Experimental Analysis of Metronome Synchronization

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Nonlinear Dynamics and Chaos
Outline

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2 Experimental setup

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4 Result analysis

5 Conclusion and future work
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- Synchronization of coupled oscillators \(\Rightarrow\) Two or more systems interact and move together.
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- Accidentally discovered by Christiaan Huygens when observed pendulums hanging from beam.

Applications

- Biology: Firefly synchronization.
- Neural/Pancreatic/Pacemaker cell synchronization.
- Computer science: Distributed power grids.
- Social science: Opinion formation. Audience applause synchronization.
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Project objective:
Investigate synchronization of mechanical oscillators.
Experimental setup

- Coupled mechanical oscillator system → Metronomes on moving platform.
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Experimental equipment

- $N$ number of metronomes $\rightarrow$ subsystems that oscillate at specific frequencies.
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- $N$ number of metronomes → subsystems that oscillate at specific frequencies.
- Light platform made of foam acting as coupling between the metronomes via inertial force transfer.
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**Experimental equipment**

- \(N\) number of metronomes \(\rightarrow\) subsystems that oscillate at specific frequencies.
- Light platform made of foam acting as coupling between the metronomes via inertial force transfer.
- 2 soda cans, acting as a base that is able to move in one dimension.
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- Camera-equipped smartphone, used to measure positions of the metronomes.
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For educational purposes, we employed \( N = 3 \) metronomes to learn how to gather data and develop the required code.
Measurement setup

Measurement equipment

- Used camera phone to capture video of metronomes.
- 2 sets of markers were used for position tracking.
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Measurement process
- Metronomes were initialized by random initial positions → Mechanical energy input via spring winding.
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Measurement process

- Metronomes were initialized by random initial positions → Mechanical energy input via spring winding.
- Marker was placed on the edge of the pendulum, used to measure position wrt point of reference.
- Marker was placed on the center of metronome, used as point of reference.
In order to derive trajectories from video, we developed a Matlab script for motion capture.

- Load video to Matlab using `VideoReader` object.
- Read frame and save as RGB matrix.
- For $i = 1, \ldots, N$ metronomes, determine corresponding positions and reference points.
- Determine $x_i$, $y_i$ positions of reference point and edge of pendulum.
- Compute $\phi_i = \arctan(x_i, y_i)$.
- Find trajectories of angles for the $N$ metronomes.
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Result analysis

Observed trajectories – Initial phase

- Metronomes are initialized from random angles.
During $\sim 30, 45$ sec. we observe anti-phase synchronization of one metronome.

Eventually, all metronomes achieve synchronization.
Result analysis

- Metronomes are moved off the base – mechanical coupling is lost.
- Synchronization is lost due to perturbations from movement.
Result analysis – Theoretical analysis

- Need to evaluate theoretical models against experimental results.
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- Develop code to compare evolution of metronome synchronization in simulation and experiment.

\[
\begin{align*}
\phi_i + b \dot{\phi}_i + g l \sin \phi_i + \frac{1}{l} \ddot{x} \cos \phi_i + \bar{F}_i &= 0, \\
(M + n m) \ddot{x} + B \dot{x} + Kx + ml \sum_{j=1}^{N} \sin \phi_i &= 0.
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- Equation of motion of platform.
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- \(N\) equations of motion (based on E-L).
- Equation of motion of platform.
- Coupling between metronomes via perturbation force due to \(\ddot{x}\).
Conclusion and future work

**Conclusion**
- Set up the experiment to achieve synchronization of metronomes.
- Investigate the measurement environment to achieve position tracking.
- Developed Matlab code for visual position tracking.
- Studied the literature for appropriate mathematical models.

**Future work**
- Analyze the fit of the mathematical model.
- Conduct experiment with greater number of metronomes.
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Thank you