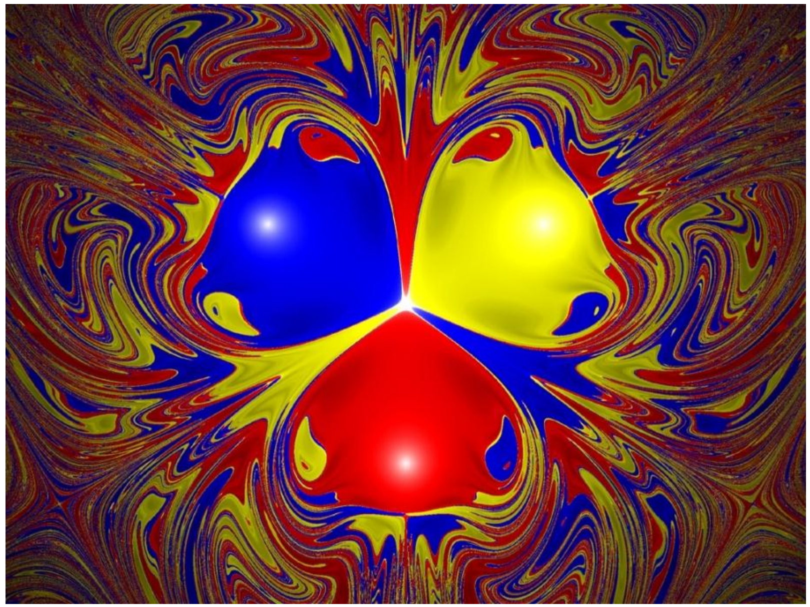
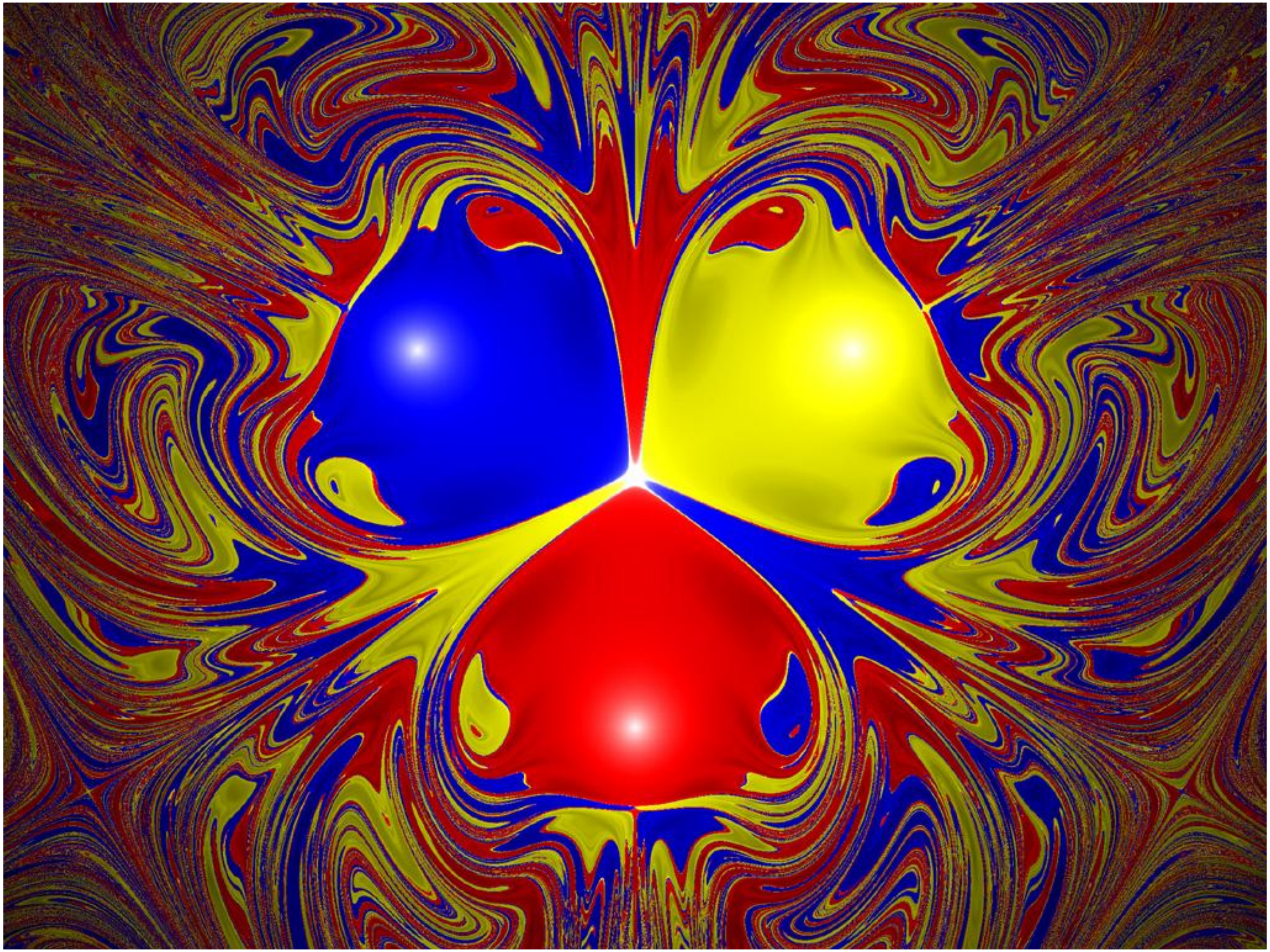
Fractal Basins in a Magnetic Pendulum System



Galen Craven, Chris Martin, Fredrik Ekström, and Jung Byun

"Designed to illustrate the random forces that effect us all. Can you make order out of this chaos? If so, you could probably write a bestseller"

-ThinkGeek.com



- www.bugman123.com

Background

- Toy system that has complex dynamics and fractal formation
- Basins of attraction / fractal boundaries
- Interesting Patterns

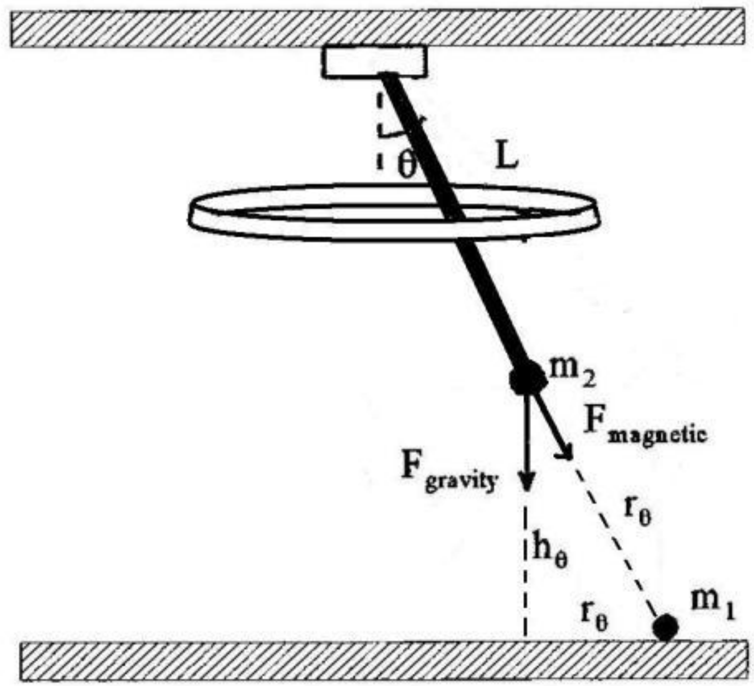
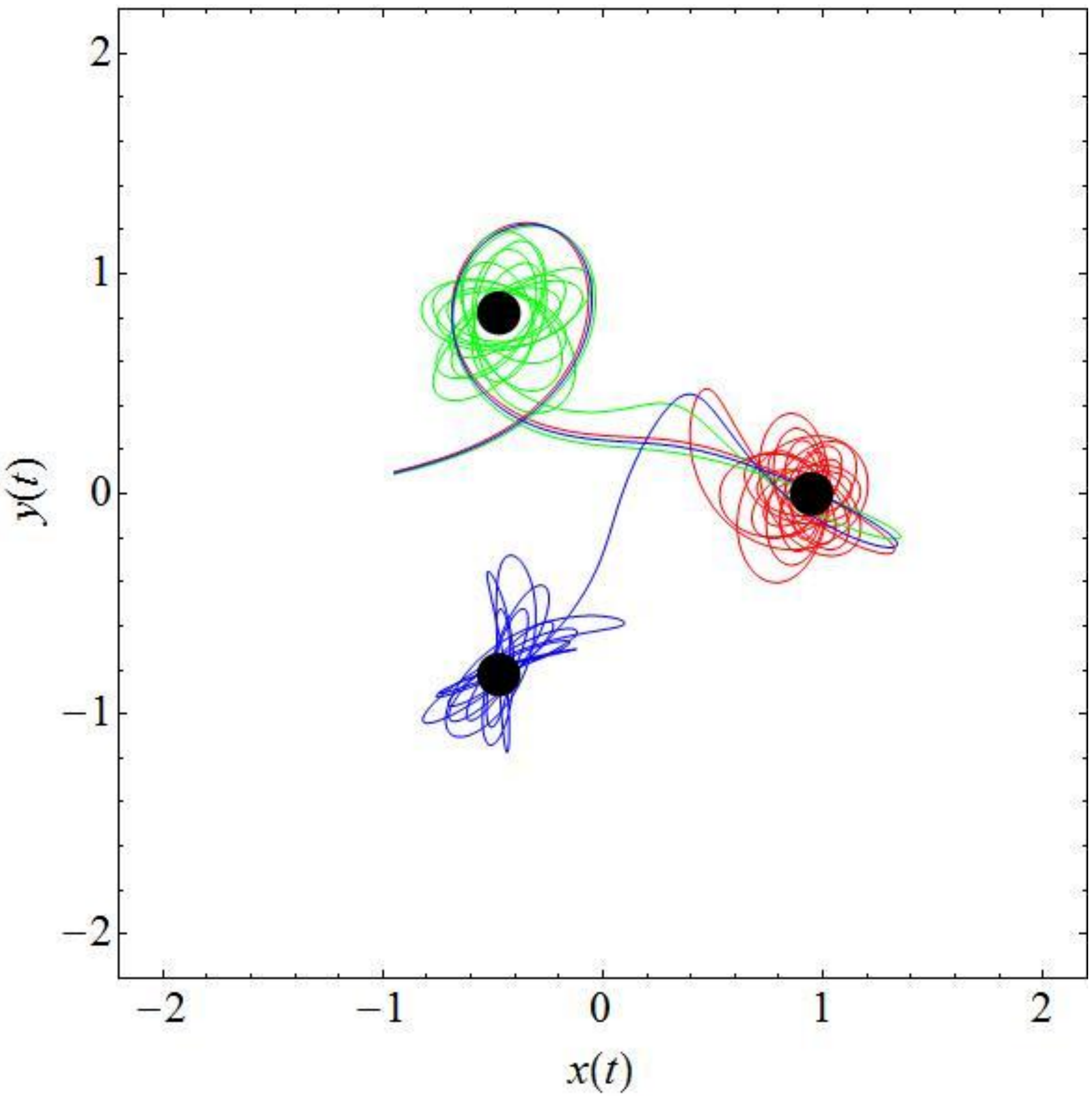


Fig. 1. Schematic diagram of the physical pendulum and the forces acting on it.

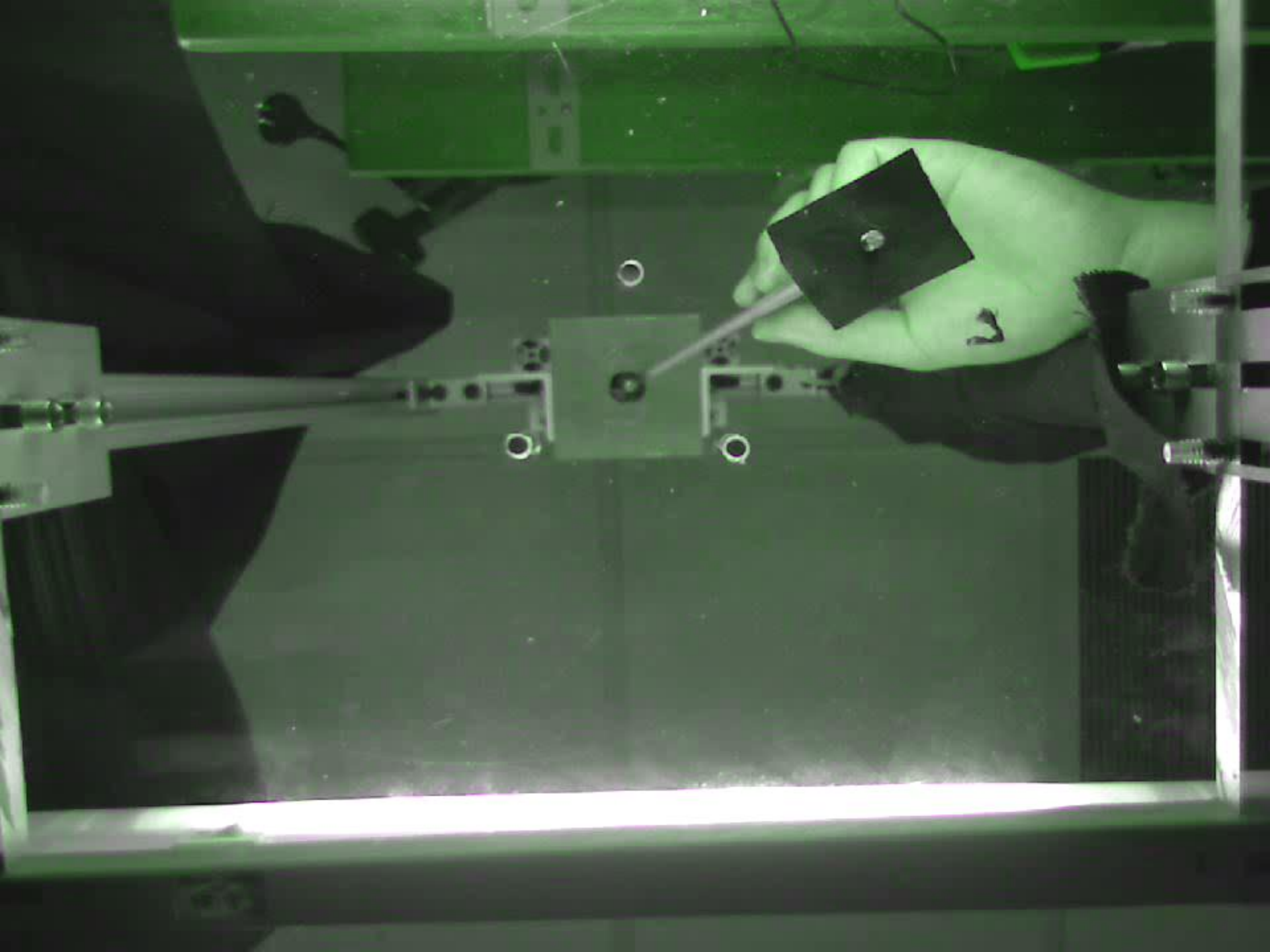
- Gravitational force
- Magnetic force
- Damping

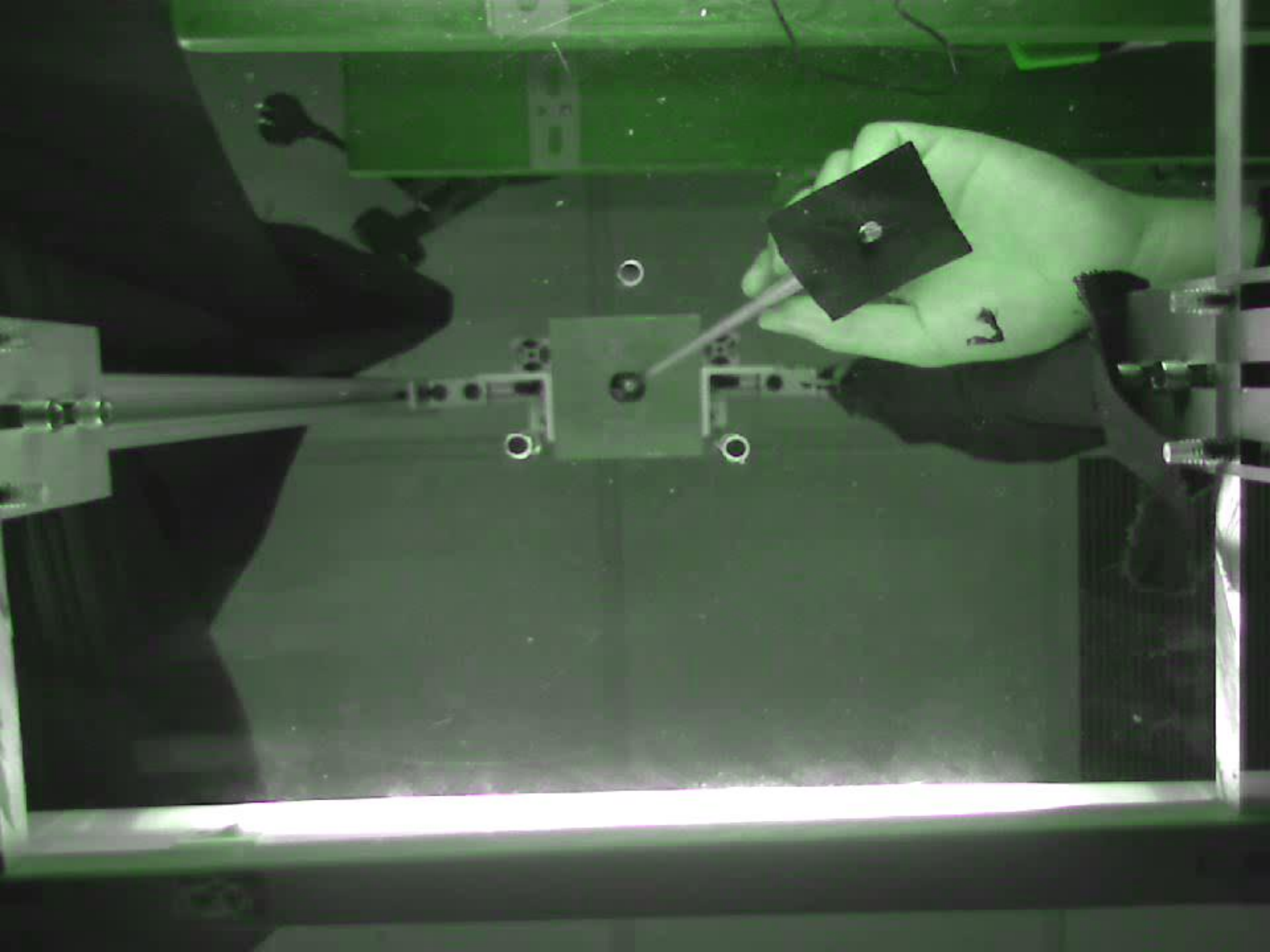
Final State Sensitivity

- The magnets are stable fixed points
- Simulation of trajectories from almost same initial conditions
- Example videos of a few runs (from almost same initial conditions)



2
1
0
−1
−2
y(t)
−2
−1
0
1
2
x(t)



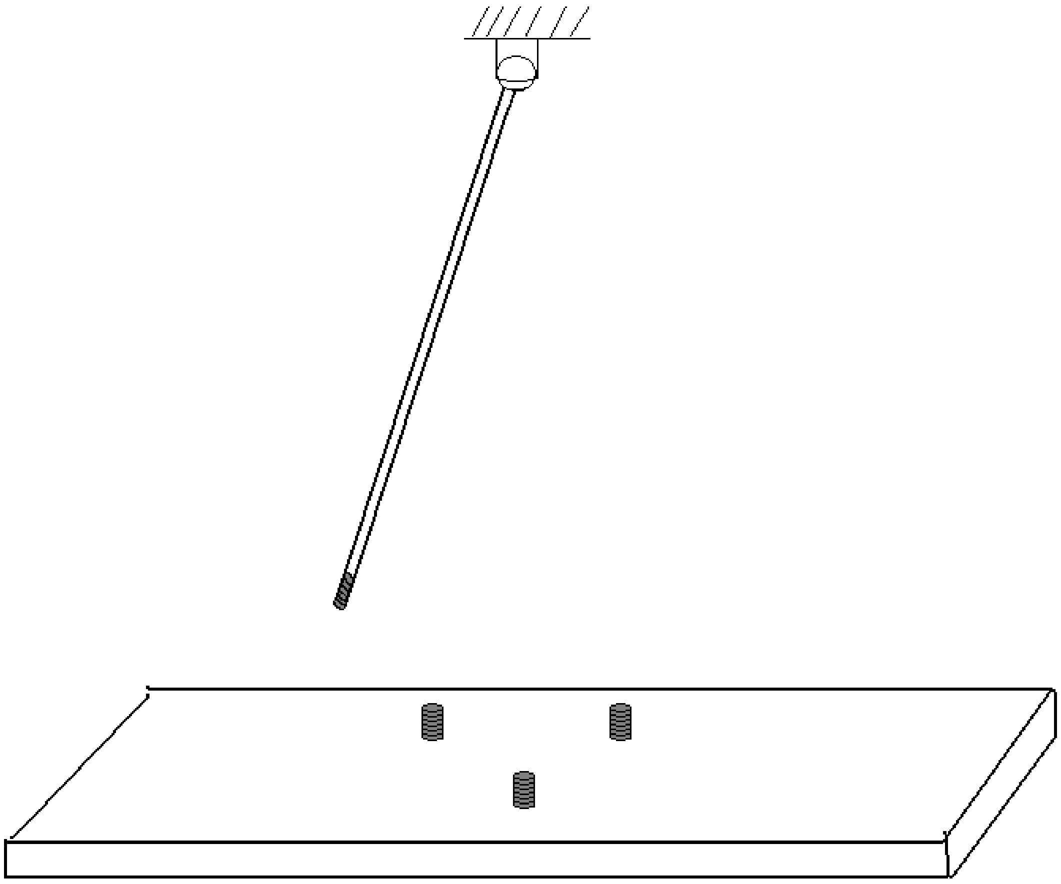


Goals

- Recreate fractal using experimental data
- Investigate the dynamics of the system
- Optimize parameters for simulations

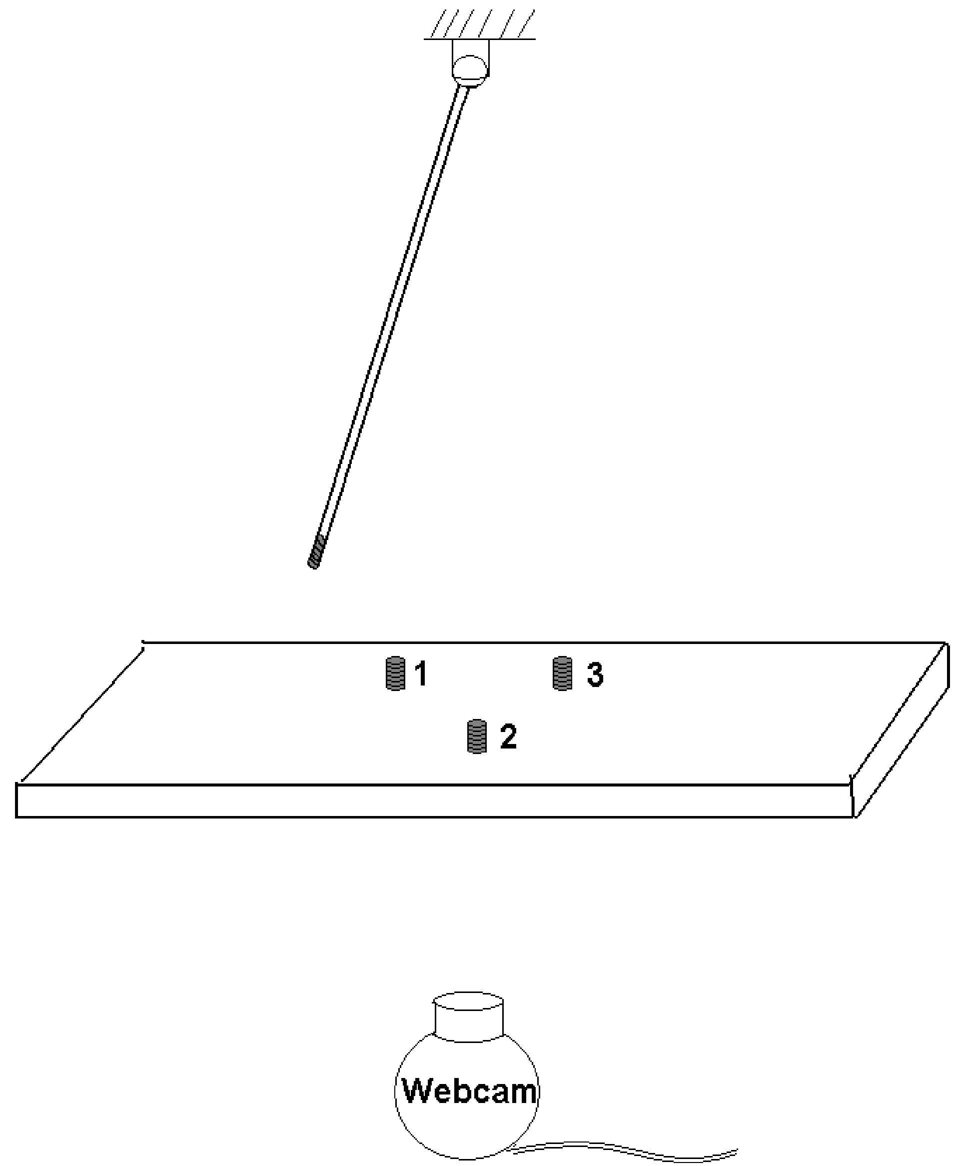
Experimental Setup

- Features
  - Pendulum
  - Free 2D pivot
  - Magnet on Rod
  - Base magnets
  - Correct Polarity “base attracting pendulum”



Measurements to Take

- Initial Conditions
  - Measured with Picture of initial state
- Final Position
  - Final position of pendulum
  - (1,2,3) or (r,g,b)

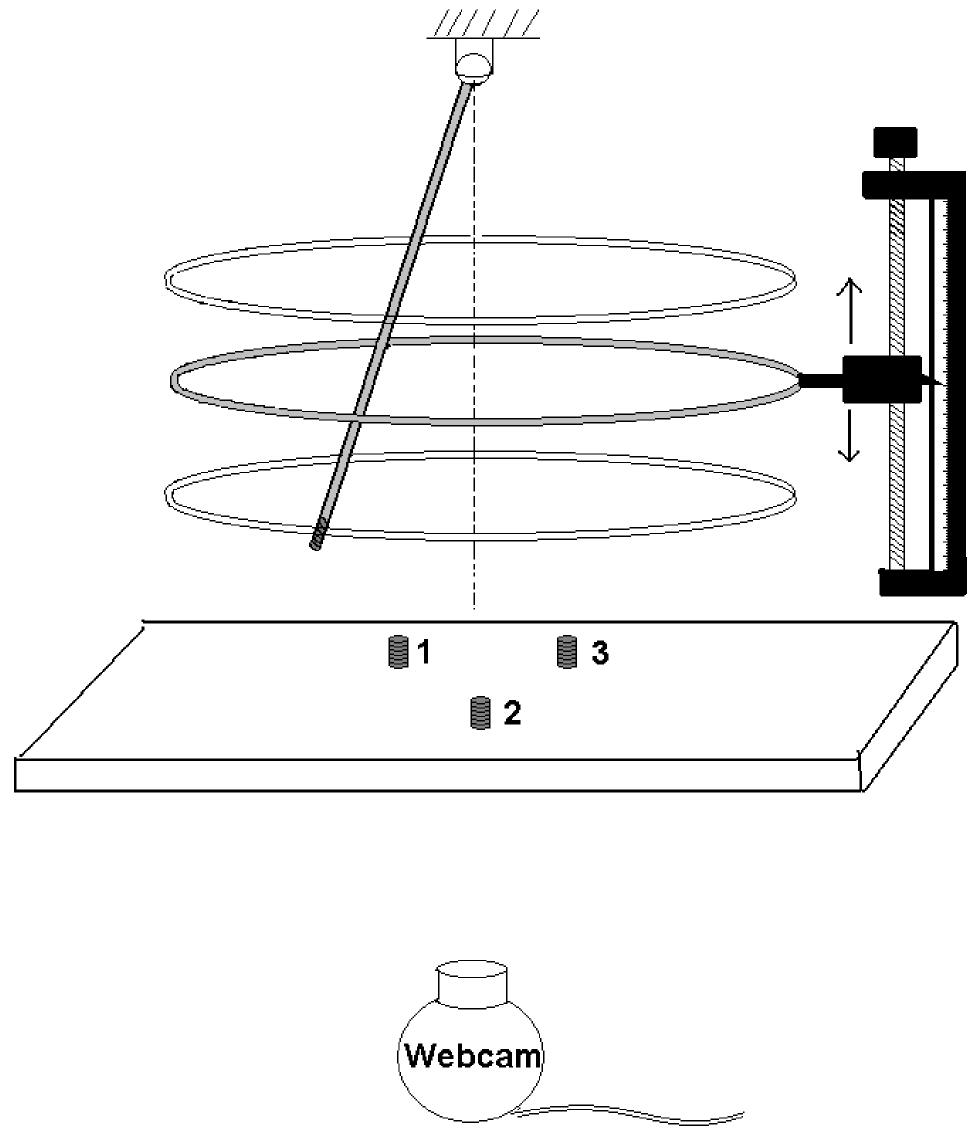


Things to Control

- Need initial Velocity to be zero

$$V_o = 0$$

- Stabilize on aluminum ring
- Better than trying to hold steady



Special Considerations

- Magnet Strength
- Length of Pendulum
- Mass of Pendulum
- Separation of Magnets
- Orientation of magnets
- Distance of Base magnets from Pendulum
- Friction in pivot joint

Physical Experiment

Data Acquisition

- Static Pictures
- Whiteout on base to help

- Final Position tagged onto photo

Difficulties

- zero velocity initial condition
- Light intensity variations

Image processing

- Need to find position of white dot
- Tried using a threshold and compute centroid, but did not work
- Marked position manually instead

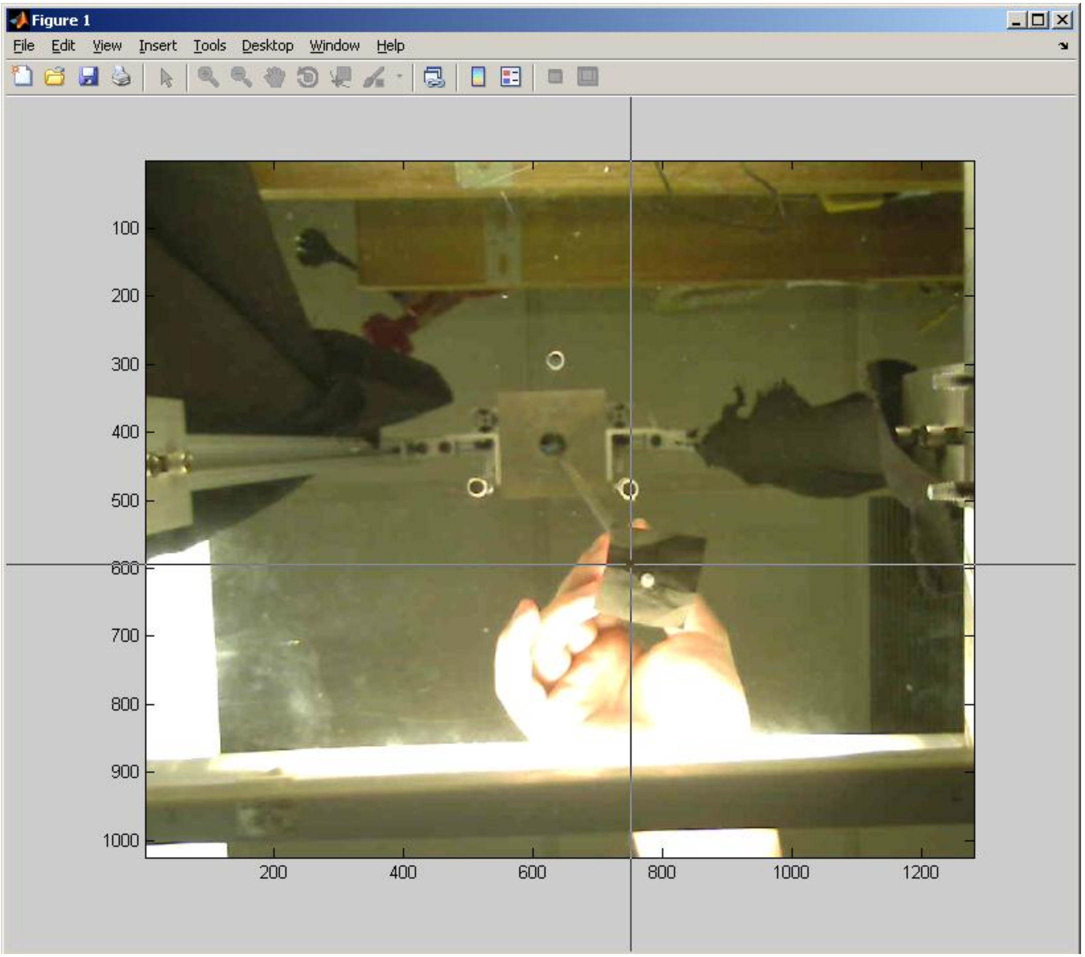


Figure 1
File Edit View Insert Tools Desktop Window Help
100
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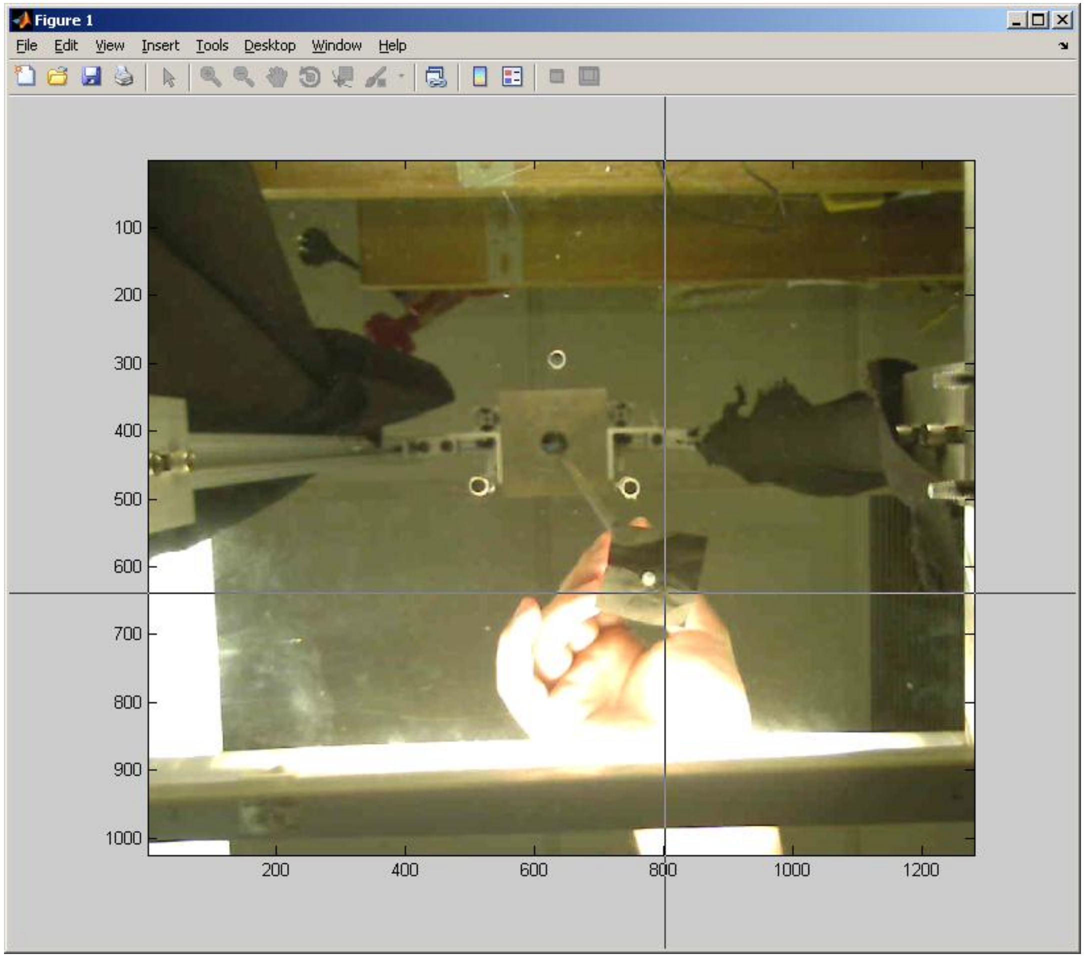
1200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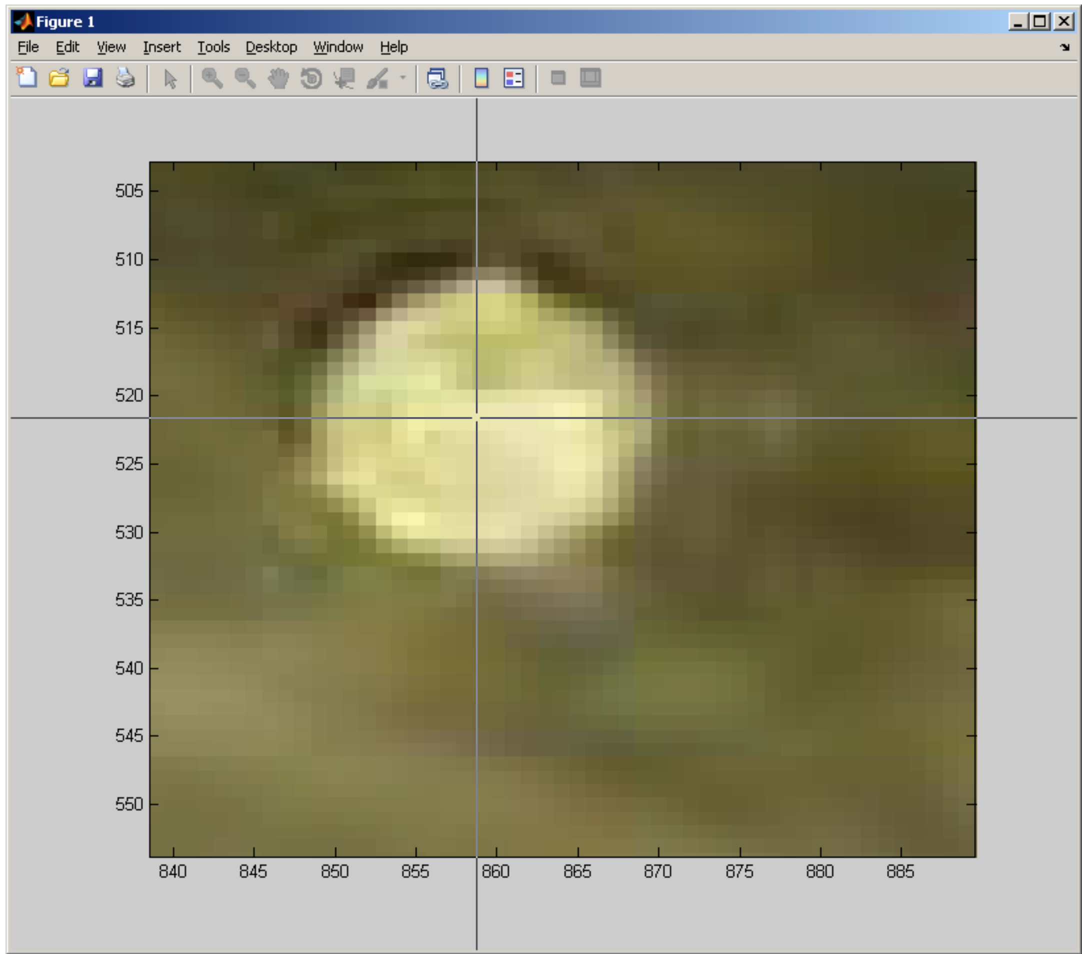
Figure 1
File Edit View Insert Tools Desktop Window Help
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870
875
880
885

Model

- Gravitational force: $$mF_g = -C \begin{bmatrix} x \\ y \end{bmatrix}$$

- Damping force: $$mF_d = -R \begin{bmatrix} x' \\ y' \end{bmatrix}$$

- Force from magnet:

$$mF_i = A \frac{1}{\left((x_i - x)^2 + (y_i - y)^2 + d^2\right)^{3/2}} \begin{bmatrix} x_i - x \\ y_i - y \end{bmatrix}$$

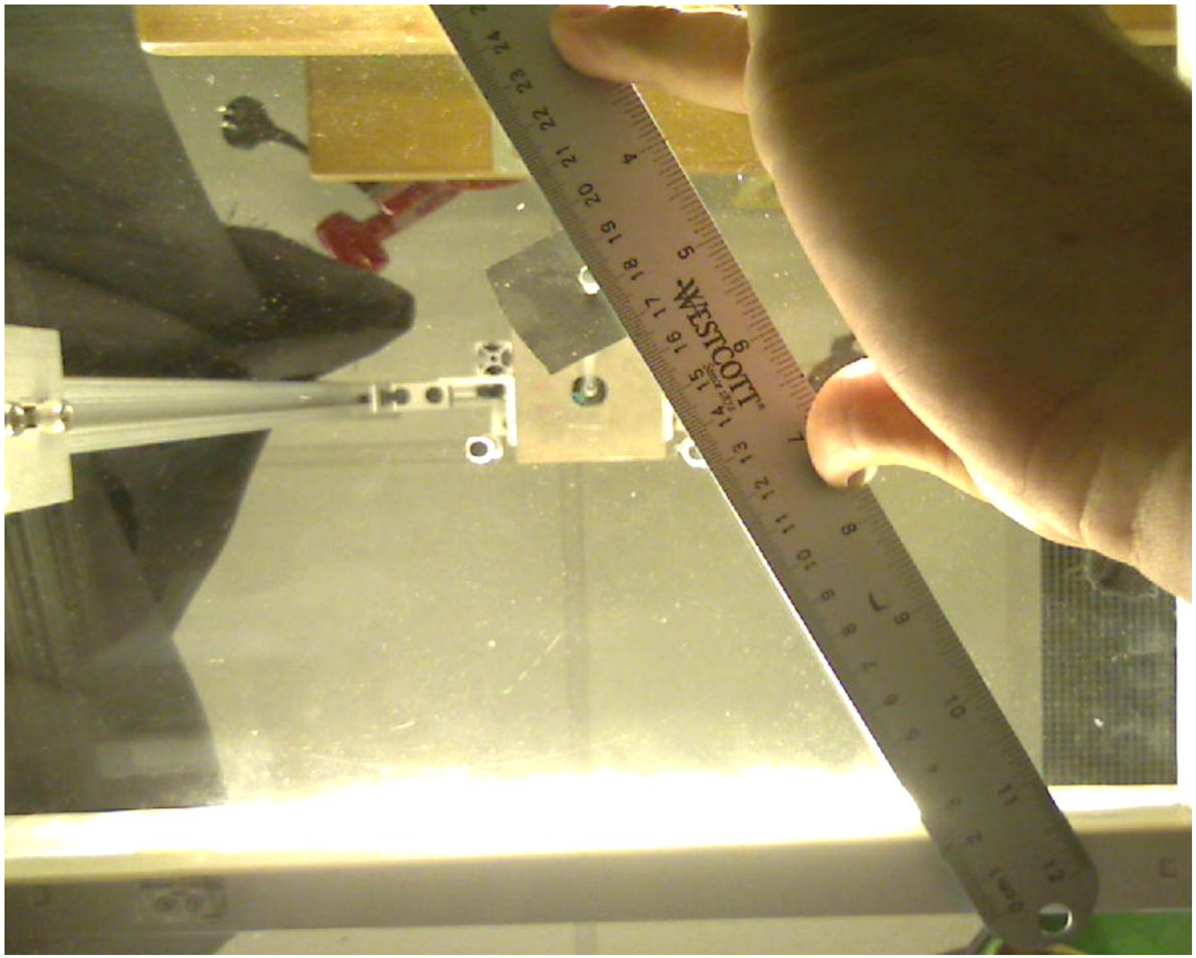
H. Peitgen, H. Jurgens, D. Saupe, Chaos and Fractals: New Frontiers of Science (Springer, 2004)

Model

$$x'' = -Cx + A\sum \frac{x_i - x}{\left((x_i - x)^2 + (y_i - y)^2 + d^2\right)^{3/2}} - Rx'$$

$$y'' = -Cy + A\sum \frac{y_i - y}{\left((x_i - x)^2 + (y_i - y)^2 + d^2\right)^{3/2}} - Ry'$$

Parameter estimation



Parameter estimation

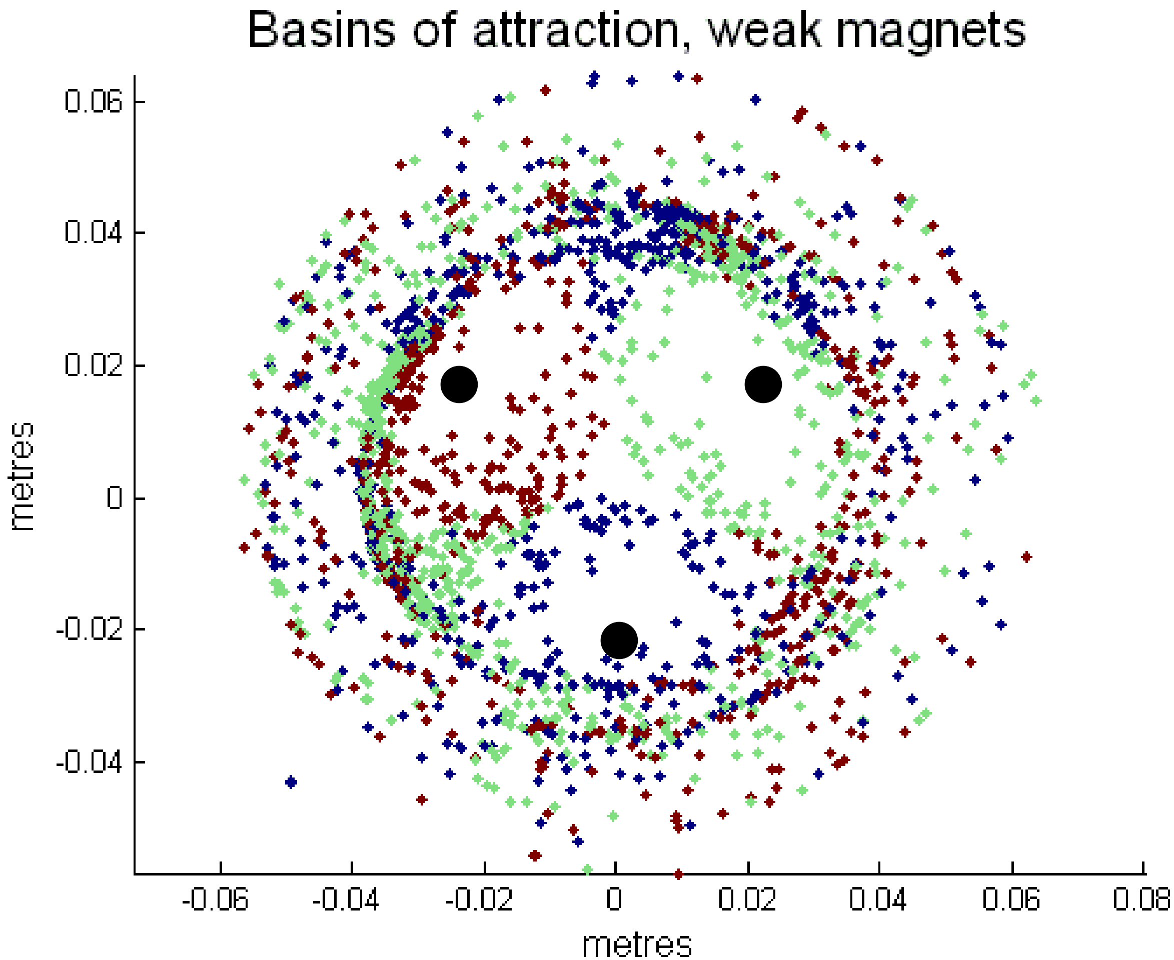
- Equation: $x'' + Rx' + Cx$
- Solution: $x \propto \exp(-\alpha t)\sin(\omega t)$
- Gives *R* and *C*
- *d* and *A* trickier

Parameter estimation

- Minimize

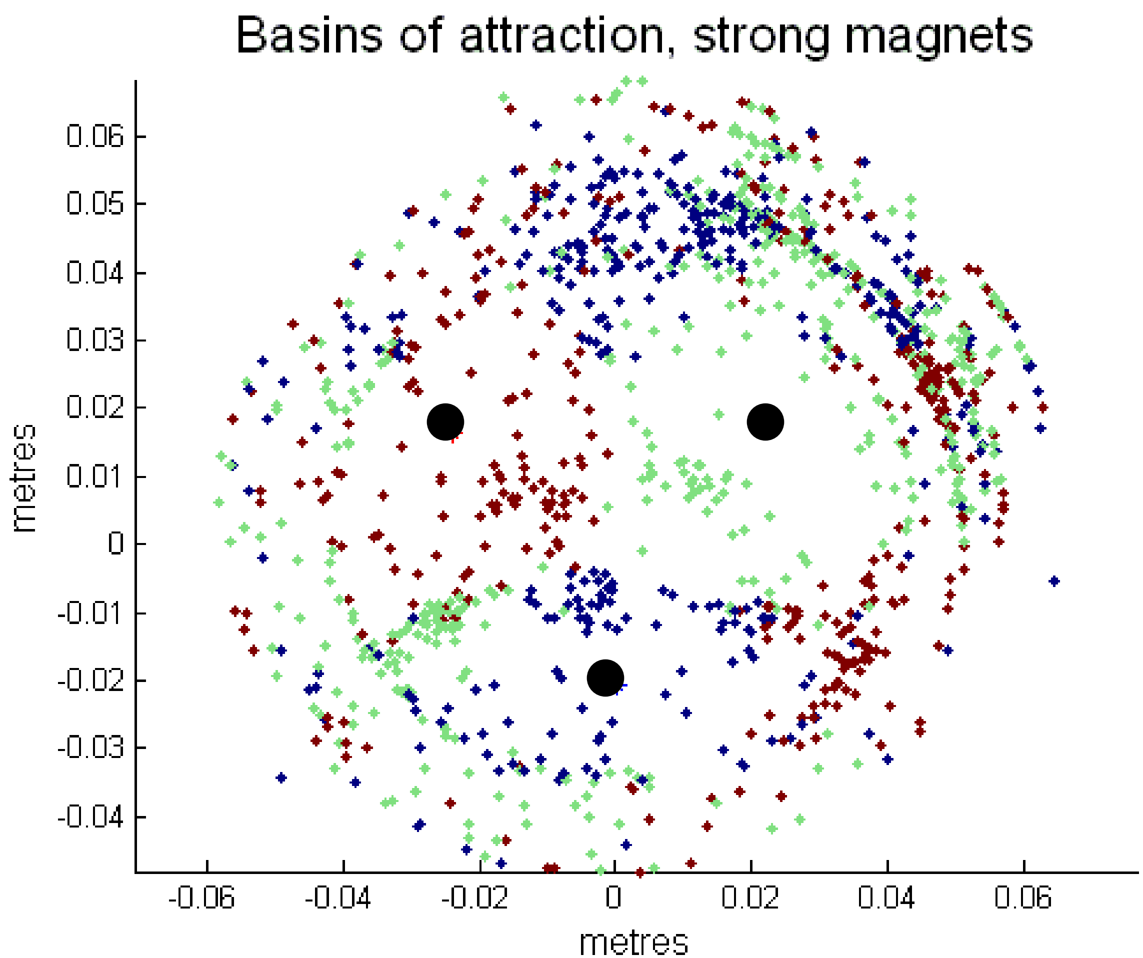
$$N(A, C, R, d) = \#\,\mathrm{misses}$$

- Improved agreement a bit but not much



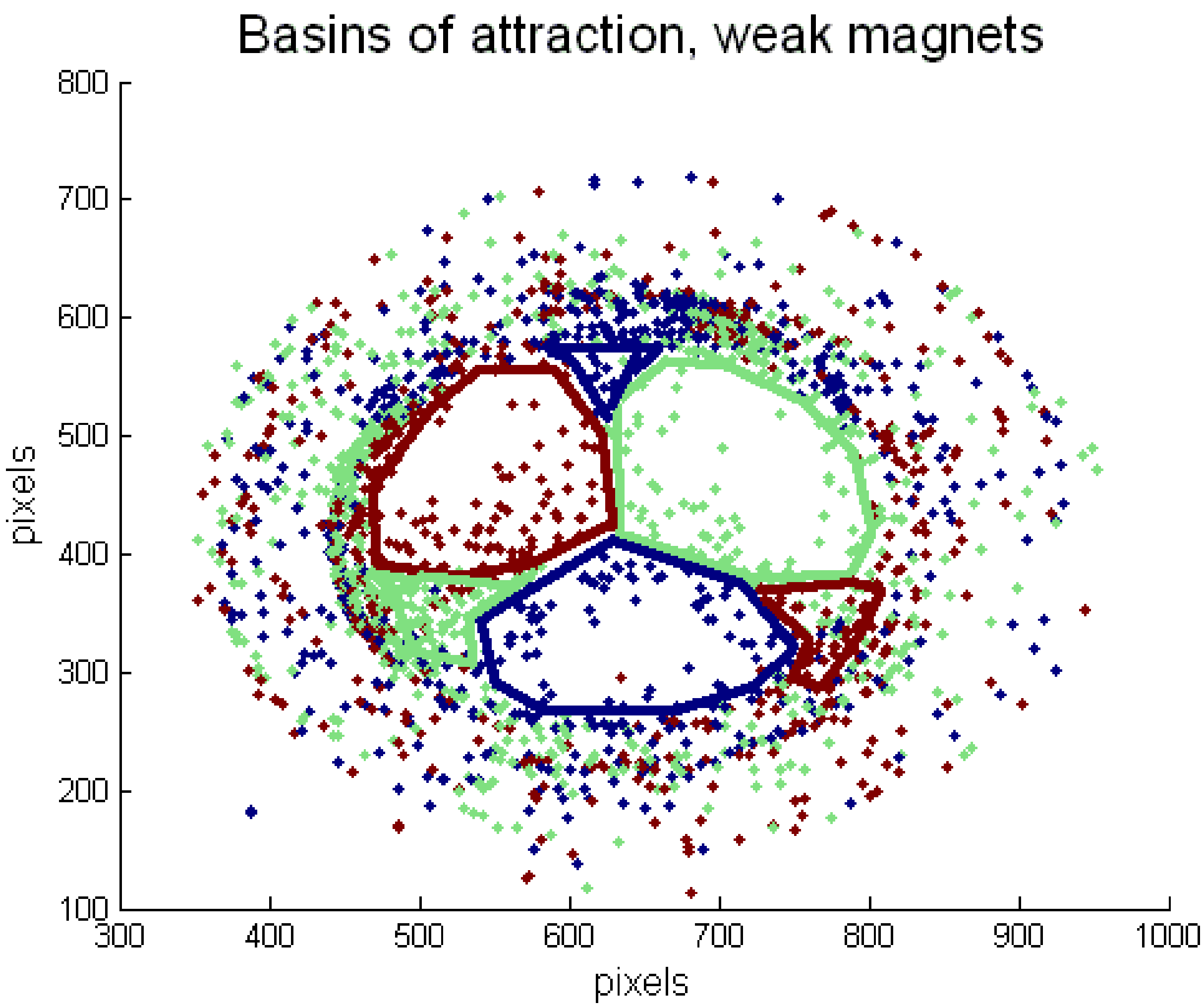
d ≈ 6 mm

Points ≈ 2300

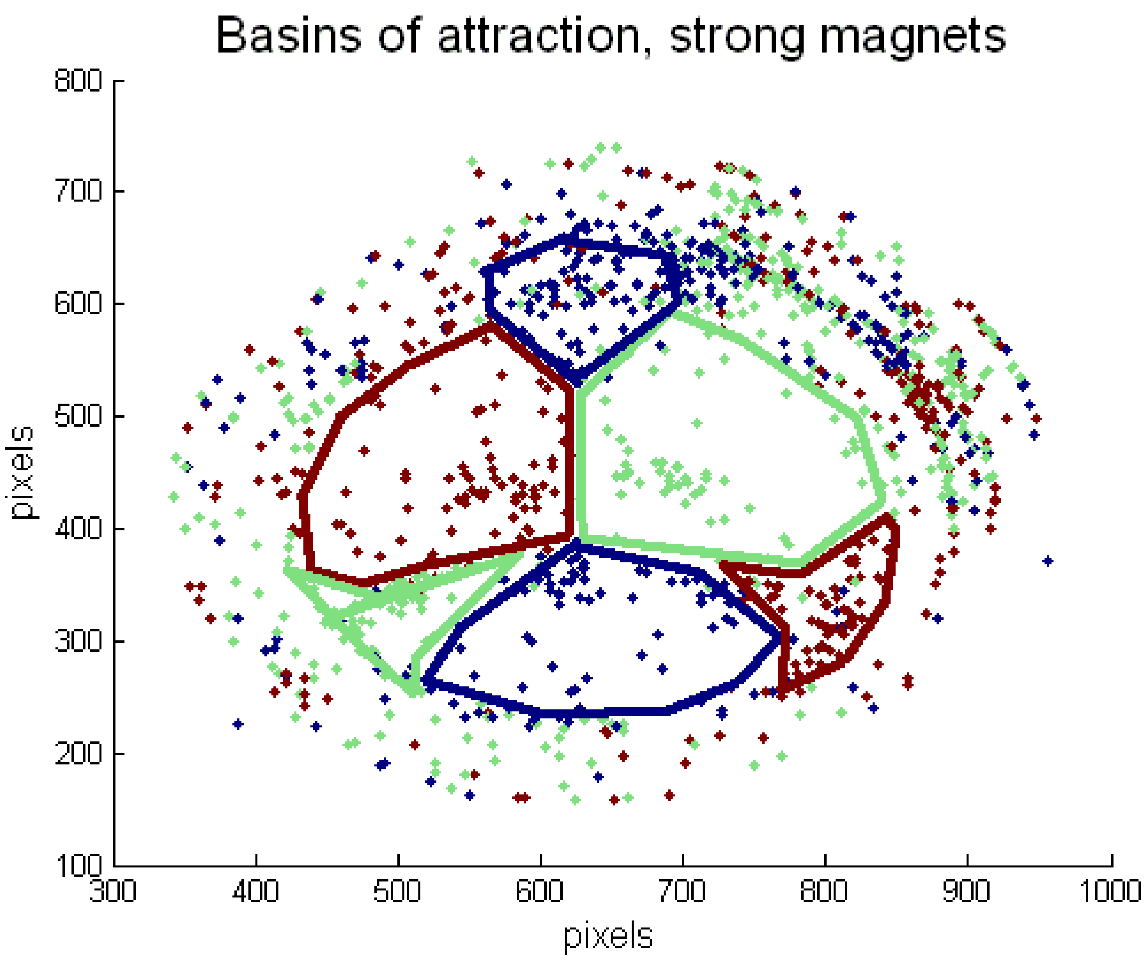


d ≈ 4 mm

Points ≈ 1200


Basins of attraction, weak magnets
800
700
600
500
400
300
200
100
300
400
500
600
700
800
900
1000
pixels
pixels

d ≈ 6 mm


Basins of attraction, strong magnets
800
700
600
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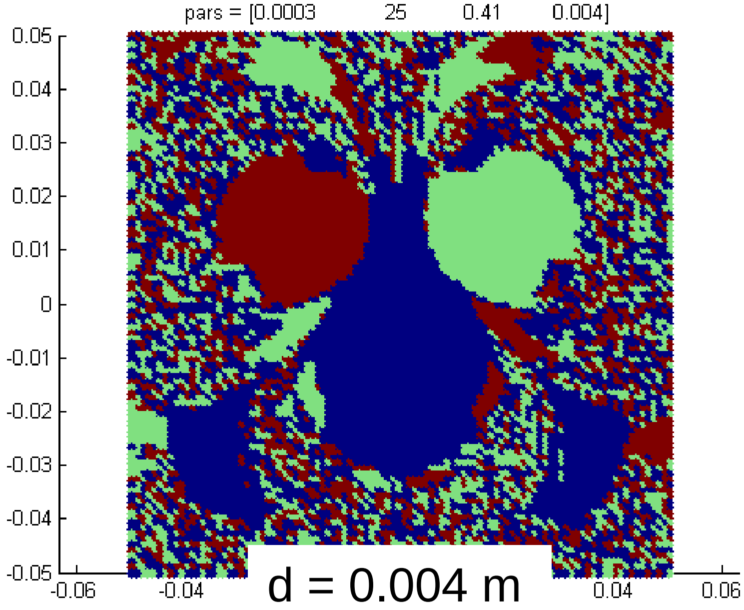
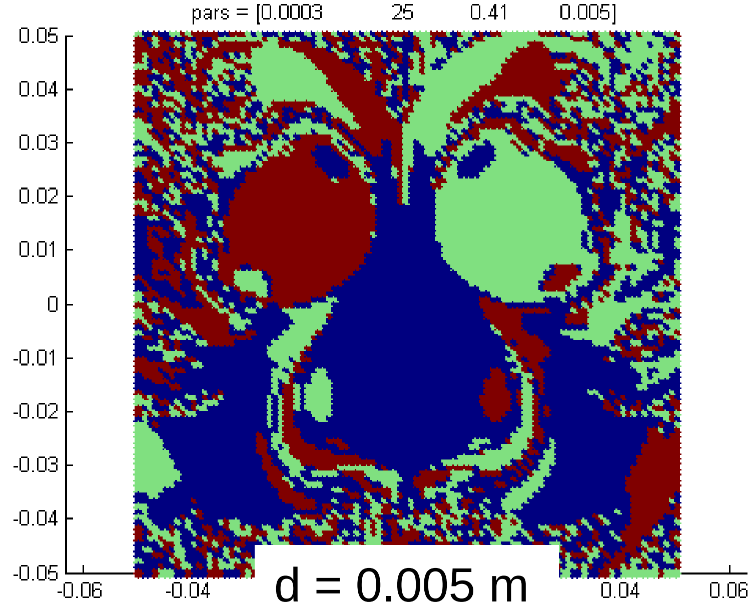
pixels
pixels

d ≈ 4 mm

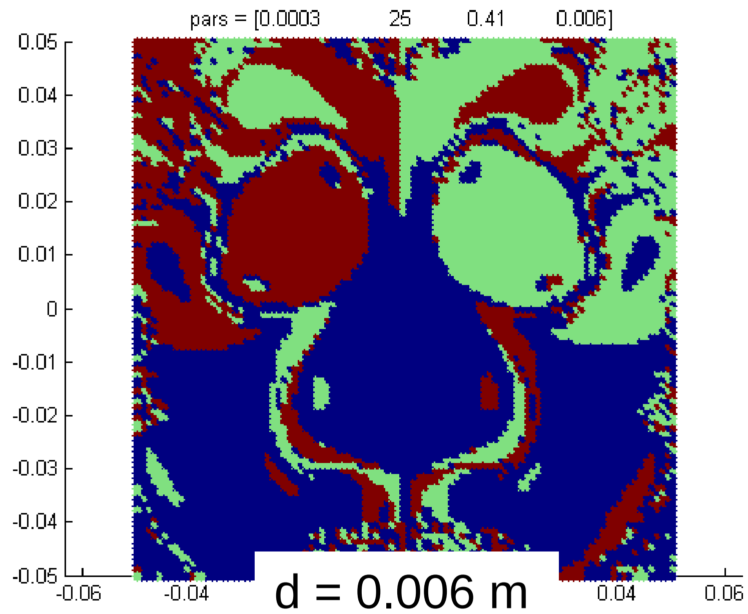


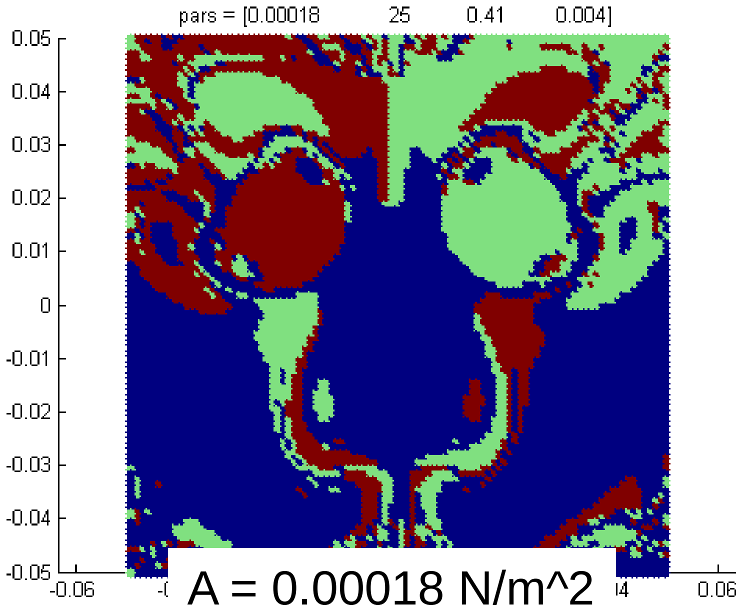


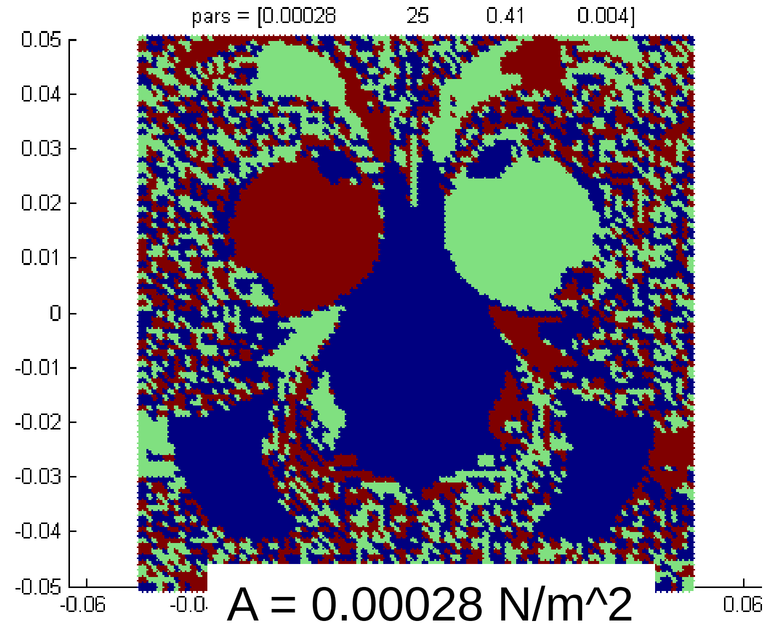
Simulations

Varying d

- C = 25 N/m
- R = 0.41 N/m·s
- A = 0.0003 N/m^2



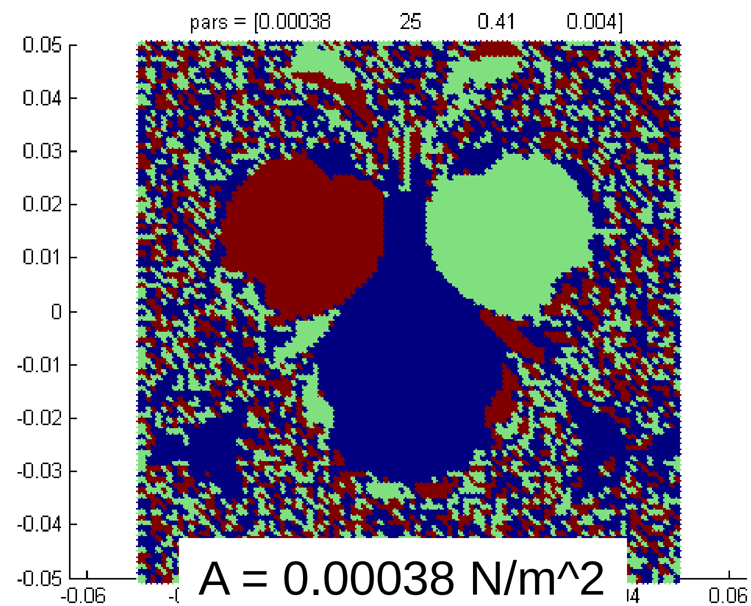


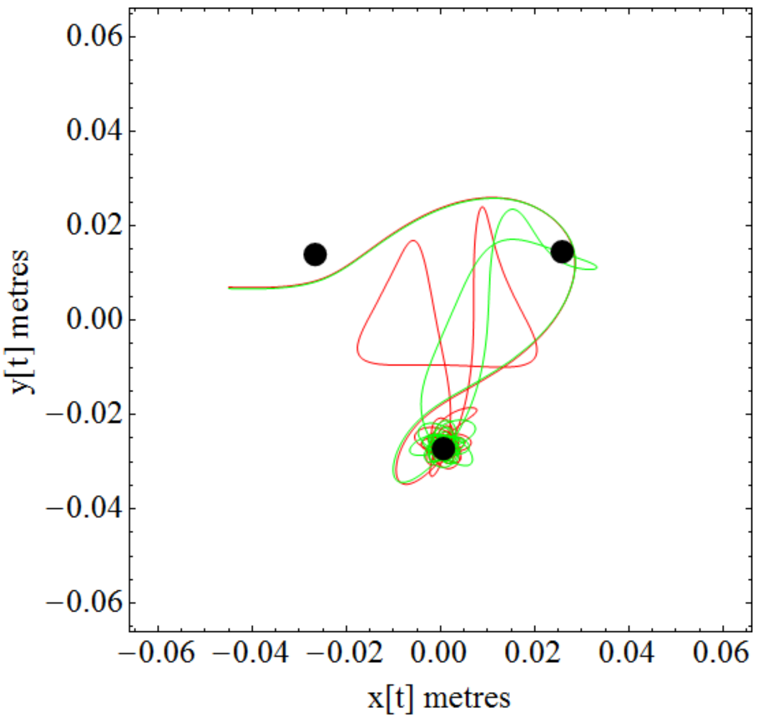


Simulations

Varying A

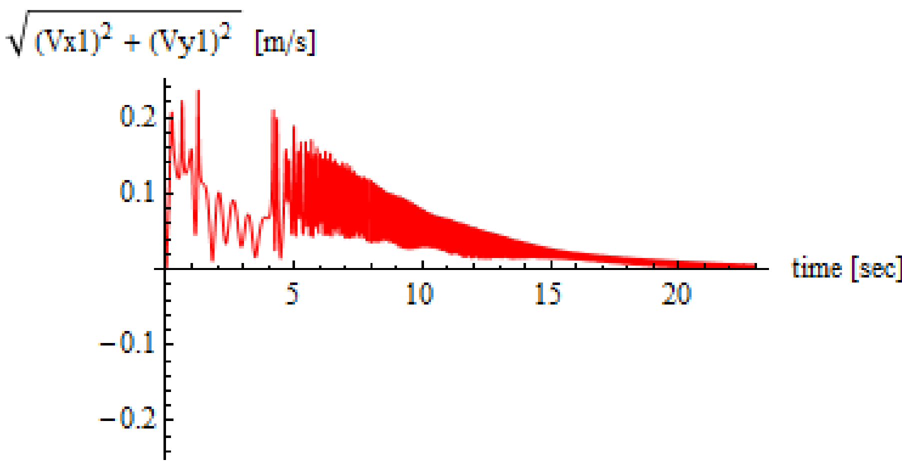
- C = 25 N/m
- R = 0.41 N/m·s
- d = 0.004 m

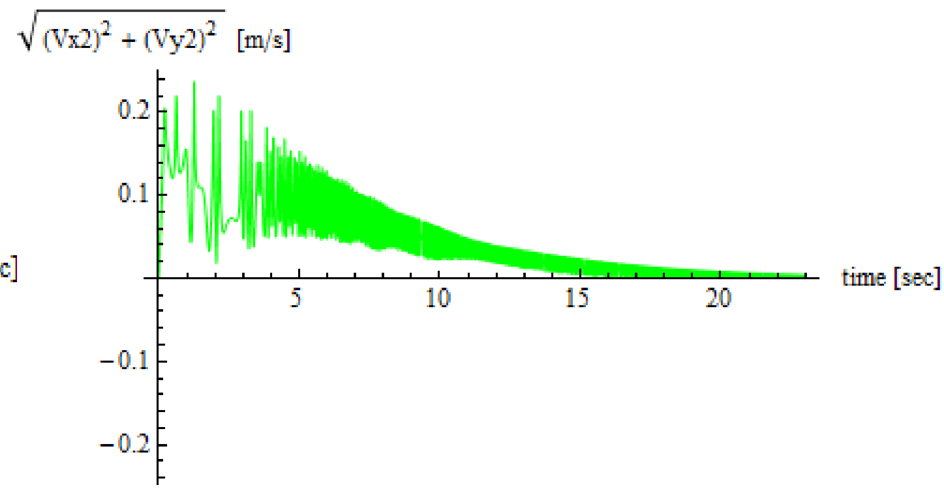




Time Dependence of Final State
(How long it takes to reach final state)

- C = 25 N/m
- R = 0.41 N/m·s
- d = 0.005 m
- A = 0.0001 N/m^2


$\sqrt{(Vx1)^2 + (Vy1)^2}$ [m/s]
0.2
0.1
−0.1
−0.2
5
10
15
20
time [sec]


$\sqrt{(Vx2)^2 + (Vy2)^2}$ [m/s]
0.2
0.1
−0.1
−0.2
5
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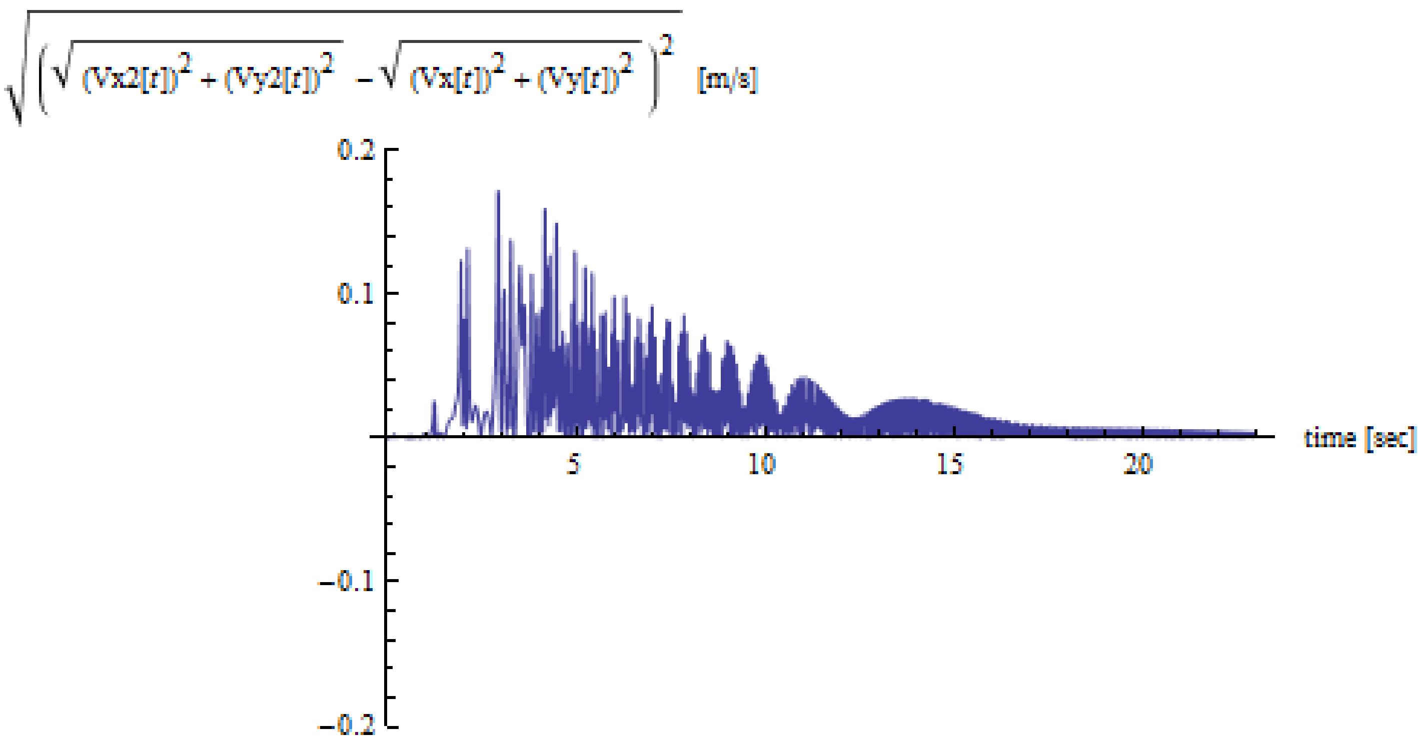
time [sec]


$\sqrt{\left(\sqrt{(Vx2[t])^2 + (Vy2[t])^2} - \sqrt{(Vx[t])^2 + (Vy[t])^2}\right)^2}$ [m/s]
0.2
0.1
−0.1
−0.2
5
10
15
20
time [sec]

Conclusions

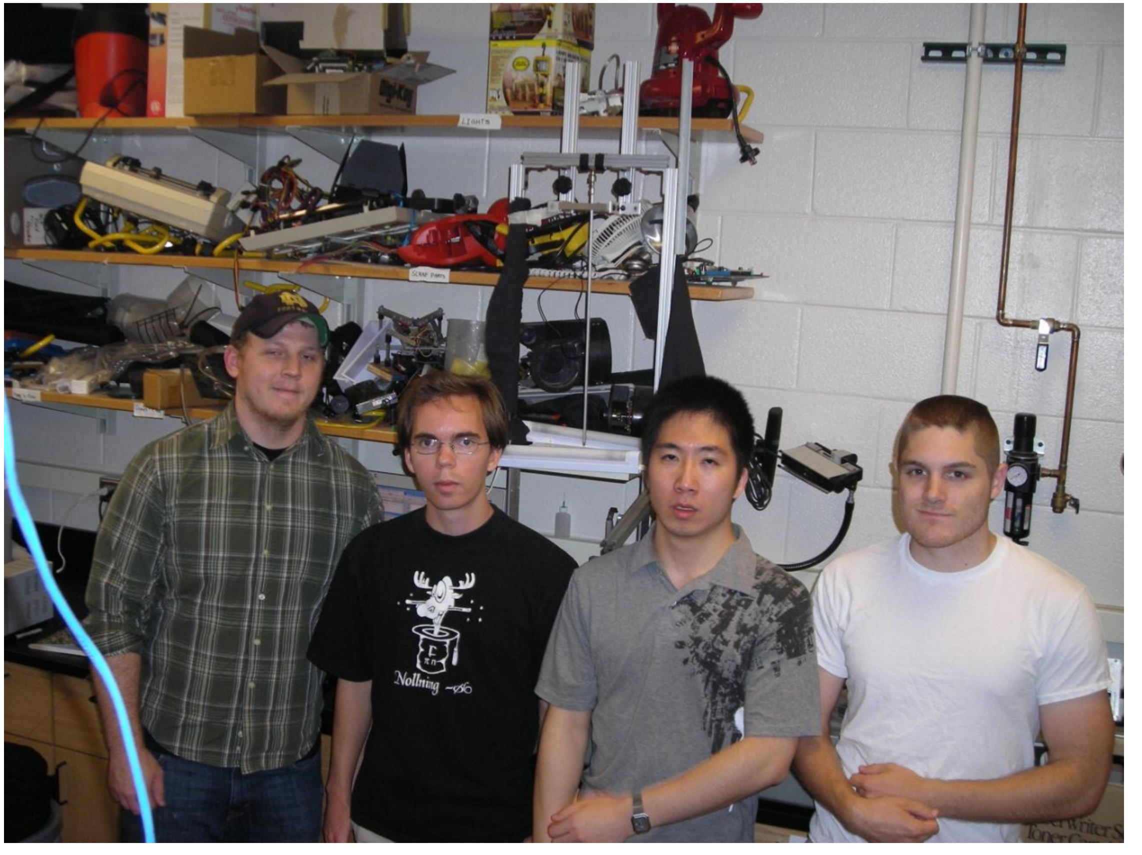
- Varying Opinions
  – Opinion 1: Experiment is so flawed that the measured points must be rejected. Simulation accepted
  – Opinion 2: Data is good. Reduction of dimensionality in simulation model is a poor approximation
- Agreement
  – Chaotic systems are difficult to probe experimentally

Improvements

- Automate: More points and less variance in initial velocities
- Working parameter optimization algorithm
- Automatic centroid tracking

Contributions

- Fredrick
  - Experimental Design
  - Data Acquisition
  - Data Analysis
  - Simulation Code
- Chris
  - Experimental Design
  - Data Acquisition
  - Data Analysis
- Jung
  - Experimental Design
  - Data Acquisition
  - Data Analysis
- Galen
  - Experimental Design
  - Data Acquisition
  - Data Analysis
  - Simulation Code



Acknowledgments

- Andrei
- Nick
- Dr. Goldman

Thank You