Vortex Cannon Dynamics

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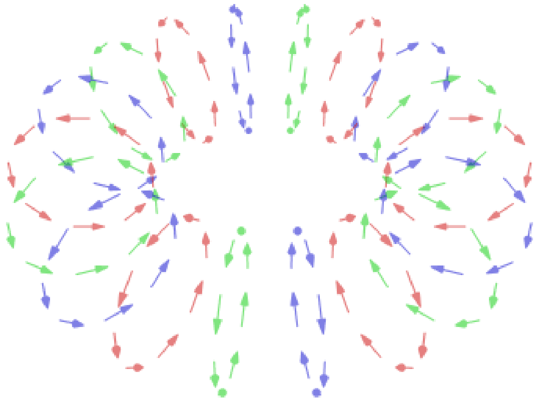
PHYS4267 - Nonlinear Dynamics & Chaos

Introduction

- **Goal:** Observe vortex rings and analyze their complex airflow dynamics through observation
  - Formation of rings result from:
    - Viscous friction between dense, high velocity fluid and a mass of stationary fluid creates a poloidal flow
    - This can be created by a mass of fluid being pushed through a small opening by some impulse.



- **Motivation**: Nonlinear Dynamics of Vorticity
  - Navier-Stokes equation [1].
  - Circulation related via Stokes' Theorem [3].
- **Method**: Slug Model
  - Create Control aperture diameter and displacement of the piston we can determine necessary conditions of forming a vortex ring from the slug model, and calculate the circulation of the vortex.

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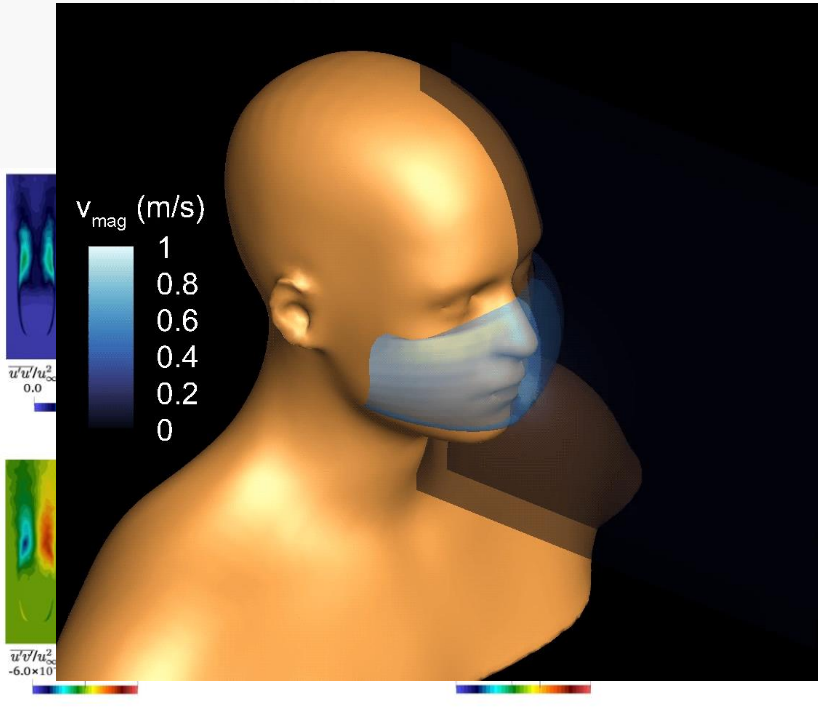
Examples of Vortex Rings Systems

COVID-19 and Face Coverings



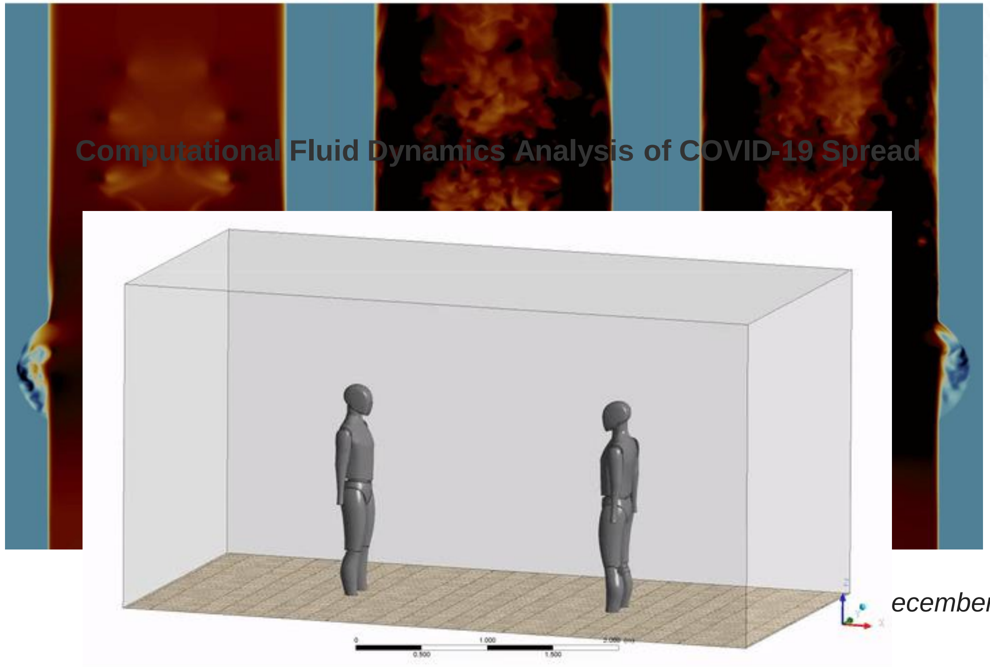
A computational simulation of a cough shows the airflow velocity of droplets moving through a simple face mask.

Ren, W.; Reutzsch, J.; Weigand, B. Direct Numerical Simulation of Water Droplets in Turbulent Flow. *Fluids* **2020**, *5*, 158.

Mittal, R., Ni, R., & Seo, J. (2020). *The flow physics of COVID-19*. *Journal of Fluid Mechanics*, 894, F2. doi:10.1017/jfm.2020.330

Human Heart - Turbulent Blood Flows

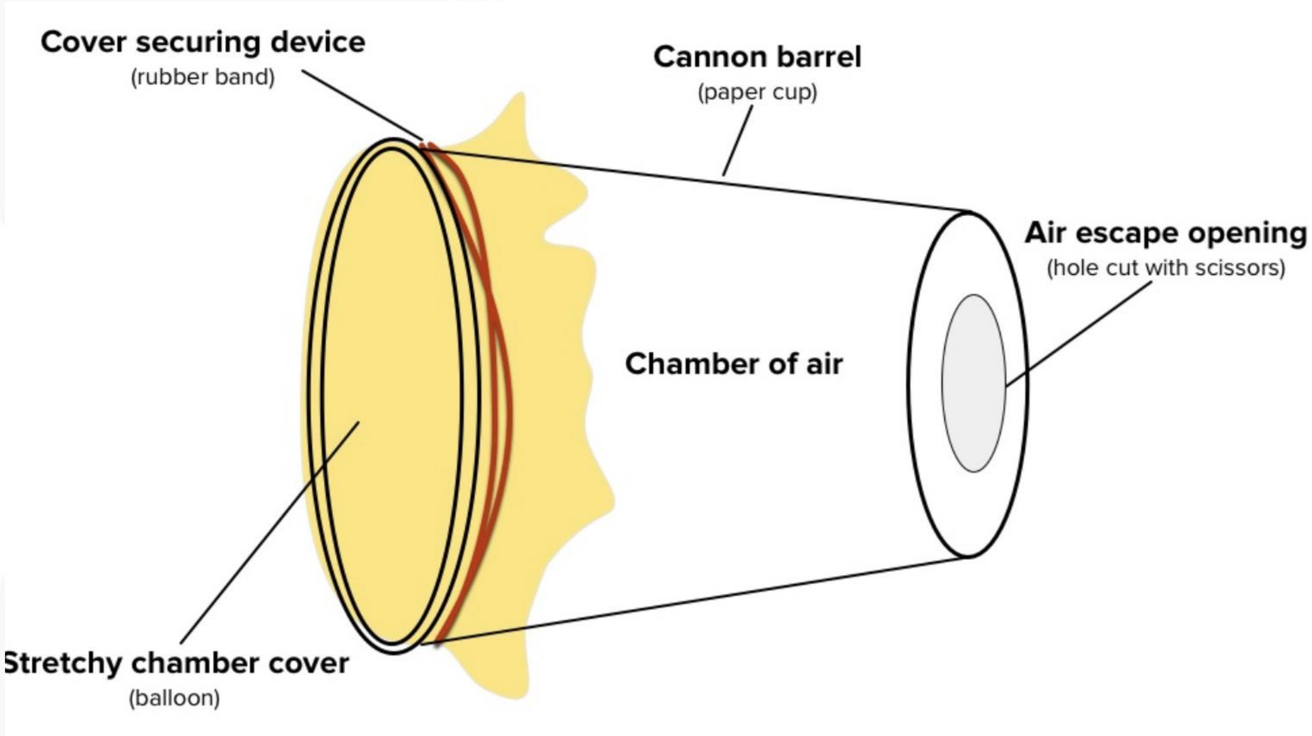
Computational Fluid Dynamics Analysis of COVID-19 Spread

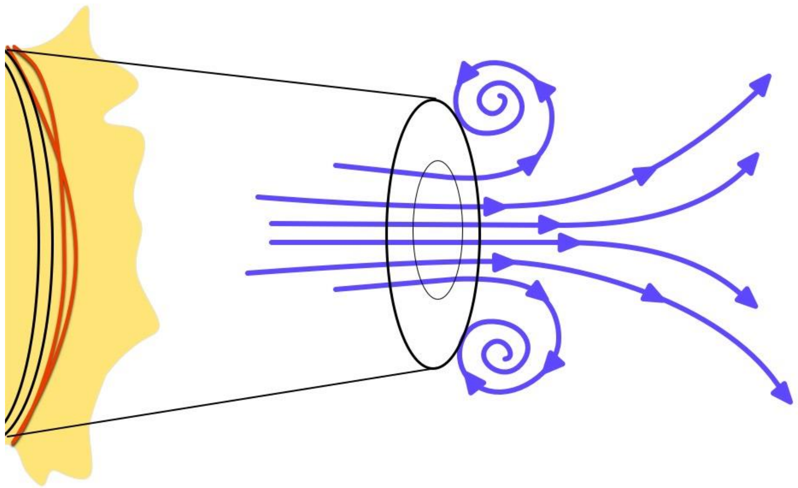


ecember

Mehran Salehi, *Senior CFD Analyst, Southland Industries* (2020)

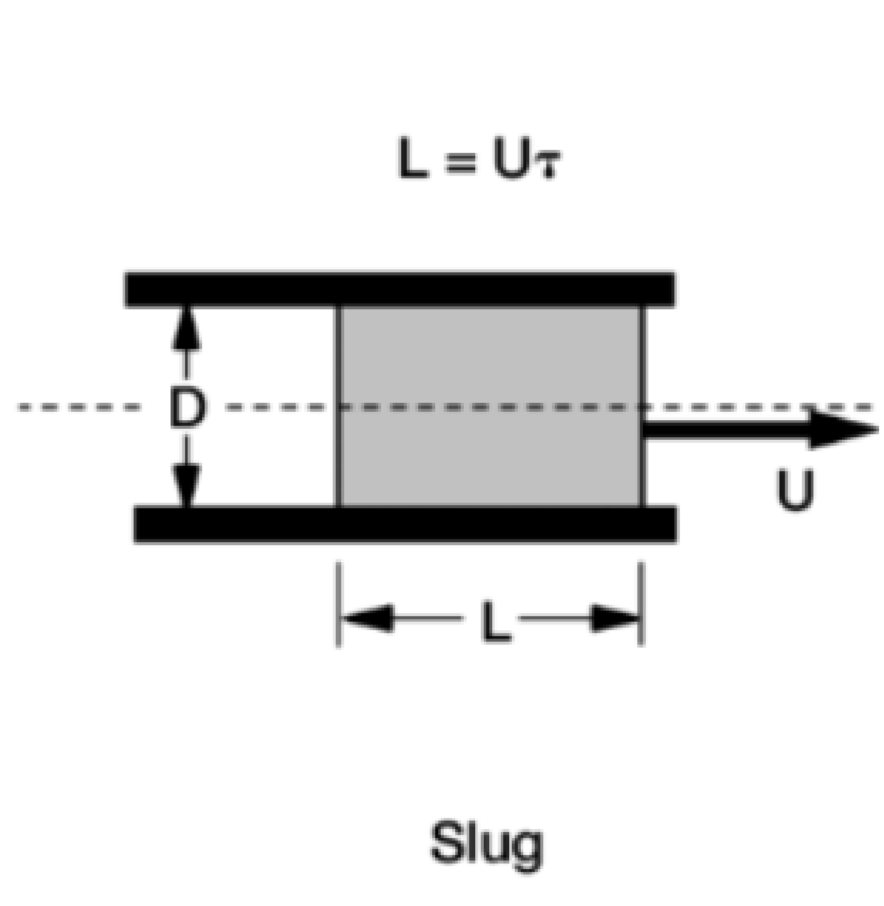
Vortex Ring Simplified Design





https://www.sciencefriday.com/educational-resources/design-a-better-vortex-cannon/

Model - Slug/Vortex Cannon

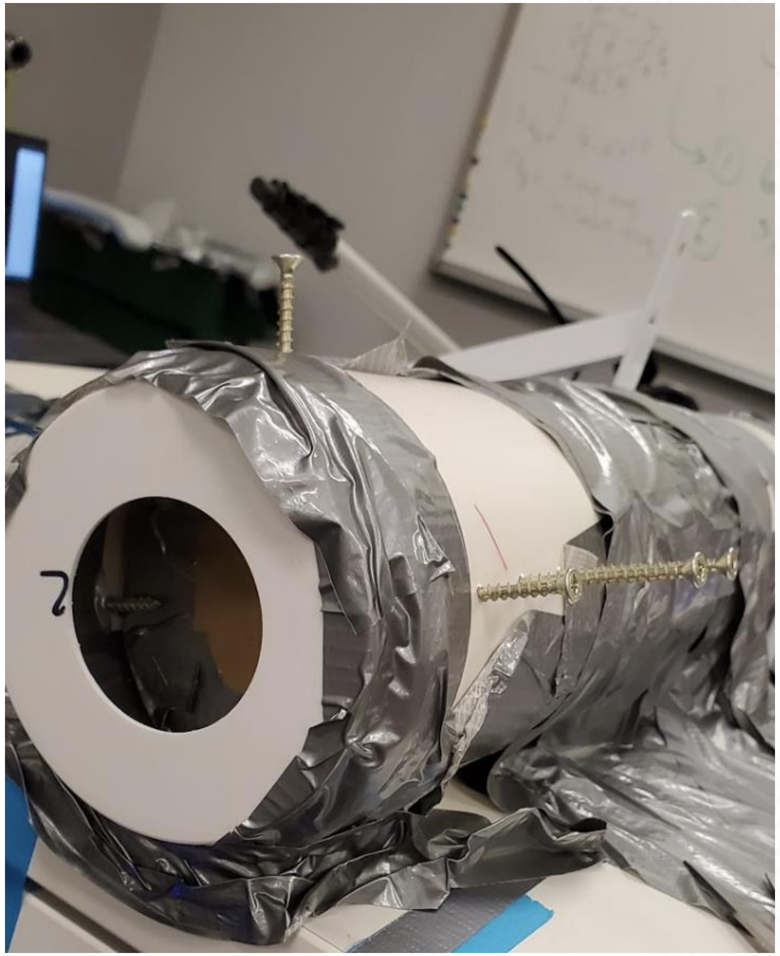


Ideal Slug model

- Shusser & Gharib (2000)
    - simpler version than normal toy diaphragm version
- Energy equation
    - Reviewed in literature
- Velocity
    - Experimentally measurable

Experimental Apparatus Materials

- Vortex Cannon
  - PVC pipe
    - Outer - 24 inches long, 4 in. diameter
    - Inner: - 48 inches long, 3 in. diameter
  - Screws to vary aperture diameter
  - Cardboard
  - Duct tape
  - Plastic apertures of varying diameter
  - Bungie Cables ands
  - Zipties
- Fog machine
  - AGPTEK 500W Portable Led Smoke Machine with Lights (Red, Blue, Green) & Wireless Remote Control
  - 16.66 cubic ft/sec (almost instantaneously fills cannon with gas)
- Laser
  - Construction leveling laser
- Cameras
  - 3 webcam, phone, Home Canon



Vortex Cannon without Aperture

- Bands attached on either side
  Attached to 2 variable points
  - Change stiffness of band
  - Change of velocity of piston

- Kept steady with duct tape

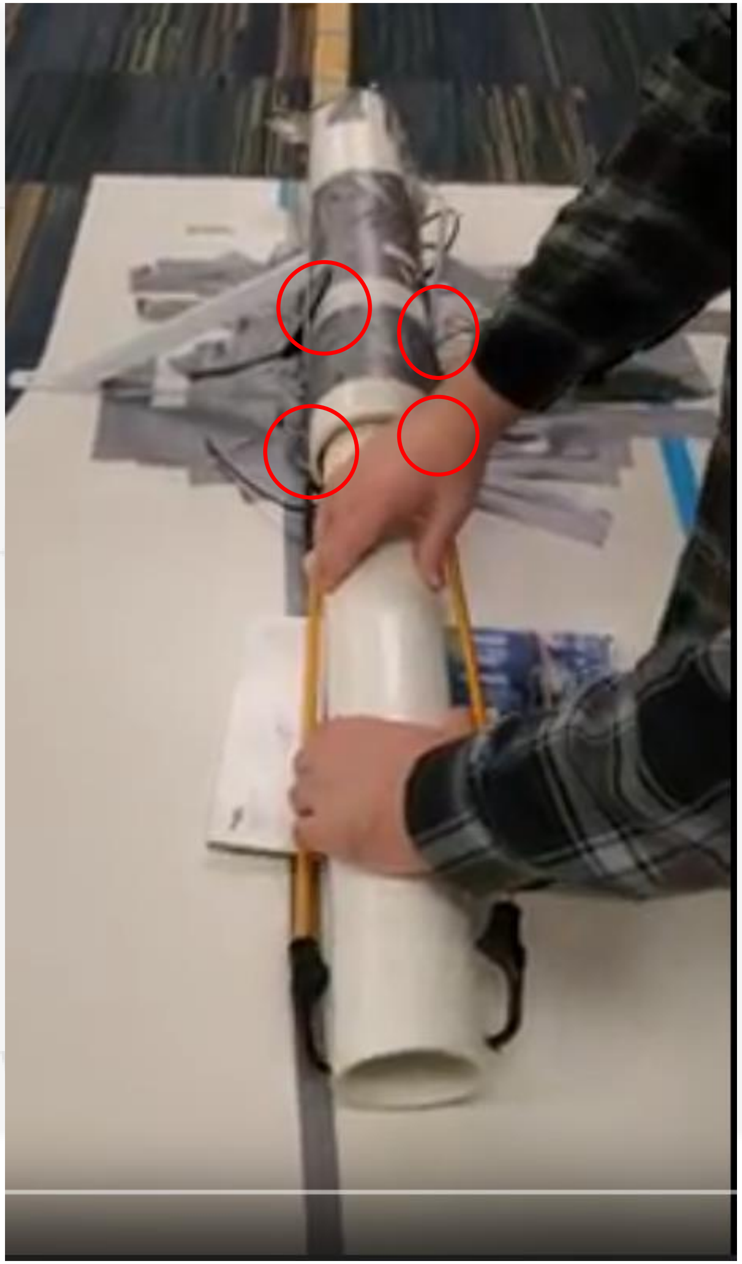
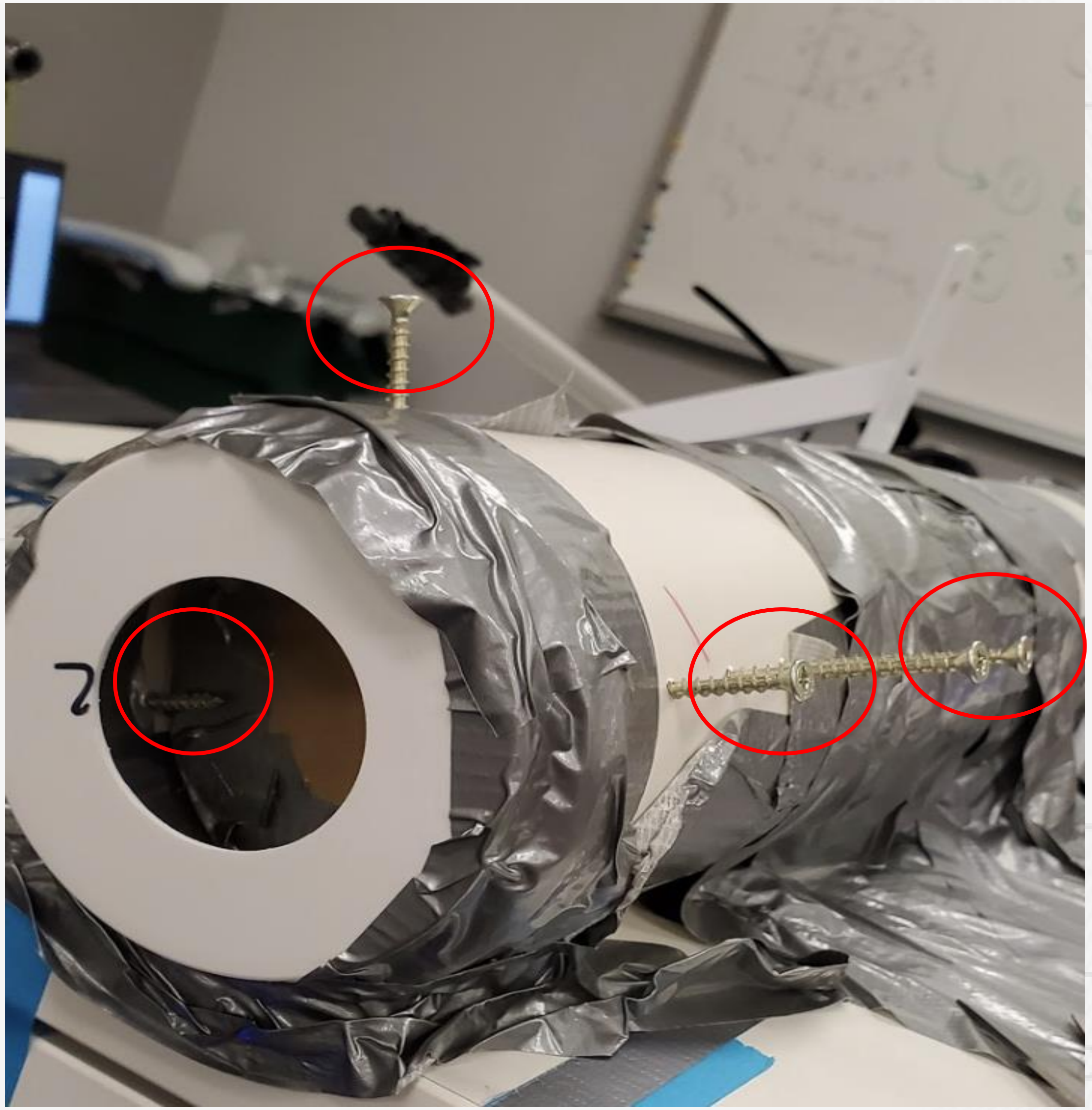


Figure 4. band attachment points

- Stopping points, l_0
  - Screws
  - Stopping before cap at 0.25 in. 1in, 3in
    - Not to break the cap

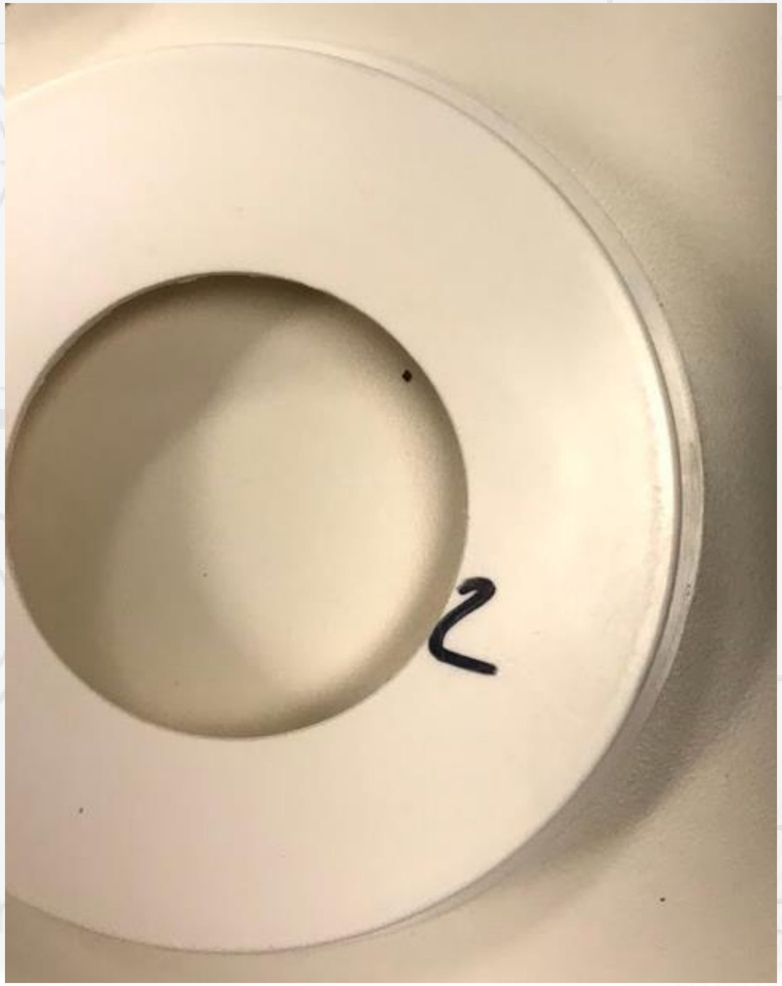


Screws to stop the piston from destroying the cap, and varying l_0

Cannon without aperture

Experimental Procedure

- Wooden planks were aligned with the cannon and measured with a tape measure.
- The planks were then marked with electrical tape @ every 12 inches.
- The nozzle was set and loaded onto the cannon
- Aperture Diameters,
  - D (in) = ¾, 1⅛, 1½, 2
  - Inner diameter = 4.5 in.
  - piston diameter= 3.6 in.



D = 2 Aperture

- Displacement of piston was then set
    - Initial piston displacements: 0.5 ~12 ± 0.25 in.

- Chamber was filled with gas and allowed to come to as close as possible to equilibrium

- Cannon was then fired at the set displacement by releasing the piston allowing the bands to push the gas through the aperture. The cannon was then reset, and a new displacement was increased by approximately 0.5-2 in.

- Once we were confident of the displacement that would yield the bifurcation, we then switched to dark and used the laser to capture cross section and air flow dynamics probing slightly above and below the critical displacement.
    - We also used super cool laser safety goggles during this part. Safety first!

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Qualitative Data

displacement	D =1 1/8	tenseband	D=2 (tense)	D = 2 (new band)	D = 2 (tense) (	D = 1 1/8 (tense)	D = 3/4 in (ter	D1 *1/2	D = 3/4	D = 2
0.5	ring	ring	x	ring	x	x	ring	ring	ring	x
1	ring	ring	x	ring	x	ring	no ring	ring	no ring	x
1.5	ring	ring	x	ring	x	ring	x	ring	x	x
2	ring	ring	ring	ring	ring	ring	x	ring	x	ring
2.5	no ring	no ring (ghost)	x	x	x	no ring	no ring	ring	no ring	x
3	no ring	no ring	x	ring	x	no ring, splitting patte	split ring	ring	split ring	x
3.5	no ring	x	x	x	x	x	ring	ring	ring	x
4	no ring	no ring	ring	ring	ring	x	no ring	ring (messy)	no ring	ring
5	x	x	x	x	x	x	no ring	split two	no ring	x
5.5	x	x	x	x	x	x	x	split two	x	x
6	no ring	no ring	ring	ring	ring	x	no ring	no ring	no ring	ring
6.5	x	x	x	ring	x	x	no ring	no ring	no ring	x
7	x	x	x	ring	ring	x	x	no ring	x	ring
7.25	x	x	x	x	ring	x	x	x	x	ring
7.5	x	x	DAMAGED	ring	x	x	x	no ring	x	ring
8	x	x	x	no ring	x	x	x	x	x	no ring
8.25	x	x	x	no ring	x	x	x	x	x	x
8.5	x	x	x	no ring	x	x	x	x	x	x
9	x	x	x	no ring	x	x	x	no ring	x	x
10	x	x	x	no ring	x	x	x	x	x	x
12	x	x	x	no ring	x	x	x	x	x	x
l = 1/2 in	l = 1/2 in	l = 1/2 in	l=1/2	l=1 in	l=1 in	l =1 in	l=1inch	l = 1inch	l=3in	l=3in

Qualitative Results - Initial

- Explored the least upper bounds of vortex ring formation
- Rings stopped forming only after increasing the volume.
- Increasing width of the aperture diameter increased amount of displacement until no rings formed
- Observed azimuthal instability at end of ring life cycle, however was difficult to capture.
- Changing l_0 did not change the displacement

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Qualitative Results

What happened here?

- Rings at 3-3.5in displacement seen sporadically

Increased circulation?

- Numerical data still in process of being analyzed

Or perhaps could be smaller eddie formation!

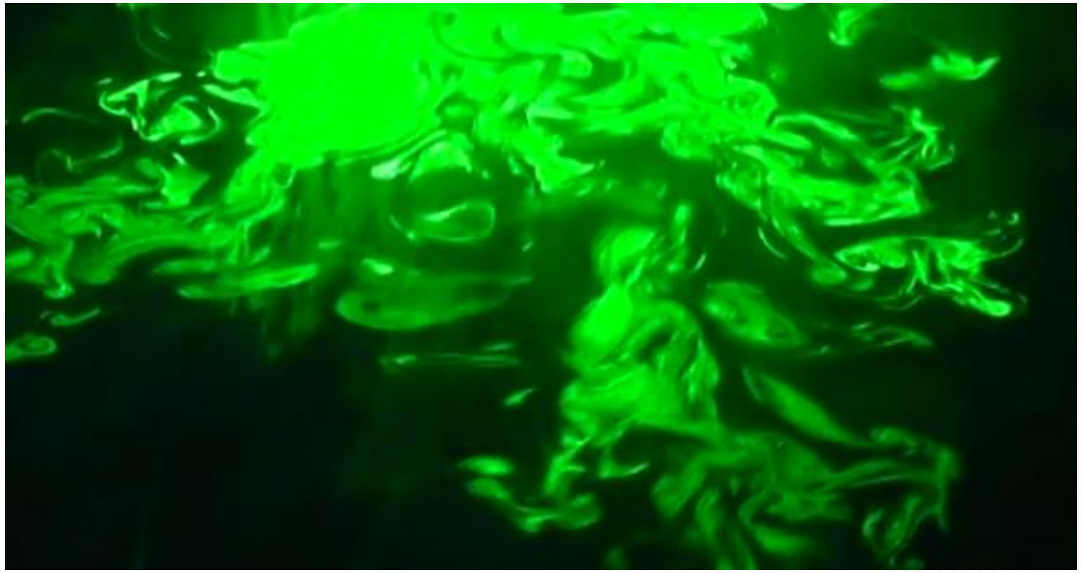
- Smaller eddies could form the ring at 3

displacement	D = 3/4
0.5	ring
1	no ring
1.5	x
2	x
2.5	no ring
3	split ring
3.5	ring

Kolmogorov’s Theory of Inertial Turbulence

- Eddy - the swirling of a fluid in a turbulent flow regime - See swirls to right

- Larger eddies break up, and transfer their energy to smaller eddies until Reynolds Number is sufficiently small



Big whirls have little whirls
Which feed on their velocity;
And little whirls have lesser whirls,
And so on to viscosity
in the molecular sense.

- L.F Richardson 1922

Could Turbulent Behavior lead to stable rings?

1-

2 -

3 -

4 -

5 -

6 -

Quantitative Methods

3 videos taken using marked displacements to extract velocity from video. Side video can retrieve velocity

Top down video can give us dynamics, especially with the laser.

Firing Cannon video taken with webcam

- marked with 1 inch displacements

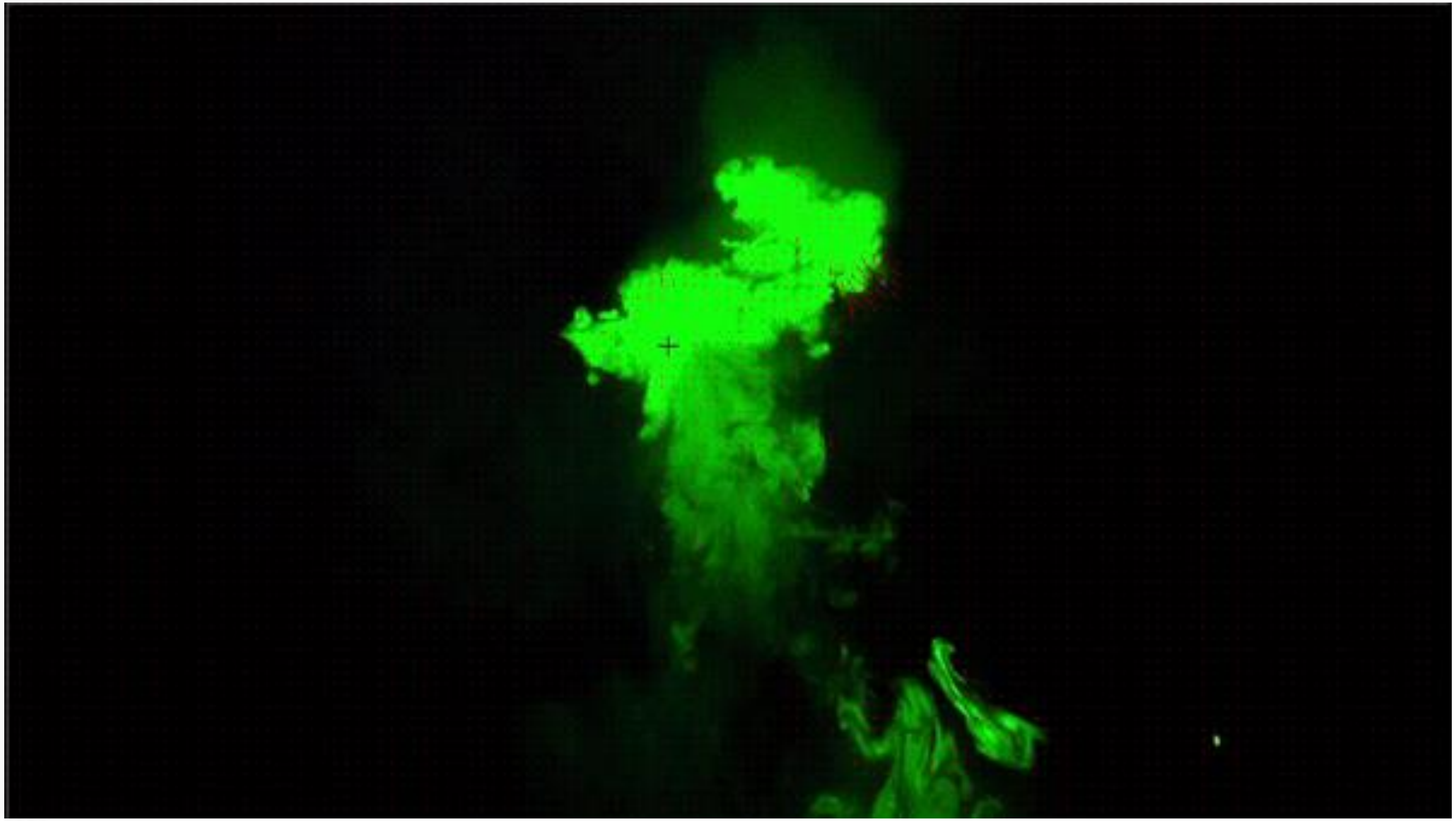
Laser Top Down Video

Could we use optical flow (a computational algorithm) to get gradient fields? Calculate a function of velocity from the video?

(Heads out of plane in this video - but we have others level with plane!)

Optical Flow Progress

Not fine grained enough though - algorithm parameters need to be tuned. The little red arrows can be thought of velocity vectors



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Quantitative Results - Previous Work:

Circulation (Gamma) - the line integral of a vector field around a closed curve. Circulation_fluid/circulation_pipe on y axis

L/D - in our case displacement (Δx) divided by diameter of aperture

VORTEX RINGS 245

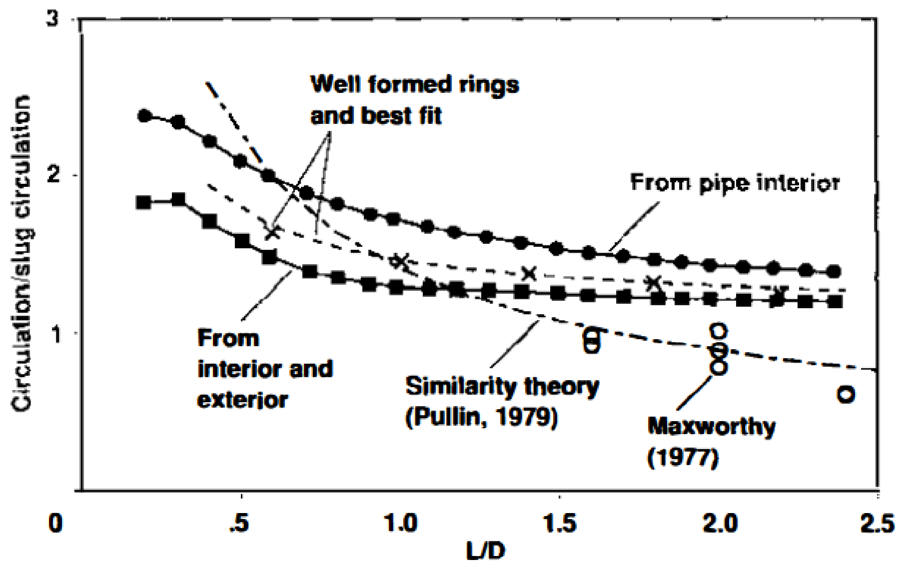


Figure 3 Circulation of vortex rings formed at a pipe referred to the circulation predicted by the slug-flow model, Equation (2.5). Unless indicated, the data are from Didden (1979). The multiple data points of Maxworthy (1977) at the three L/D values are for different Reynolds numbers, increasing downward. Well-formed rings seem to have a larger circulation (*crosses*) than the total flux at the pipe (*solid squares*) due to slight inadequacy in measuring the flux at the pipe.

$$\frac{\Gamma}{\Gamma_{slug}} = 1.14 + 0.32(L/D)^{-1}, \qquad L/D > 0.6.$$

Line of fit - Dashed line - Didden 1979

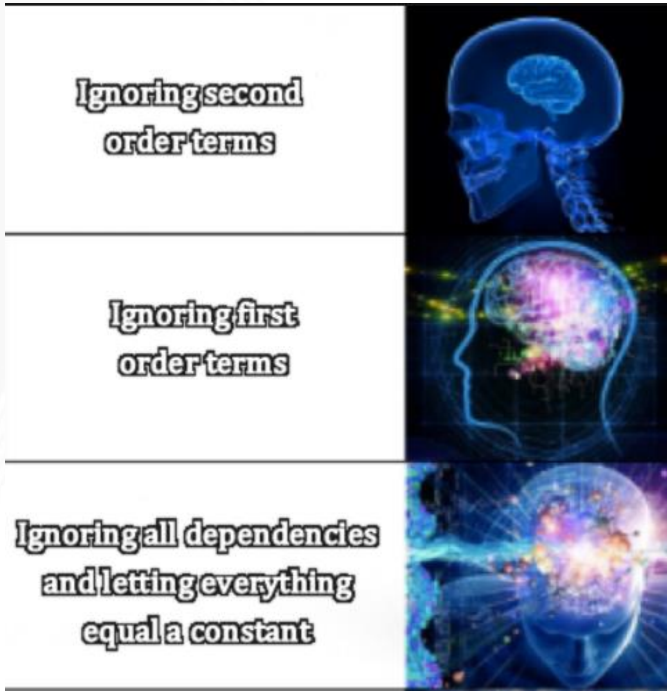
$$\frac{\Gamma}{\Gamma_{slug}} = 1.41(L/D)^{-2/3},$$

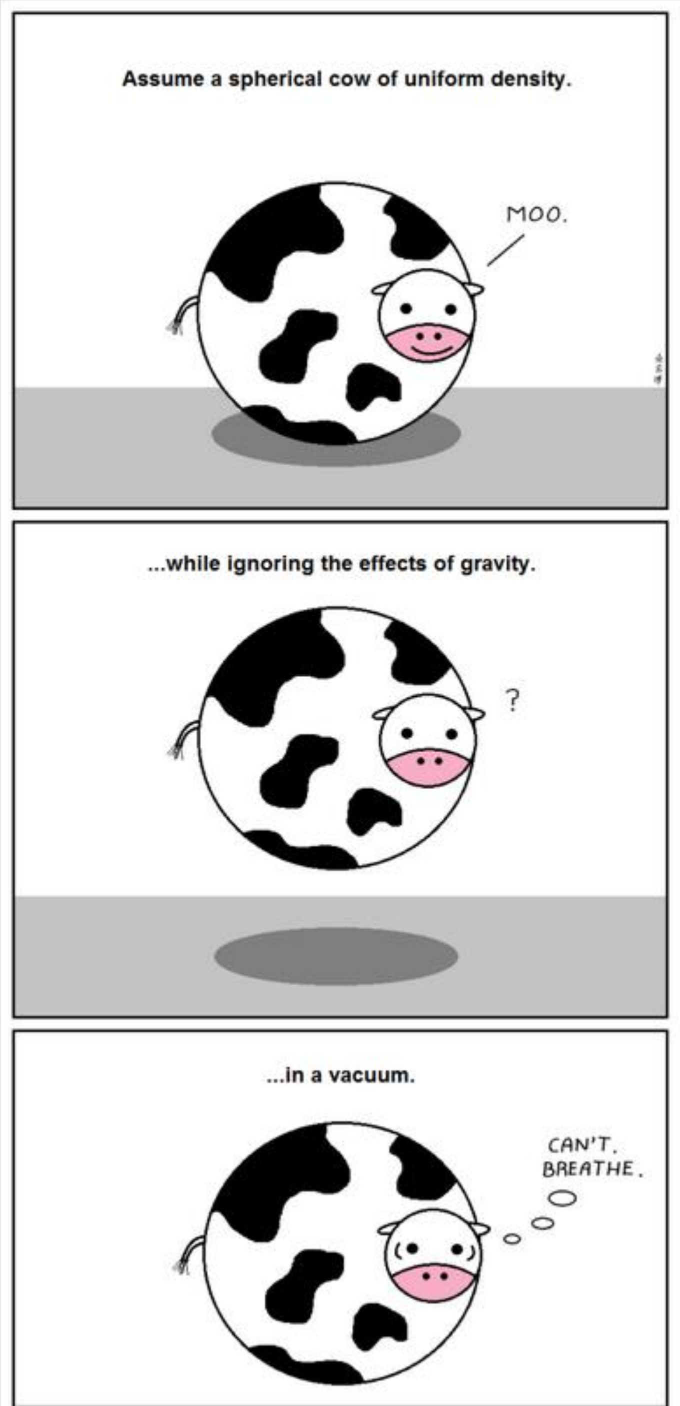
Line of fit - Chain line - Pullin 1979
(better for higher reynolds numbers)

Shariff, and A Leonard. 1992

Initial Quantitative Results

- Data crunching, and optical flow algorithms are hard and lengthy and need really good cameras
- Can we really roughly approximate using our video data to fit to the curve before?





Circulation Approximations

Vorticity (curl) is the second term of Taylor expanded velocity (first order, rotational term, “rigid body”), can also be derived from Navier-Stokes.

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}(\mathbf{x}_0) + D(\mathbf{x}_0)(\mathbf{x} - \mathbf{x}_0) + \frac{1}{2}\boldsymbol{\omega}(\mathbf{x}_0) \times (\mathbf{x} - \mathbf{x}_0) + O(|\mathbf{x} - \mathbf{x}_0|^2)$$

Using an approximation of vorticity (ω) given a velocity field u(x,t).

$$\frac{\Gamma_{slug}}{dt} = \int_0^{t_{piston\ stop}} \omega u_x d\sigma \approx \int_0^{t_{piston\ stop}} \frac{\partial u}{\partial \sigma} u_x d\sigma \approx \frac{1}{2} U_p^2(t)$$

Gharib, Rambod, Shariff 1998

(Didden 1979). Invoking the parallel flow assumption (namely, that the radial flow velocity, v, at the nozzle/orifice exit is zero for all time), the azimuthal vorticity is approximated by $\omega_\theta \approx -\partial u/\partial r$ and equation (2) integrates to

$$\frac{\partial \Gamma}{\partial t} \approx \tfrac{1}{2} u_{cl}^2(t), \qquad (3)$$

Krueger, 2005

2 Points

Using 2 points at D=2, (Δx=7.5 and Δx=8) and frame by frame video analysis (this takes forever) and a simple box integral using dt as 1/fps

displacement	D = 2 (new band)
0.5	ring
1	ring
1.5	ring
2	ring
2.5	x
3	ring
3.5	x
4	ring
5	x
5.5	x
6	ring
6.5	ring
7	ring
7.25	x
7.5	ring
8	no ring

Our results:

- L/D = 3.75 and $$\frac{\Gamma_{out}}{\Gamma_{slug}} \approx 0.533$$
- L/D = 4.00 and $$\frac{\Gamma_{out}}{\Gamma_{slug}} \approx 1.146$$

These actually fit to each of the curves!

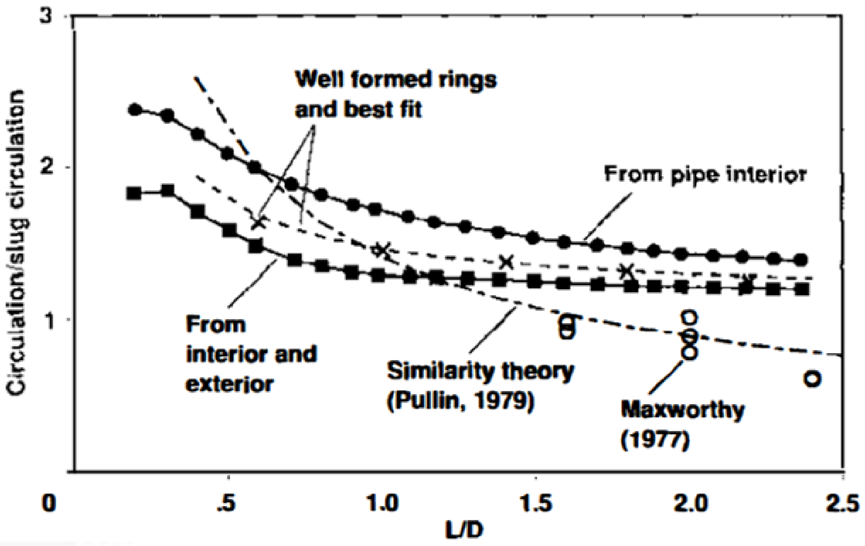
Curve Fit

For the Pullin 1979 curve (expected for high reynolds numbers) at L/D 3.75

$$\frac{\Gamma_{out\ expected}}{\Gamma_{slug\ expected}} = 1.41(3.75)^{-\frac{2}{3}} = 0.5841 \qquad \frac{\Gamma_{out}}{\Gamma_{slug}} \approx 0.533$$

The other Didden curve fits this L/D 4.00 point closer and not the top curve, which there was no ring!

$$\frac{\Gamma_{out\ expected}}{\Gamma_{slug\ expected}} = 1.14 + 0.32(4.00)^{-1} = 1.22 \qquad \frac{\Gamma_{out}}{\Gamma_{slug}} \approx 1.146$$



Didden used fluids (we used gas!) while Pullin used similarity theory which is better fit for higher Reynolds numbers

Information pulled from Shariff, and A Leonard. (1992)

Future Analysis

- More Optical flow to get vorticity directly
- Translational velocities and displacements of vortex rings (fairly simple)
- Obtain approximate Reynold numbers by dividing circulation Γ by the viscosity (ν) of the fog machine gas
- Keep parsing more data!

Sources of Error and Uncertainties

- Breakage of PVC pipe @ initial length had to repair the cannon. Stronger, more durable materials would have prevented load from overwhelming the pipe.

- A band became unusable due to overstretching and had to replace with new set of longer bands. Did not appear to affect results however may have altered translational velocities

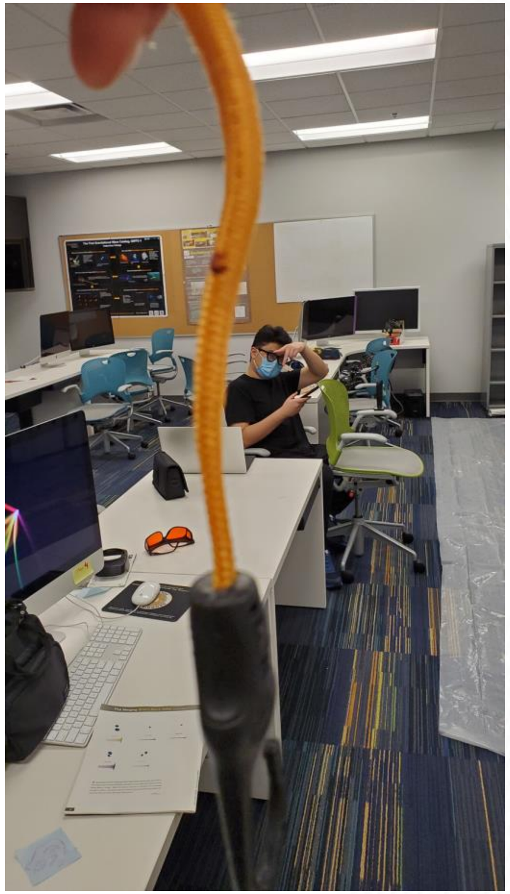
- Screws snapped repeatedly. Duct tape helps, but causes screws to bend instead.

Broken Screws

Sources of Error and Uncertainties

- Camera that recorded the displacements of the piston only has 18 fps, a more precise camera would’ve allowed for a better estimate of the piston velocity. A high fps camera for everything would be better

- Displacements were marked on the piston itself. A automated mechanism for stabilizing the piston, and cannon while simultaneously recording the displacement would reduce this uncertainty



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