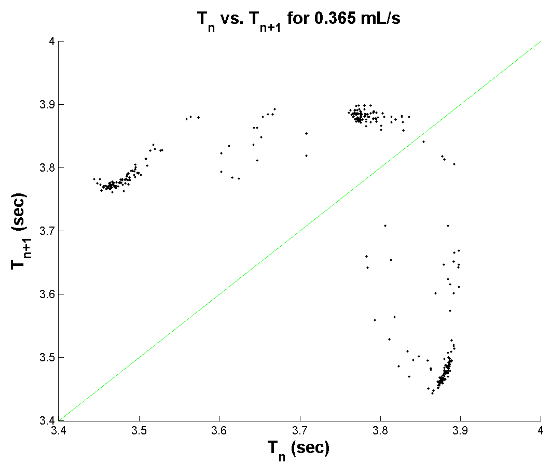

FIG. 17. period-1 T_n vs Drop

time series of the sam data. The Poincare map shows three clusters, indicating that the period cycles between three values. To illustrate this example, consider starting with a period of about 3.9 seconds, in the bottom right cluster. The next period axis (Tn+1) dictates the new period will be about 3.5 seconds. Taking this as the new current period and checking the resulting next period, the point falls into the far left cluster. The Tn+1 axis now dictates that the next period will be about 3.8 seconds. Returning to the Tn axis places the period in the upper, middle cluster. Finally, the Tn+1 axis dictates a return to a period of 3.9 seconds.

Comparing the three period data to the one and two period data sets, a fair amount of points lie outside of the three clusters. These points can be explained by looking at the plot of period bands versus drop. There are regions where the three horizontal lines are distinct and then regions of apparent chaos. This is actually the system exhibiting transient chaos. Since period-3 is a periodic window within the range of chaos, the system may sway either way if it is on the border.

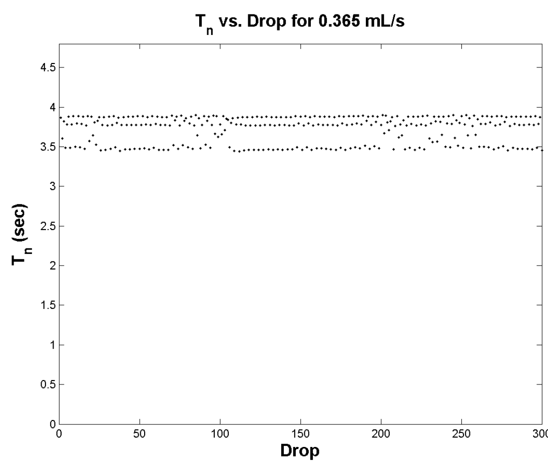

FIG. 18. period-1 T_n vs Drop

VII. CONCLUSION

From the data gathered by the experiment, period doubling and chaotic behavior were clearly seen. While the data shows some accuracy, additional modeling and experimental work remains to be done in order to better understand the system. One major isues lies within the flow regulator, which has a relatively low level of fidelity and could be improved to better see the regions of periodic doubling, chaotic windows, and the transition to the chaotic regime. The experimental error analysis showed a major goal gathered from the project, which is the importance of detail and experimental accuracy. The flow rate strongly dictated the phenomena seen during the testing, and therefore the ability to finely control flow rate and provide uniform flow fate are critically important. One method to improve the control of flow rate to provide finer changes than currently provided by the machine would be to shunt the flow by using a larger tube to divert the flow to alter the resulting flow rate seen by the dripping faucet. This would allow the user to observe the 70-90 flow-rate unit (FRU) region that exhibit the most interesting characteristics. Finally, the nozzle size played a major role in the dripping behavior. A larger nozzle size minimizes the impact of imperfections of the tube itself, allowing harmonic oscillator dynamics.

In addition to modifications in the experimental setup, current research papers offer more complex and well developed models that provide even more accurate correlations with experimental data. These models could provide the user with a much better picture of the overall dynamics of the system. Data collection also proved to be important, as satellite drops and debouncing the laser-photodiode in order to remove the double-counting of drops were very important to obtaining accurate data. In conclusion, predictions of two routes to chaos by Dreyer and Hickey were confirmed through both experimental and simulated data. The data showed period doubling, periodic windows, and chaos. While the data was not quantitatively examined, it showed a strong qualitative math against the predicted values obtained from literature and previously completed research.

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