# Fire dynamics experiments in a 1D spatial discretization*

Mateo Reynoso, Lezheng Fang, and Ben Musci†
*Authors' institution and/or address*‡

(Dated: December 10, 2020)

In the attempt to understand forest fires, we tried to recreate fire dynamics by exploring the propagation of a fire front in a discrete lattice of matches. We looked for critical parameters that would change the behavior of the fire propagation, such as the separation of the matches or a relative height difference between matches. We tried to explored how the fire front velocity of the flame is related to the matches spacing. Additionally, We explored how the introduction of a relative inclination for the matches introduces two ways of propagating and how their characteristic velocities compare.

## I. INTRODUCTION

Under the current state of the world, with so many wild forests burning down, it lead us to investigate more about the dynamics of wildfires. Previous physics researcher have shown that the spread of wildfires contains extremely rich dynamics, including fractal formation, self-organization critically etc. Initially we wanted to understand the pattern formation that arises from the propagation of the fire front under certain critical conditions as explored from real wild fire data and simulations [1][2].

The problem with exploring wildfire dynamics, is that it requires an analysis on a great scale and many different interactions. Thus, with the many physical processes one can solve the problem more accurately at the cost of speed. In literature the fire propagation can be described by coupled differential equations [3]. Then our research attempt has importance, since we are trying to look at a more simplistic model that could give us insight about the problem in the big scale. Possible, with better understanding of the basic mechanics of the fire propagation, we were looking for assumptions that could be made such that some assumptions can be made to simplify the problem can be simple or a better understanding of the basic fire propagation mechanism to reduce the complexity of computer models.

In a rather naive 2D model with both space and time discretized, there are established studies showing that the fractal pattern of fire spreading exists when the forest density is in a critical range. This has been studied through experimental work utilizing matches [4]. However, their journal only described qualitatively the patterns that emerged from their experimental set up, their motivation was mostly from an education standpoint. Their work showed that the experiment could also be compared to simulations that obeyed simple rules for the fire propagation, that executed in iterations rather than in actual time steps. These computational models have also shown to recreate diverse patterns under some control parameters, such as the forest density [5].

In our research our initial intention was to analyze the fractal patterns left by the fires. We implemented a 2D grid matches to recreate the dynamics of the fire fronts in actual fires, as Punckt [4] had previously suggested. However, after facing the difficulty of getting clean data from the flames propagating in two directions over our grid, we moved to start our analysis in simpler cases. We realized if we could understand better the propagation in 1D, it would gives insight into how to treat the 2D case. Our research then quickly turned into intensively analyzing the fire dynamics of the matches in a 1D scenario. Our motivation and interest became to understand under what assumptions could the matches properly represent the fire dynamics of a wild fire.

In other words, if we could utilize the matches to recreate fire dynamics knowing the fire propagation behavior when we utilized matches. Additionally, our 1D discretization of the fire gave us insight into what dynamics remain the save between a step computation of a wild fire and our experiment with the matches. This way we could establish better rules for the fire propagation in the computations.

As mentioned before, as our initial interest in the arise of fractal formation started shifting to understanding what factors influence the behavior of the propagation, we noted that the use of the matches itself affected the dynamics. Analyzing the data as we will later discuss, the role of the gunpowder at the tip of the matches turned to define the type of behavior of the fire propagation.Understanding this behavior can lead us to recreate

* A footnote to the article title
† Second.Author@institution.edu
‡ http://www.Second.institution.edu/~Charlie.Author

certain behavior depending on what properties we are trying to find in the front wave of the fire.

## II. METHODS

Our experiment required to control the separation of the matches to see how the behavior of the fire propagation changed as the match lattice changed. A rectangular piece of Styrofoam was utilized to create the spacing between the matches, it provided us enough repeatably to perform the experiment multiple times. In retrospective creating a board with the desired spacing would have benefited the experiment, by reducing our human error while placing the matches.

For our experiment we defined 3 characteristic separations, 6.5 mm, 8.0 mm, and 10.0 mm. The lower bound of the spacing was set by the fact that we did not wanted the matches to be in immediate contact with each other, but also we desired a small distance, where our error in spacing the matches was not greater than the separation itself. On the other hand, the upper limit on the spacing was adopted because it was an adequate separation where the flame would consistently spread without reaching the critical lattice size, where the flame would not propagate at all. Additionally, we set the upper range having in mind our human errors placing the matches. If we had worked to close to the limit, we could set up a part of the lattice where the fire would not propagate and ruin the data collection.

For the data collection we used a smartphone capable of recording at 120 fps. We recorded the flame's cross section that was perpendicular to the flame propagation and the vertical axis. This allowed us to get a sense of the front wave speed of the flame and capture the spacing of the matches clearly. Multiple videos were recorded of the fire propagating having the 3 characteristic spacings as the independent variable.

On top of our analysis on the spacing of the flame, we added a second scenario to explore. In reality the terrain where wild fires occur is not completely flat. While the matched remained perpendicular to the surface The effect of gravity on the flame would change its orientation and inevitably change the dynamics. We were interested in learning how the changing of slope would alter the dynamics of the wild fire. In this case we added a second parameter to control, that would indicate the angle between the matches and the vertical axis. How we introduced the tilting of the plane can be seen in figure 1.

Finally we worked towards building a 2D grid with the same spacing in both directions. In this case, we could not place the camera with a side view, so we tried instead a top view with some inclination. The problem with these set up was, that the video was more noise and the tools from our previous analysis could not be utilized.

## III. IMAGE PROCESSING

Considering the time constrains of our project we could not invest to much in developing an elaborate scheme for the image analysis of the videos. In our case we utilized MATLAB to analyze the frame. Additionally, we used Image J to check the validate our assumptions and simplistic analysis.

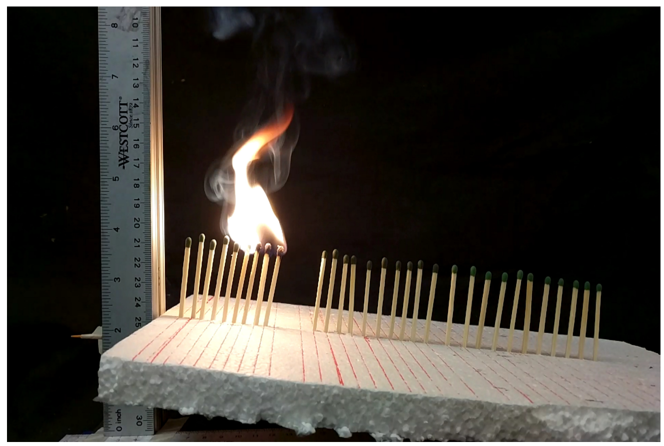

FIG. 1. Unprocessed data obtained from a phone recording with frame rate of 120 fps.

For the 1D pictures, we wanted to extract the shape of the flame and some measure of its relative intensity. To do this we applied a formula that selected pixels according to how they compared to the standard deviation of the rest of the picture. In the case of our video, we expected the flame to dominate in the green and red channels, so we applied the algorithm (see 1) through the red colors of the picture.

The next step was establishing a method for getting the front speed of the flame as it propagated. For our code we assumed the shape of the flame as a rectangle, then we could make the approximation that the front flame speed doubles the velocity of the center of mass of the flame. From the image of the flame processed we could easily select the area where the flame shape was extracted to avoid any noise, and extract the center of mass of the intensity of pixels. Certainly, what the pixel density we captured with the camera did not represent the heat density of the match. This is something that

**Algorithm 1:** Here is the pseudo-code that we utilized to get rid of all the elements of the background and obtain the shape of the flame. By setting all the other pixels to 0, we ended with a gray scale map of the flame.

**Result:** Obtain the shape of the flame
**for** $k \le N$ **do**

$$A(A \le 2k/N\mathrm{stdev}(A)) = 0$$

**end**

could be looked with more detailed, but for our purposes we were tracking the speed and the pattern of the flame that was visible to us. Converting the units of the pixels into $cm$ and the number of frame into time according to the sampling of the video recording, the position and velocity of the flame was obtained for our analysis.

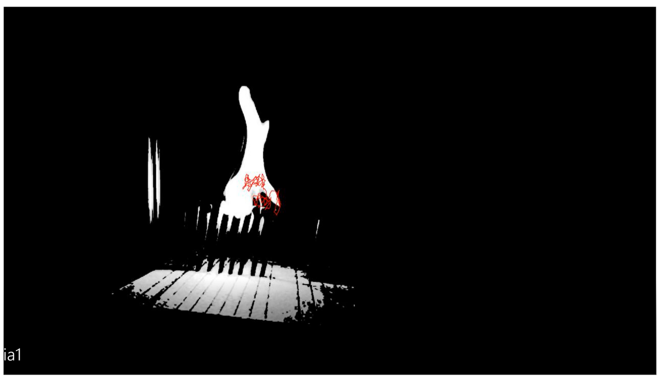


FIG. 2. The resulting frame after the processing. Most of the flame is isolated an its density interpreted as a gray-scale, and left only with some noise from the reflections of the flame.

To see if our assumptions were valid we utilized more sophisticated software like Image J [6]. This public access software also had the tools to recognize the front flame from our video, as its speed. After comparing the front flame speed to those we calculated with our code, we confirmed that our assumptions were valid. Our speed results, despite the inherent noise in the background were valid Under our time and computing power constrains.

Additionally we used the fast Fourier transform to analyze the behavior of the fire fronts. Despite the fact that we expected to observe time scales on range of $0.5 - 2.0$ Hz, the sampling of 120 fps gave us window more than enough range to inspect its behavior in term of frequencies. Rather than utilizing the Fourier transform to identify spatial frequencies in the flame images, we utilized it on the data acquired from the center of mass of the flame.

## IV. DATA ANALYSIS

### A. Fire propagation in 1D flat spacing

#### 1. *One dimensional lattice analysis*

To start building our understanding, we start with the 1D math lattice. Utilizing the data from the position of the center of mass in the direction the wave was traveling. Looking at the graph 3 we could observe two behaviors. A constant slope related to the velocity of the flame, which could be cleaned from the signal that is on top using a low-pass filter. The second pattern in the graph turned to be something more interesting. Using the fast Fourier transform we were able to identify this as "beating" when the flame propagated to the next match. Re-watching the videos we observed that as the flame moved from one match to the next it went back and forth. It originally made sense to us that this frequency was related to the spacing of our lattice of matches.

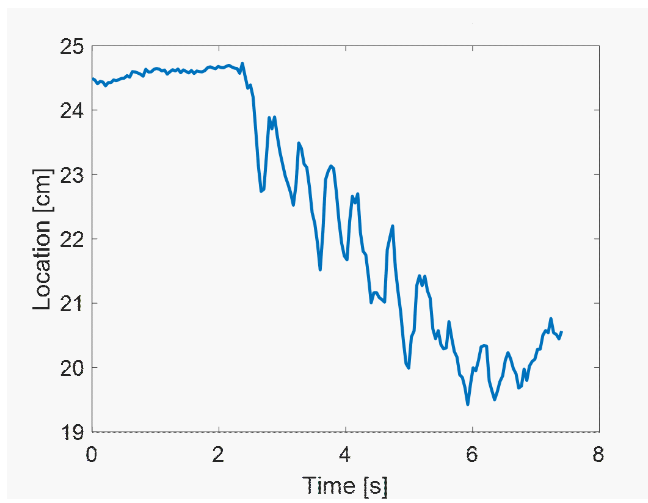


FIG. 3. The position of the center of mass was plot as a function of time utilizing the scaling factors provided by the a ruler in the video and the recording frame rate from the video.

After applying the Fourier transform we observed that there was the clear peak associated with this movement 4. Interested in knowing what determined this particular frequency, we went to measure the speed and this frequency, which we will name from now on as average time per match. After repeating this analysis for 8 different set ups with the same spacing, we found a linear relationship between the average fire-front speed of the flame and the average time per match [1].

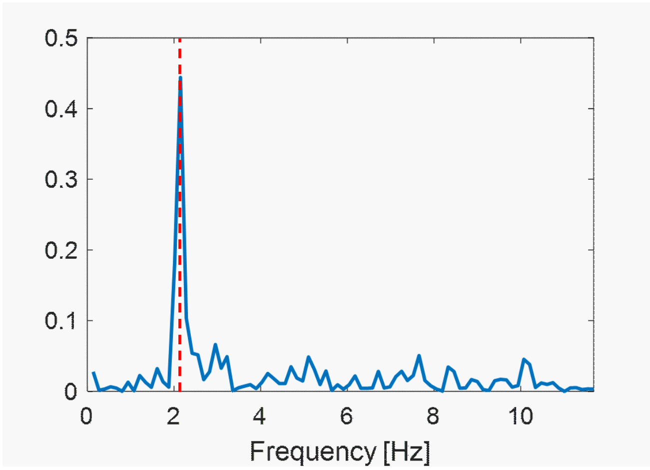


FIG. 4. Utilizing the fast Fourier transform and the sampling of our video recordings we were able to identify the Frequencies that influenced the propagation of the flame

Making a quick stop to note, that we could not understand the relationship between these two. It could be that because we are working in a lattice, these two are forced to be related, since the speed along the lattice will depend on how long it takes for the beating. Additionally, at the end of the beating it allows the flame to be further away as if there was none, making the spread faster. This is a behavior that should be noted in case someone is trying to recreate a fores fire, since the chemicals on the matches will make the propagation faster.

To confirm our hypothesis we tried to see how the fire spread using the back of the matches. In this alternative case, the spread was much slower and the we could not observe this oscillation in the position or velocity of the fire-front of the match. This effectively makes a difference in what can be consider the fire-front. In the case of the matches, since the fire propagates faster, the width of the fire-front is effectively larger than the case when the fire is burning without all the chemicals of the tip of the match.

### *2. Models for 1D lattice*

Continuing the analyzes on the average fire-front speed we can guess that the form of the solution is a superposition of a the front wave moving at a constant velocity and a modulation that represents the beating of the flame as the chemicals in the next match are ignited. The position for the fire front could be written as

$$x(t) = v_{\mathrm{f}} t + A\cos(\omega t) \tag{1}$$

where $v_{\mathrm{f}}$ is the average velocity of the flame and omega is the oscillation due to the ignition of the chemicals of the matches.

The solutions already represents a solution to the wave equation, at least for the 1D dimension case. In this wave case we could assign a wave number and its direction and write the solution for this simple case. However, this suggest different comparing it to a diffusion equation [3] with a driving term in the form

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} T) + f(T) \tag{2}$$

as the model for the propagation of fire in a wild forest.

In our case, the fact that solution looks like it belongs to the wave equation rather than the diffusion equation could possible reside in the fact that we are only looking at this from a 1D perspective [7]. Also this could be from the fact that the damping of the the diffusion equation would not be perceived in a 1D case.

Later we hypothesized that this effect was caused from the explosive reaction of the chemicals at the top of the match. Repeating the experiments with the matches upside down, we tried

Looking at the two different cases we wanted to understand how this effect of the chemicals on the propagation of the fire changed the dynamics of the fire front. Primarily we could see that the velocity of the fire front was related to the frequency of this beating frequency. Working on a lattice the faster the But also the beating allowed the definition of a larger fire front an easier transition of the fire from a math to another match.

It would be interesting analyzing a broader range of spacing to see by how much does the explosive reaction of the matches change the critical distance between the matches that the fire can extend, keeping in mind if the temperature of the match burning in both cases is similar. But despite this, we can still translate our findings of the average time per match into as a conversion of what the time between two iterations of a forest fire simulation [5] of the form

$$F = \sum_{i,j} T_k \tag{3}$$

[1] I have not added a graph for this linear relationship, since we were only allowed to have 6 figures and I am already pushing it.

Where $F$ is the probability of match to ignite depending of the state of its immediate neighbors (in an X) labeled by indexes $k$. This way we could turn a simulation ruled in steps, into something that is recorded as a function of time.

In the same model we could try to understand what is the time between iteration of other fore fire models with more complicated rules, such as those of also considering the neighbors in the diagonal, or having different stages for the burn rate of the trees

#### 3. *Future work in 2D lattice*

As we seen before, in a infinite lattice 1D, with transitional symmetry, we could expect the particular case for the diffusion equation give a similar equation to that of the wave equation. However, if we wanted to work in 2D we would need to be more careful with how we identify the fire front speed. Probably a better solution would be to move to a velocity field. It could also seen that in some way our simplistic model could behave according to a more sophisticated model as the set of coupled differential equations proposed as a model for wild fires [3] with the form of

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \boldsymbol{\nabla} \cdot (k \boldsymbol{\nabla} T) - \vec{v} \cdot \boldsymbol{\nabla} T + A(S e^{-B/(T-Ta)} - C^{(T-Ta)}), \tag{4}$$

and

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -C_S S e^{-B/(T-Ta)}, \qquad T > Ta, \tag{5}$$

where the extra parameters control the reaction time, heat loss and heat advected by the wind.

We could try to compare our date with a better model by performing our experiment in 2D. This will greatly increasing the complexity and introduce more dynamics to analyze. However, this would require a better set up, to get rid of the noise and more advanced processing techniques, such as optical flow for PIV (particle image velocimetry). In the case of successfully obtaining a velocity of the field for the fire, in some resemblance to the data analyzed in 1D, we could get a greater insight characterizing the speed of the fire front traveling along the different directions of the lattice.

### B. Fire propagation in relative uniform slope separation

After looking at the dynamics with the different spacings, we wanted to explore how the relative height difference of matches would affect the travel speed compared to the flat cases. Considering the effect of the buoyant force on the flame, its orientation would be given by the angle between the foam and the vertical axis. This effectively reduces the effective flame to the next match. This behavior can be described geometrically as

$$d_{\text{eff}} = d \cos(\alpha), \tag{6}$$

where the effective distance $d_{\text{eff}}$ of the matches is the projection of the spacing $d$ along plane perpendicular to the $z$ axis. This as shown in figure 5, makes it easier to move to the next match since it is closer.

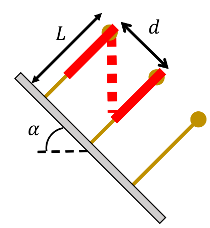


FIG. 5. Geometric view of how the inclination changes the characteristic distances of the match lattice. In red we can see an additional method of propagation, where the red dotted line represents the flame. Once the fire has burned through enough distance of the match it is able to light the next match.

Calculating the propagation speed for the different inclination lead us to in fact, confirm this intuition from the geometry. As the angle increased the velocity increased as shown in figure 6.

In this case knowing the velocity at which the flame travels for a certain spacing, when the plane is tilted we can make an approximation for what is the average time per match, given by

$$\Delta t = \frac{d \cos(\alpha)}{v}, \tag{7}$$

where $v$ is the average fire-front speed given by the spacing without any inclination.

When the inclination is not exaggerated, the geometrical shape of the flame can be ignored, by assuming it to look like a rectangle. As the inclination increases, the shape and the temperature change of the flame over space is more complicated and our approximation deviates. Despite our simplistic assumptions, our model agrees over

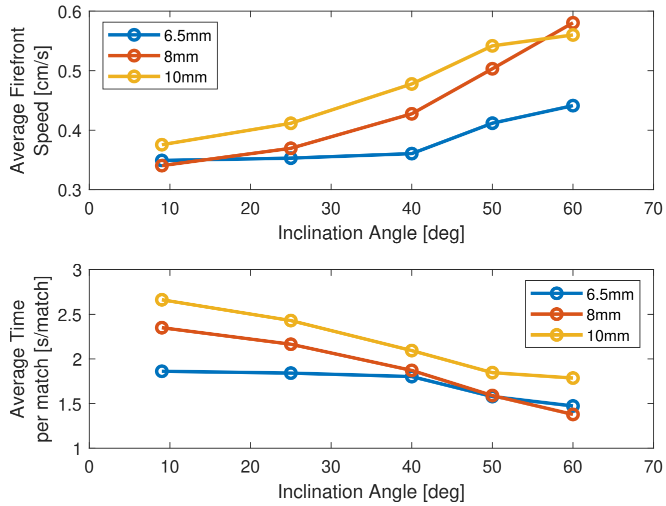


FIG. 6. The average fire-front speed and time per match was plot as a function of the inclination angle.

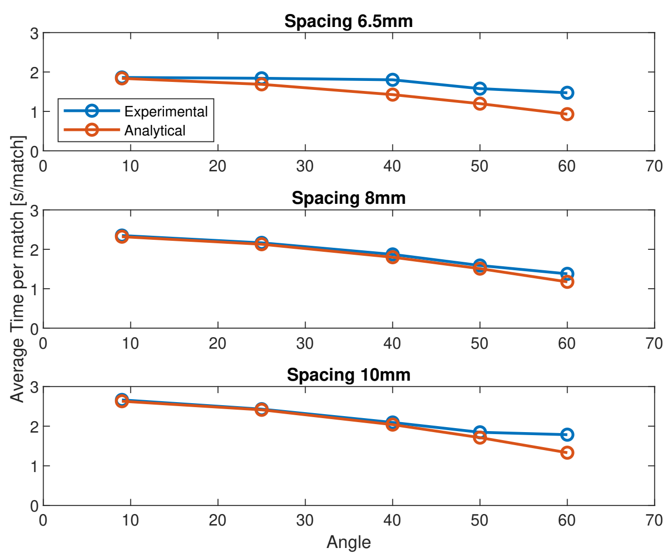


FIG. 7. The Average time per match from the experimental data was plotted as a function of the inclination angle. It was also plotted against what Average time per match our model for horizontal propagation was expecting, for all the spacings measured.

a great range of the inclination as seen in figure 7. Towards the larger angles, there are other effects that could be accounted in the propagation.

One of the other effects for the inclined propagation that we accounted for, was that the fire propagation could change if the fire is able to burn through enough of the match to light the next one. Under a critical angle, the flame will be positioned below the next match. This would be another mechanism that would allow the fire to propagate given that the flame is tall enough, with

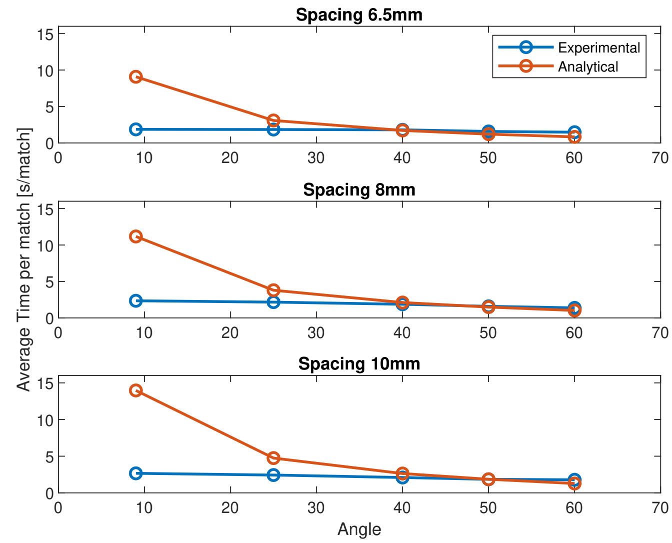


FIG. 8. The Average time per match from the experimental data was plotted as a function of the inclination angle. It was also plotted against what Average time per match our model for vertical propagation was expecting, for all the spacings measured.

its own characteristic frequency of average matches per time.

$$\tan(\alpha) = \frac{L}{d}, \tag{8}$$

where the Inclination angle is related to ratio of the length $L$ of the match and the spacing of the lattice.

In the same way as we previously define a average time per match in terms of the inclination, we can utilize the new geometric relation 8 to obtain

$$\Delta t = \frac{d}{v\tan(\alpha)} \tag{9}$$

In this case the model gets quickly out of control, because the assumptions of the flame get on the way. As the plane gets more horizontal, the average time goes to infinity, as we do not expect this transition to happen.

Therefore, we expect again, some critical inclination that represents the limit on which this can happen depending on the length of the matches, the distance of the grid and the size of the flame.

As we have seen before, the previous effect dominated more the fire propagation than this one, since there was not that much correlation between what average time per match we were observing between our model and the experiment. It would be up to future work to set different experiments with a different spacing to see if our model would capture this different frequency, instead of the spacing shorting.

## V. CONCLUSIONS

In this case we demonstrated that we need to be careful how matches alter the propagation of fire dynamics, since the chemical ignition changes their dynamics. In this case, being aware that the phosphorus increases the speed of the propagation of the matches could be use to get closer to what one could be trying with simulate with the matches. work with the matches, since the chemical ignition changes their dynamics.

Additionally, we have demonstrated how an inclination of the terrain can introduce a change to the propagation of the fire. In our paper we have explained to ways in which the flame can propagate from one matches to the next in terms of geometry. This can lead to a better understanding on how to model wild fires when the terrain is not on the same elevation or introduce rules for computer simulations of forest fires.

## VI. FUTURE WORK

Summarizing the future direction that this project could take can be divided in the analyzing the fire-fronts for more complicated scenarios and generalizing it to a 2D lattice. This way we could understand how the wave moves in the different directions of the lattice and what kind of initial conditions change the propagation. Additionally, registering the ways the fire can propagate can understand the way we model the equations.

The other options is to explore how to relate our discretized model and how it can transition to a stepping model.

As we described in the experimental method section, we can understand how likely a match is to be excited by its neighbors. A more sophisticated model would be Punckt, et al suggestion paper from the paper "Wildfires in the Lab: Simple Experiment and Models for the Exploration of Excitable Dynamics" [4]. The impact factor $F$ of the neighboring matches could be given by the more advanced form of

$$F = a\sum_{k} Te^{(-\alpha-\beta)} + b\sum_{h} T^{(-\alpha-\beta)} \tag{10}$$

where the elements from $k$ belong to the neighboring elements in the $x$ or $y$ direction and the $h$ elements are the cells in the diagonals of the central cell.

The constants $a$ and $b$ determine how much does the temperature of the neighboring cells affect the state of the principal cell. The important part of this model, are the parameters $\alpha$ and $\beta$, which could be experimentally determined from how the elevation and a change in the spacing of the matches affect the propagation of the fire between a neighborhood.


[1] G. Caldarelli, R. Frondoni, A. Gabrielli, M. Montuori, R. Retzlaff, and C. Ricotta, Percolation in real wildfires, Europhysics Letters (EPL) **56**, 510 (2001).

[2] T. Beer and I. Enting, Fire spread and percolation modelling, Mathematical and Computer Modelling **13**, 77 (1990).

[3] J. Mandel, L. S. Bennethum, J. D. Beezley, J. L. Coen, C. C. Douglas, M. Kim, and A. Vodacek, A wildland fire model with data assimilation, Mathematics and Computers in Simulation **79**, 584–606 (2008).

[4] C. Punckt, P. S. Bodega, P. Kaira, and H. H. Rotermund, Wildfires in the lab: Simple experiment and models for the exploration of excitable dynamics, Journal of Chemical Education **92**, 1330 (2015), https://doi.org/10.1021/ed500714f.

[5] U. Wilensky, *NetLogo*, http://ccl.northwestern.edu/netlogo/ (Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL, 1999).

[6] C. A. Schneider, W. S. Rasband, and K. W. Eliceiri, Nih image to imagej: 25 years of image analysis, Nat Meth **9**, 671 (2012).

[7] F. Olivar-Romero and O. Rosas-Ortiz, Transition from the wave equation to either the heat or the transport equations through fractional differential expressions, Symmetry **10**, 524 (2018).