# Experimental Characterization of Chua's Circuit

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A chaotic system is aperiodic, deterministic, and very sensitive to initial condition. There are many applications for chaotic system, such as in atmosphere, human body, or communication. Chaotic system can be used to ensure a more secure communication by masking the information with chaotic signal. The chaotic signal can be produced by a simple autonomous circuit. The simplest autonomous circuit is Chua's circuit. Its ease of construction and manipulation makes the Chua's circuit the most favorable circuit used to study chaos. In this experiment, a single Chua's circuit was constructed and voltage data was collected. Nonlinear dynamics of the Chua's circuit data was analyzed by means of plotting phase space to determine the route to chaos, computing the Lyapunov exponent to characterize the chaos, making iterative map to show patterns of consecutive peaks, reconstructing the attractor by Time Delay Method to predict the 3D phase space

## I. INTRODUCTION

Chaos is aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions[11]. There are many applications for chaotic system, such as predicting nonlinear flow in atmosphere, understanding the dynamics in cardiac arrhythmias, handwritten character recognition. There was even studies show that chaotic system made by circuits can produce novel musical sounds[3]. However, in this experiment, we focus more on applications of chaotic system in communication. Chaotic signals are usually broad-band, noise like, and difficult to predict[5]. Therefore chaotic systems are suitable to carry information. In digital communication, sinusoidal carriers are used because they have optimal bandwidth efficiency and a relative ease of reconstruction of the original signal. However, their high power spectral density would cause a high level of interference and enhance the probability of interception by other receivers[1].

Chaotic carrier can solve this problem. A chaotic transmitter can send the desired information masked by chaotic signal. This masked signal is then transmitted to the destination, and the signal might be intercepted on the way but would not be decoded without the masking chaotic signal. At the destination, another chaotic signal generator will be synchronized with the chaotic signal generator at the transmitter and thus can produce the exact replica of the masking chaotic signal. The masked signal will then be demodulated by subtracting the masking signal from it to reveal the desired information hidden within[6]. Also, broadband information carriers enhance the robustness of communication and are less vulnerable to interference from narrow-band disturbance. This is the basis of spread-spectrum communication techniques, such as the code division multiple access (CDMA) protocol used in the Global Positioning System (GPS) and in the third generation of mobile telephones[2]. In order to achieve masking information in chaotic signal to obtain secure communication, the chaotic oscillators, transmitter and receiver, must be synchronized. Initially, we want to achieve synchronization between two or more Chua's circuits acoustically[8]for the following reasons. Chua's circuit, introduced to the world by Leon Chua in 1983, is the simplest autonomous circuit that can exhibit bifurcation and chaos, because it satisfies the three criteria for displaying chaotic behavior: containing one or more nonlinear components, one or more locally active resistor, and three or more energy-storage devices. It has been a very suitable subject for studying dynamical chaos by means of both laboratory experiments and computer simulations[8]because Chua's circuit's behavior can varies just by changing some parameter and be seen easily with an oscilloscope. Acoustic coupling enables us to adjust various parameters to achieve synchronization, e.g. wave amplitude, feedback time delay, characteristic of media between the oscillators, etc. However, after setting up our circuits and the acoustic component, we found out that the parts we had are too weak to communicate with the other circuits. Because of the time constraint in this class, we didn't have time to solve this problem and achieve synchronization between circuits. In the process of building up the Chua's circuits and adjusting their parameters to find chaotic signal, we found some interesting nonlinear behavior of the Chua's circuit. So we decided to change lane and focus on characterizing the nonlinear dynamics of the Chua's circuit. People have been characterizing Chua's circuits by computer simulation, so we want to do it from an experimental point of view.

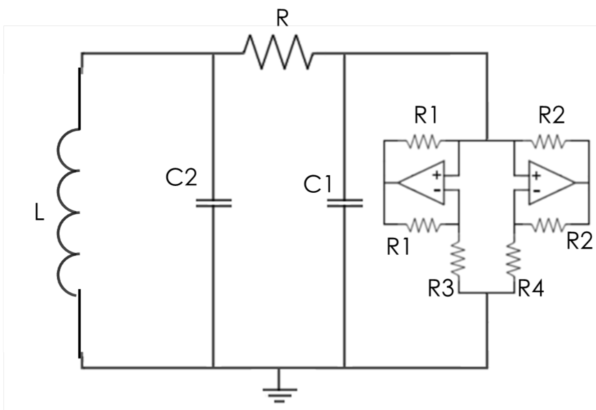


FIG. 1. Circuit diagram of Chua's circuit

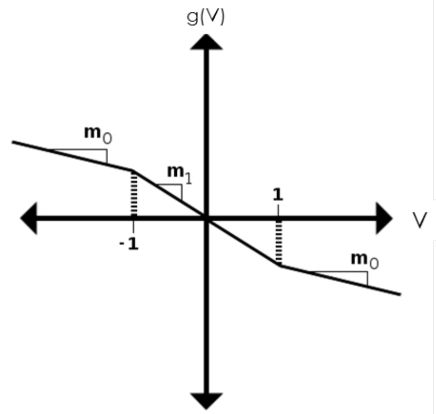


FIG. 2. V vs I of nonlinear resistor

## II. METHODS

One of the reasons that Chua's circuit was ideal for studying chaotic behavior is its ease of construction. The circuit was made by the most basic components: resistors, inductors, capacitors and operational amplifier ( op-amp). There are many different versions of Chua's circuit. The design we chose was best for our purpose of acoustically synchronizing the circuits (see FIG.1). The nonlinear part of the circuit was made by six resistors and two op-amps; we called it the nonlinear resistor. It produces a piece-wise function of the resistor in response to different voltage across it (see FIG.2). With the current of the nonlinear resistor, g(V), the equations governing Chua's circuit can be derived from Kirchhoff's law and voltage law:

$$C1(dV1/dt) + g(V1) = (V2 - V1)/R \quad (1)$$

$$C2(dV2/dt) + (V2 - V1)/R = I \quad (2)$$

$$L(dI/dt) + rI + V2 = 0 \quad (3)$$

C1, C2 and V1, V2 are capacitors and voltage across the two capacitors. R is the resistance for the varying resistor, the potentiometer. I, L and r are the current, inductance and resistance of the inductor respectively. The parameters we used are the following: L=15mH, C1=5nF, C2=100nF, R1=220 Ω, R2=22k Ω, R3=2.2k Ω, and R4=3.3k Ω. We built the circuits on bread board and used computer to sample the voltage across the two capacitors at a rate of 48kHz.

## III. RESULTS

### A. Route to Chaos

When we change the resistance of the potentiometer, dynamics of the voltages across the two capacitors change as well. We started from 0.9 Ω because that's the lowest resis-

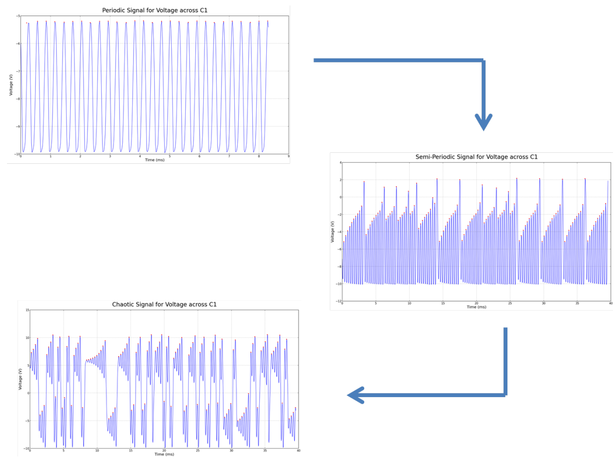


FIG. 3. Route to Chaos in Time Series

tance of the potentiometer and slowly increased the resistance. At first, the voltage showed periodic oscillation, but when the resistance reached 1.9k Ω, the voltage became aperiodic, and the phase space plot of the two voltages formed double scroll. When we keep increase the resistance to 1.982k Ω, the phase space plot exhibited a screw attractor formation. When we increased the resistor further, the dynamics went back to periodic oscillation. Our resulting route to chaos for Chua's circuit confirms the result from previous experiments[8].

## B. Lyapunov Exponent

The Lyapunov exponent shows how two nearby points diverged from each other over time. If positive, then the two trajectories diverge exponentially, and the system becomes chaotic. On the other hand, if the exponent is negative, then the two trajectories converge exponentially and chaos is not observed. We first pick two points that are close to each other on the trajectory in the phase space, and then measure the distance between them over time. From the time series of the signal (see FIG.6), we can see that the two points started out in sync with each other, but diverge into two different trajectories very quickly. Because we expect exponential divergence from the distance of the two trajectories, we plot the separation distance (D) as log(D) vs time. As we expected,

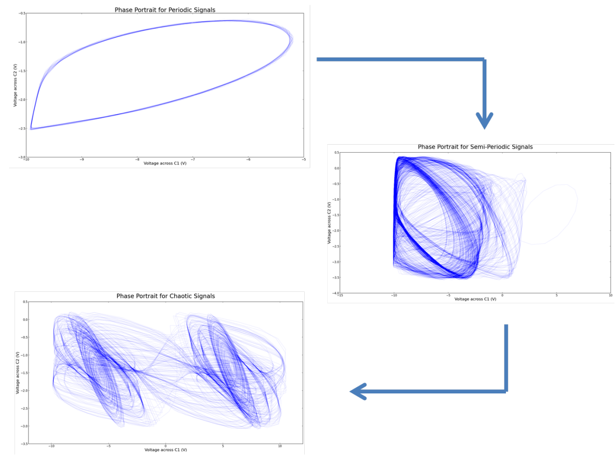


FIG. 4. Route to Chaos in Phase Space

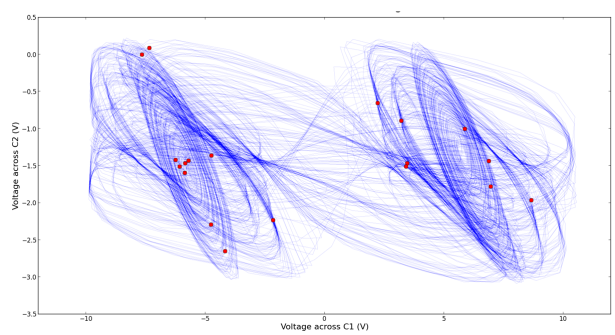


FIG. 5. Initial Positions

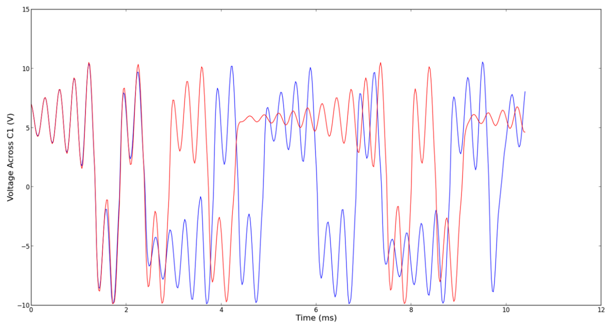


FIG. 6. Trajectories of Close Points over Time

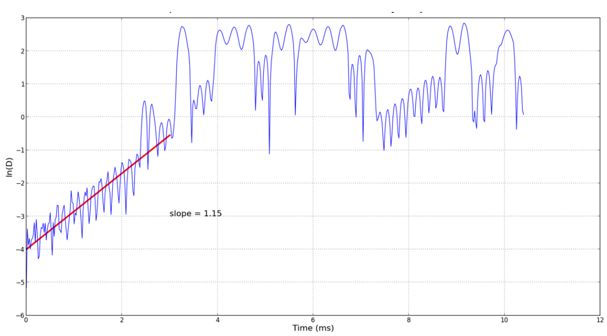


FIG. 7. Line Fitted Separation Distance of Nearby Trajectories

the plot is linear at the beginning. The slope of the linear part is the Lyapunov exponent (see FIG.7). Lyapunov exponent is sensitive of the initial conditions, so we average over several measurements. Since we have two attractors in our double scroll, we obtained two different average Lyapunov exponents: 1.01 for one attractor and 1.65 for the other. Because of the initial condition sensitivity, it is hard to compare our data with those found in literature.

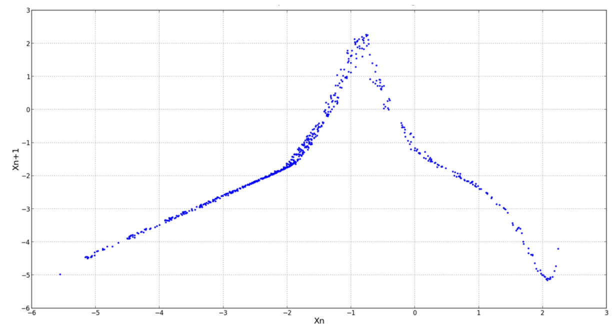


FIG. 8. Iterative Map for Semiperiodic Signal

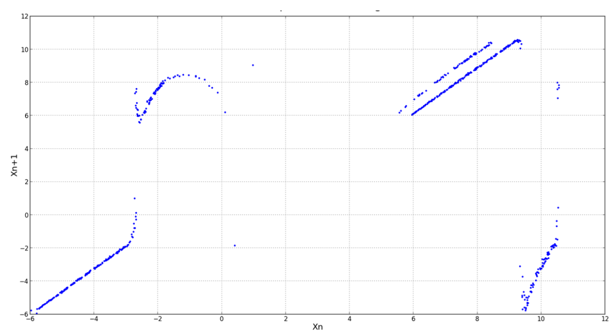


FIG. 9. Iterative Map for Chaotic Signal

## C. Iterative Map

Another examination we did was looking at the peaks of the voltage in time series. Plotting the values of the peaks vs the next one gave us the iterative map. When the circuit's voltage was semi-periodic, when it had screw attractor, the iterative map was a tent plot, which coincided with the Lorenz map[11]. When the circuit had gone chaotic, the iterative map became out of order too. Although there was some linearity in the map, the plot in whole did not reveal any more patterns (See FIG.9).

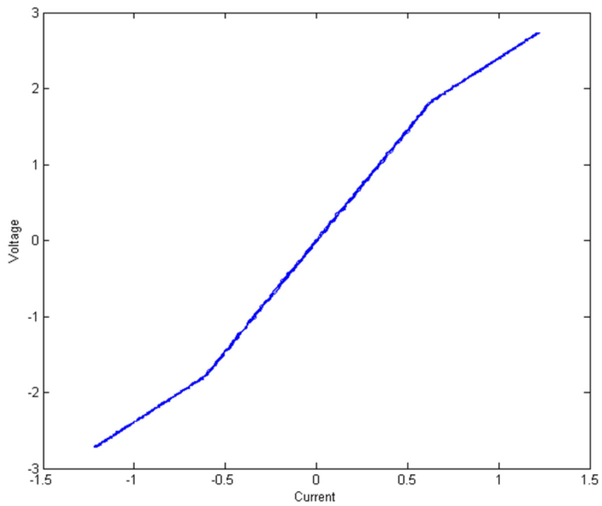


FIG. 10. Piecewise Function for Nonlinear Resistor

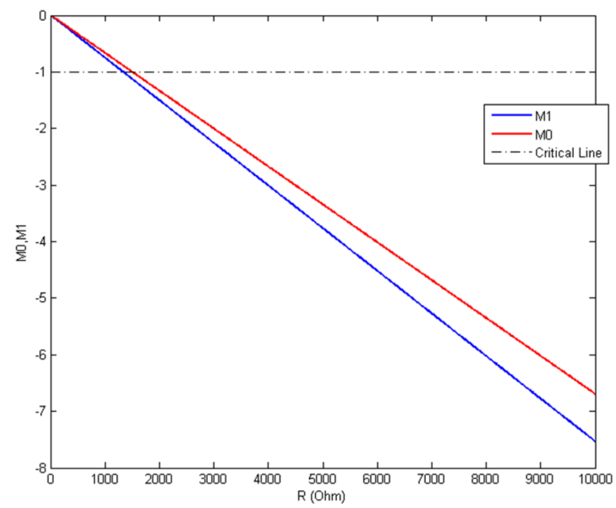


FIG. 11. Check the Critical Condition of m1 and m0

## IV. DISCUSSION

### A. Lack of Period Doubling

There are other routes for the Chua's circuit to head for chaotic behavior. For one, the voltage oscillation could go through period doubling[4]. We tried to manipulate or circuit to achieve period doubling, but failed to do so. So we went back to do some control about the parameters. In Chua's paper, he mention that the parameters have to satisfy $m1 * R < -1$ and $-1 < m0 * R < 0$, where R is the resistance of the potentiometer and m1 and m0 are the nonlinear resistance (slope in FIG.2). After finding the current-voltage plot for our circuit (see FIG.10), we found that there were only a very small window of the resistance that allow our circuit to have period doubling (see FIG.11). However, the potentiometer we have does not have such a fine precision to achieve that. Meanwhile, we used computer simulation to confirm that with the appropriate parameters, our version of Chua's circuit can also produce period doubling (see FIG.12).

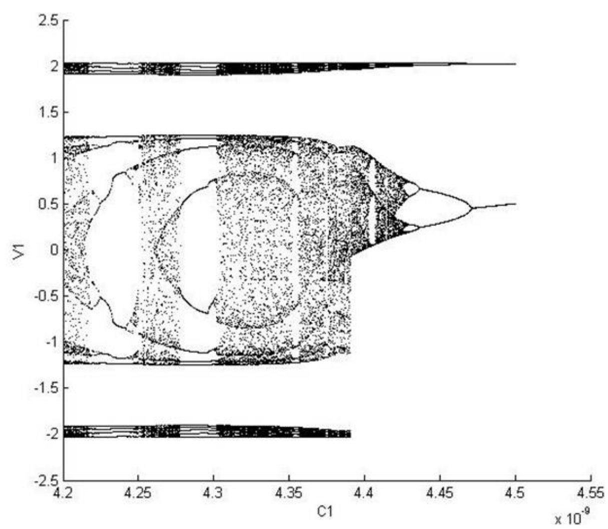


FIG. 12. Bifurcation Diagram showing Period Doubling

## B. Attractor Reconstruction

Beside the two voltage readings we recorded, people had been measuring the current going through the inductor to construct three dimensional phase space portrait of the Chua's circuit. We didn't plan to do that at the designing of our experiment, so we couldn't do it with the resource given. However, we did use the data we have, namely the voltage data from the two capacitors, to construct the three dimensional phase portrait. The method we used was the Time Delay Method[7]. This method allowed us to use a single measurement over time to construct a 2D phase space by plotting a signal value with its time-delayed value. If we extend the method to three-dimension using twice-delayed signal as the third variables, we can construct the 3D phase space for our Chua's circuit. We used the mutual information method to find the best delay time for our circuit[9]. After reconstructed the 2D plot from our chaotic signal data, a comparison between the reconstructed plot and our actual plot from data was made (see FIG.13). They had the same shape, but not the same. Because this double scroll attractor is three dimensional in nature, we thought it might be the difference made from observing at different sections in space. We tried to look at the phase space from multiple angles, but still could not find the exact slice of the part we constructed. The 3D reconstruction was compared to our computer simulation and the result was confirming our technique (see FIG.14).

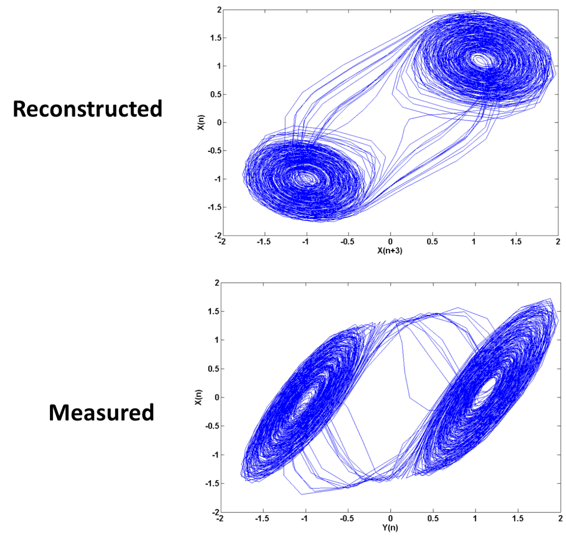


FIG. 13. Reconstructed Attractor Compare with Data in 2D

## C. Correlation Dimension

The last characterization we did was finding the correlation dimension. We chose the correlation dimension because it takes account of the density of points on the attractor[11]. In order to find the dimension, we plotted $log(R)$ vs $log(C)$, where R was the radius used to count the points and C was the correlation dimension (see FIG.15) by the following equation: $C(R) = lim[1/N^2 \sum H * (R - |Xi - Xj|)$. After fitted to a straight

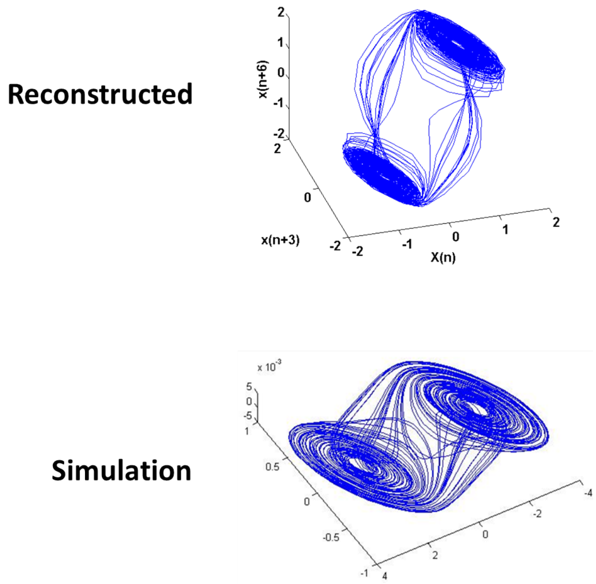


FIG. 14. Reconstructed Attractor Compare with Simulation in 3D

line, the correlation dimension of the Chua's circuit came out to be 1.8. The correlation dimension for computer simulated data was also found to be around 1.8 (see FIG.16). This value is similar to what other people found for Chua's circuit[10].

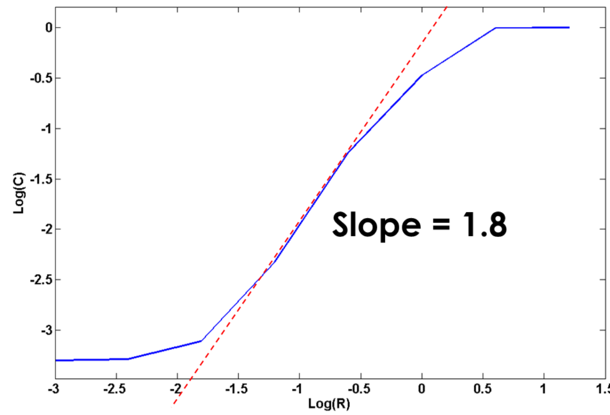


FIG. 15. Correlation Dimension vs Radius for Experimental Data

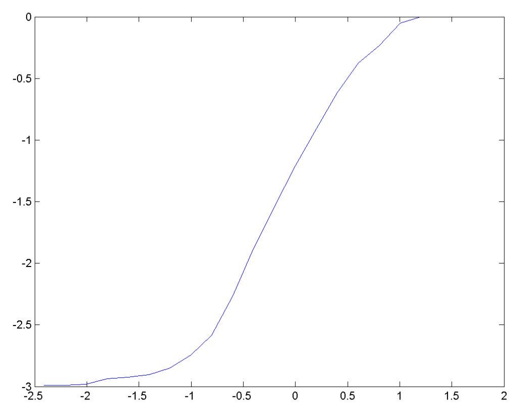

FIG. 16. Correlation Dimension vs Radius for Simulated Data

## V. CONCLUSION

In summary, we have identified one of the routes to chaos for the Chua's circuit, which is going from periodic behavior to screw attractor and finally lead to chaos. We also found the Lyapunov exponent of the Chua's circuit to be around 1.33, averaged between the left (1.01) and right (1.65) attractor. Last but not least, the correlation dimension of the circuit was computed to be 1.8. With the help of computer simulation, we were certain that Chua's circuit can produce period doubling with the correct parameters. To further investigate the nonlinear dynamics of Chua's circuit, more data need to be collected from different configurations of the circuit. With a change of the resistances inside the nonlinear resistor, the Chua's circuit would be able to display period doubling. With the proper equipment, the current passing through the inductor can be measured and counted for the third data set. An even bigger goal for the future is getting back to our initial objective and synchronizing multiple Chua's circuits acoustically, and thus help the security of communication.

## VI. ACKNOWLEDGEMENTS

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