Microlab project final report

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1 Introduction and Motivation

Combustion is a complicated process, involving multi-step reactions with uncertain stoichiometries. The result of combustion, apart from the chemical products, is a large change in temperature and pressure. Combustion chambers are designed to withstand these fluctuations. While the Navier Stokes equations, which govern fluid mechanics, do not admit a known general solution, resulting pressure waves can be modeled mathematically. However, the highly nonlinear nature [4] of thermos-acoustic phenomena reduces one's confidence in purely theoretical simulations. Thus, experimental treatments of thermo-acoustic oscillations are performed in the industry. The generated harmonics are found to depend on the geometry of the combustion chamber, and may have a negative impact on the life of these instruments. For instance, high-frequency oscillations may cause significant fatigue, which can lead to increased occurrence of accidents and breakage. Thus, the control of thermo-acoustic oscillations is an important sub-field of combustion research. Several geometric, passive methods are in use for the rejection of these oscillations. More recently, active oscillation-rejection has been explored, by closed-loop feedback into the chambers.

While full-scale experiments are useful, they are also expensive. Meanwhile, simulations may be unreliable. Therefore, a simpler experimental model is desirable. The simplest of these is a Rijke tube [6]. This device is composed of a tube, with a metal mesh inside (located below the center of the tube). The mesh is heated, either electrically, or using a flame. Once the mesh is red-hot (and the flame is turned off, if a flame is used), this causes a convective flow within the tube. Transport of air through the mesh heats up this air, changing the pressure locally. Air-transport in the opposite direction does not cause the same change in pressure. Pressure waves are reinforced by the heated gauze, and standing wave is set up with antinodes near the ends of the tube.

Rijke tubes were discovered by P.L Rijke in 1859 [7]. His initial investigations used a flame and wire mesh, while later experiments used electrical heating. Lord Rayleigh also described this effect, and marveled at the amplitude of generated sounds. There are several variations of Rijke tubes, such as Sondhauss tubes (with a closed end), and tubes operating by cooling instead of heating. These have been explored for decades. However, there are few definitive studies of coupled Rijke tubes, which would be the natural model for coupled, multiple combustors. Previous work on coupled Rijke tubes has been limited to pairs of tubes [2,5], and there is some discussion in literature of littices. The combination of 3 inter-connected tubes has not, to our knowledge, been investigated. Moreover, the control of Rijke tubes by forcing input [6,8], either open-loop or closed-loop, is limited to the control of a single tube.

We hoped to study the effects of forcing of 2 coupled Rijke tubes, and to investigate the feasibility of forcing these tubes into different qualitative behaviors, such as synchronization and oscillation-death.

2 Experimental Apparatus, Design and Data Collection

Our apparatus consisted of H-connected two tubes, composed of steel, PVC and ABS plastic, in different parts of the system. The tubes were heated electrically, and recordings were done using a pressure-transducer. The details of these materials and design choices are discussed later in this section.
The crux of our experiments pertains to the analysis of 2 coupled oscillators. The types of coupling

classically considered are reactive coupling and dissipative coupling. Reactive-coupling forces are proportional to position-difference, while dissipative coupling is proportional to velocity-difference. Ideally, one should have only one or the other type of coupling, for the characterization of a system. Our approach to coupling the two Rijke tubes made it impossible to achieve this. Thus, we could only modify both types of coupling simultaneously. We tried to modify the coupling between the tubes by either changing the length of the coupler, or by adding an acoustically-damping material into the coupler. The second, indirect way of modifying the relationship between the tubes was by changing the length of one of the tubes. This changed the frequency-ratio (or winding-number, 'p') of the system [1].

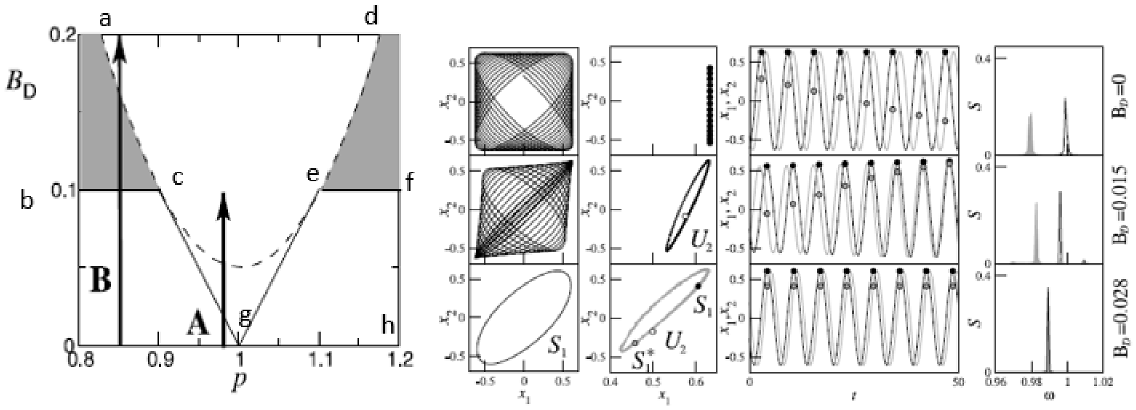

Figure 1: Synchronization Tongue, Phase Plots, Poincare Section and Fourier Spectrum

An illustration of this effect is seen in Figure 1 (adapted from Balanov [1]). Here, only dissipative coupling is considered, for a van der Pol oscillator system. Depending on the strength of coupling B_D and winding-number ('p'), one may find the system in different regions. The region a-c-g-e-d is called the synchronization-tongue. In this region, the 2 coupled oscillators have the same frequency of oscillation (i.e they are phase-locked). The strength of coupling also determines whether the oscillators are simply 'entrained' (weak coupling), or completely synchronized (strong coupling). It is difficult to experimentally distinguish between these cases, but we hope to do so upon further investigation of our results. This is because phase-synchronization may accompany frequency-synchronization, depending on the dynamics of the system.
The regions b-c-g-o and g-e-f-h are the unsynchronized oscillation regions. Here, there are two distinct frequencies. The transition from unsynchronized to synchronized (path 'A') admits a bifurcation (here, a saddle-node bifurcation). Poincare-sections indicate these results, by the appearance of a closed, 1-cycle. The Arnold Tongue (synchronization-tongue) is bordered at high coupling strengths, by regions of oscillation-death. These are regions a-b-c and d-e-f. Here, two coupled oscillators interfere destructively, and the system does not undergo oscillations. This is a desirable condition for thermos-acoustic systems, and much work has focused on keeping systems in these regions. We hoped to achieve the conditions for transitions into these regions through our experiments. The transitions may be achieved by moving left or right, by varying the winding-number 'p'. They may also be achieved by moving down by decreasing the coupling-strength. We do not have a method of increasing coupling strength for our system. Similar Arnold-tongues and synchronization-phase diagrams may be constructed for the case of forced oscillations. In this case, the coupling strength would be measured in terms of the forcing amplitude.

Our experimental design consisted of the following steps:

1) Characterization of single Rijke Tubes: This would involve measuring the pressure (and thus, the frequency spectrum) for individual tubes of different lengths. By using different lengths, we were able to tune the frequency of our tubes.

2) Characterization of coupled Rijke Tubes: This would involve coupling two Rijke tubes with a connector-tube. As both tubes were heated, we would measure the pressure in each. This would tell us which part of the synchronization-phase-diagram the system was operating in. Upon determining this, we would force the system into a different region by changing the coupling-strength or the ratio of frequencies of the two tubes.

3) Characterization of forced, coupled Rijke Tubes: This would involve using a speaker to provide pressure-input to one of the tubes as a pure tone. By varying the amplitude and frequency of the forcing, we would hope to push the entire system into a different region, with key emphasis on pushing it into oscillation-death, due to obvious engineering applications.

In addition to these experiments, we are also interested in the effects of couling 3 or more tubes together, since this is likely to give rise to rich dynamics, including chaotic regimes.

Figure 2: H-shaped coupled Rijke Tubes, Steel+PVC (Inset: Packing peanuts)

Our apparatus was constructed after many trials with failed designs. We found that using good conductors of heat was counter-productive, since a strong gradient in temperature seems to be required in order set up the convective oscillations. The tube material must also withstand high temperatures of a few hundred degrees Celsius, without melting or fuming. Thus, we rejected aluminum and plastics, and decided to use steel piping for the part of the tube housing the heater. This was despite the difficulty in machining steel. In order to get around the difficulty in machining, we used PVC pipes for the relatively cold parts of the tube. These pipes can be easily cut and used to change the length of the composite tube. The connection between 2 Rijke tubes was also made from PVC, using dimensions consistent with [2,4]. The T-joints and I-joints in this H-shaped apparatus were all 3D printed from ABS. The plastic did not melt despite being relatively close to the hot steel tubes. We also found that any gap or leakage in our connectors would dampen-out the effect. Thus, we used stage-tape to prevent leakage. Changing tubes between trials involved removing and re-applying tape. This made the entire setup somewhat unreliable, and a better design would use steel T and I joints, with well-fitting threading. We ordered our pipe-materials from McMaster Carr, with a face-value price of \$50. We were surprised to find out that shipping costs brought the price up to more than \$100. We used 3 lengths of couplers: 15cm, 24cm and 30cm. We also used 2 different lengths of Rijke tubes, with the two being equal in one case, and unequal in another. After characterizing both tubes individually, we "permanently" sealed one of the tubes to the T-connector, to avoid having to change at least one of the connections every time.

Figure 3: Electric Heater Parts: Adapter, Variac, Heater, Mica Tiles and Nichrome Wire

The heaters were constructed using nichrome wire. Originally, we used a propane torch, with a stainless steel mesh and a wide acrylic tube, to create the effect. This procedure was rejected, because the sound was found to be transient, lasting only for 5 to 10 seconds. Since our experiment hinged on measuring pressure over several tens of seconds, this would not work. Instead, we moved to an electric heater, using a hair dryer. The hair dryer was taken apart, and initially used to test the strength of the effect. Subsequently, the wire was replaced with a 25 gauge nichrome wire, wound in a specific way. Winding of the nichrome wire involved making small-diameter (several millimeters) coils of a specific number of turns (close to 200) for each heater. The coiled wire was then supercoiled around a support. The support material was critical, since it was required to withstand high temperatures of 500 degrees Celsius, or even higher. We initially used acrylic, which unsurprisingly melted and created fumes. Fiberglass has an effectively infinite melting point, but the resin which holds it together melts much earlier. Thus, fiberglass did not work either. We settled on using mica tiles, from jewelry-making kits. These tiles were firm, but pliable, and could be cut with a scissor or drilled with holes. One of the critical aspects of the heater was that the wire must not be allowed to touch the conductive metal tube surrounding the heater. Thus, we took care to supercoil the nichrome wire through grooves and holes cut into the mica, which kept it away from the metal tube. The power-supply was provided to the nichrome wire heater, using the adapter from the hair dryer. Since this was found to be too much power, we wanted a way to control this power-delivery. For this purpose, we used a variac to provide the heater with a variable input voltage. The variac was provided by the lab of Professor Dan Goldman, as were the smaller PVC connectors. The 3D printer was used from the Sponberg Lab.
Pressure-transducers (microphones) were borrowed from the Combustion Lab (by Mitchell Passarelli), as was the speaker used in our trials. The microphones were placed 2 to 3 centimeters from the mouth of each tube, to pick up oscillations in an “equal way”. Ideally, we would have placed these microphones at the same relative phase position for each tube, but we did not know where this position would be (for different frequency tubes). We also hope, in the future, to flush-mount one of the microphones on the coupler between the tubes, with a good seal. This would provide a single readout for our system. Flush-mounting is a technical challenge, which was not necessary to tackle for our pilot experiments.
The speaker was selected because it had a housing-tube, which was expected to provide good sound-isolation between the 2 tubes. This speaker tube also had flush-mounting for the microphones. However, during the experiments we discovered that the small diameter of the speaker tube and reflection-effects prevented our system from entering into Rijke tube oscillations. Therefore, we decided to postpone forced-oscillation experiments until a later date (possibly next Spring).

For changing the coupling strength, we used packing-peanuts. Data was acquired using a NI-DAQ, and processed in MATLAB. In our first step, we measured the fundamental frequency of each uncoupled tube.

One of the confounding factors in our experiments was the observation of a low-frequency signal, enveloping and modulating our expected signal. Upon further investigation, we discovered that the source of this signal was the air-conditioning system in the experiment room. A quick calculation yielded a Helmholtz resonator frequency for the room of a few Hz, which was the frequency we observed. This signal was then measured in the absence of our heaters, and was used to filter out the effects of this signal from our results.

3 Results

We found our system to operate in the regime of synchronization. This meant that we observed phase-locking between the two oscillators, relatively quickly after the heating was started (Figure). The single-tube, uncoupled frequencies were found to be 281Hz and 318Hz. In the next step, we measured the frequency of the coupled tubes. This frequency was found to be close to 244Hz for both tubes. Thus, we observed a decrease in entrainment frequency. This was somewhat expected, since the coupler, including the T-joint, effectively extended the total length of the apparatus in a nontrivial manner. However, changing the length of the coupler did not change the synchronization frequency. Therefore, the entrainment was confirmed to be an effect of the mutual interaction of the two tubes, and not simply due to a change in the length of the system.

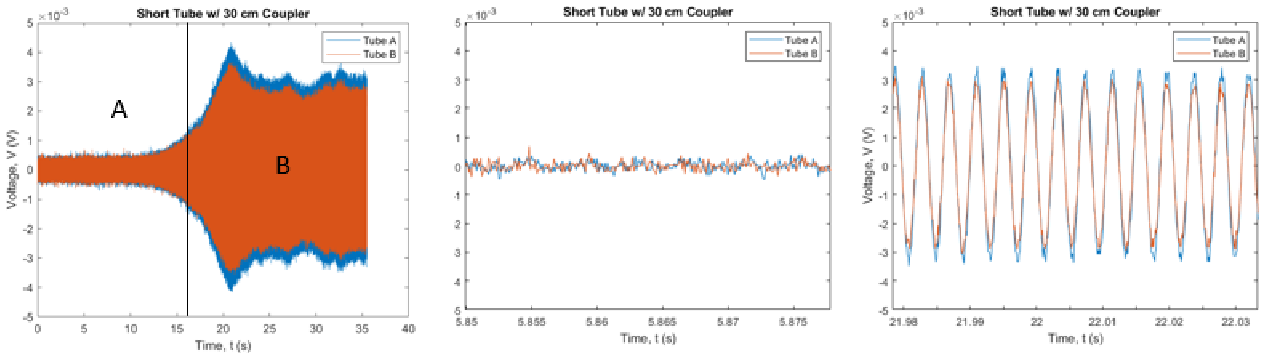

Figure 4: Synchronization: Before and After Heating

Next, we sought to characterize the phase-difference of the synchronized system. This was done by taking the Hilbert transform of our signal. The resulting phase-difference was found to drift about, until the heaters were turned on, whereupon the phase-difference settled quickly at an even multiple of pi (Figure). The strength of phase-locking was measured using the Phase Locking Value metric:

$$PLV = \frac{1}{T}|\Sigma e^{\Delta\Phi}| \tag{1}$$

This metric is effectively a vector-strength. A PLV of 1 is perfectly phase-locked system, while a PLV of 0 is an unsynchronized system.

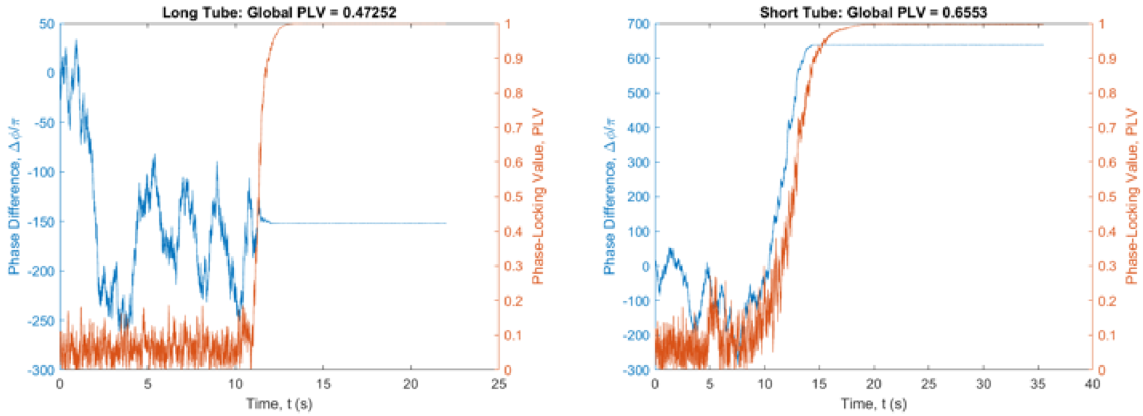

Figure 5: Phase-Locking value and Phase-Difference

Our attempts to change the damping using packing-peanuts were unsuccessful in shifting the system into an observably different regime. Change in winding number would have helped us enter the unsynchronized or oscillation-death zone if the system was in the entrainment region. Since we did not observe this, we have reason to believe that the system was in the complete-synchronization zone. In this case, our only way of exiting this region would be by decreasing the coupling strength sufficiently (which we failed to do with the packing-peanuts).

We also tried plotting the Poincare section for our pressure timeseries. This was done by taking the peak pressure for each tube, in a phase-diagram. Synchronization would imply a closed 1-cycle, or something close to that, of narrow width. While we did observe directional rotations (such as on a torus), these crossed through the annulus, which is unexpected for a Poincare section. We hypothesize that a modulatory noise might cause a change in the torus width at low frequency, to account for this observation. Since the setup had to be taken apart soon after these experiments were performed, we could not validate this hypothesis. We hope to troubleshoot this question in the coming weeks.

4 Summary and Outlook

We constructed an apparatus for investigating the effects of coupling of Rijke Tubes, and confirmed synchronization of the tubes, robust to several geometric conditions. This apparatus allows for relatively long-term experiments, including forced-oscillation experiments.
The phase-locking value of 0 degrees was an interesting observation. It may imply phase-synchronization in our system, similar to the way pairs of Huygens (grandfather) clocks on a beam tend to align themselves in antiphase [3]. Whether this is a property of the system, or the outcome of our limited experiments, remains to be seen.
We were unable to adjust the coupling between the tubes in a methodical manner, and would like to try a better-fitting material such as cotton for the damper, in the future. We are also convinced that simply using coupling tubes of much different length, and possibly smaller coupler diameter, would be sufficient to modulate the coupling. In addition, we are intrigued by the possibility of increasing the coupling-strength between tubes, by increasing the number of couplers, in a ladder-like arrangement, at regular intervals.
In playing with our system, several apparent harmonics were observed, upon partially and transiently blocking one of the tube exits. These harmonics survived for tens of seconds. How these modes are excited (assuming they are harmonics), and why they are apparently semi-stable, are questions we are excited to explore.
In principle, the same apparatus design can be extended to include a third tube. We believe that with 3 tubes, we may be able to enter chaotic regimes. In these conditions, we may be able to reconstruct the attractor from our timeseries. However, these experiments must necessarily be performed only after the 2-oscillator system has been characterized. Using a forcing input from a speaker, we also hope to controllably shift the system into different regimes. Since we are currently unable to define units for our coupling strength, as the coupling is only controlled by the damper material, for general applicability, we must decide on a metric which can clearly define our coupling

strength. By doing so, we may be able to prescribe simple control schemes in terms of geometry, for keeping the system in the oscillation-death region.

5 References

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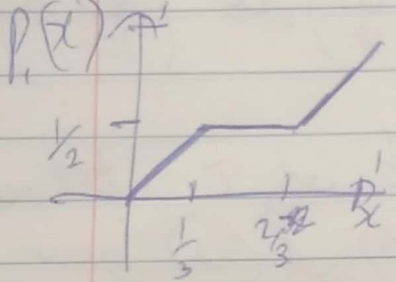
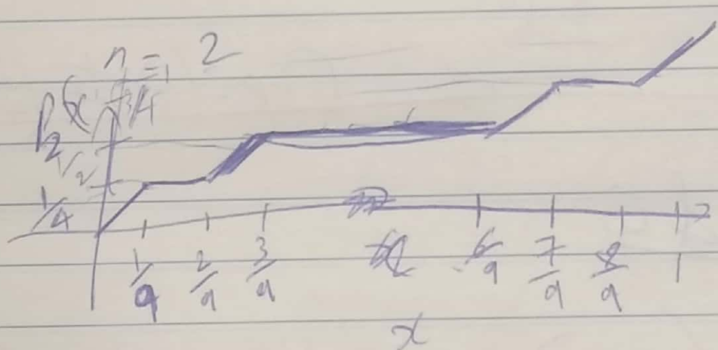
11.2.6

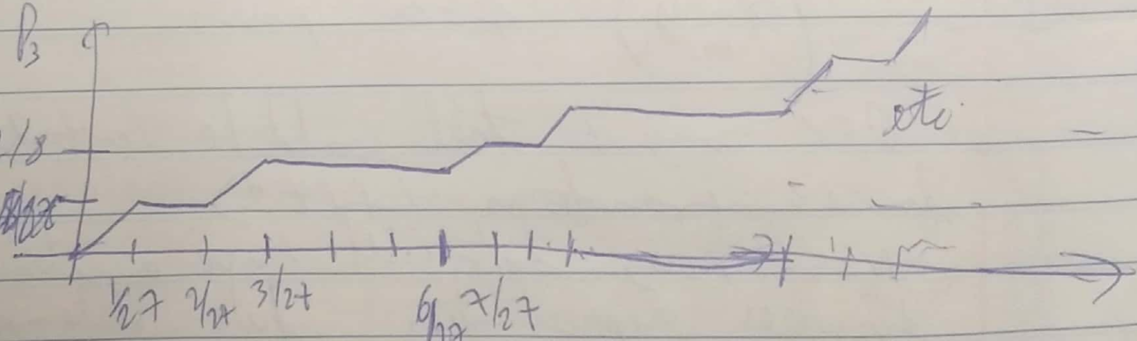
a) $P_0(x) = \frac{x}{1} = x$ $\boxed{\text{Proportional to length of } x}$

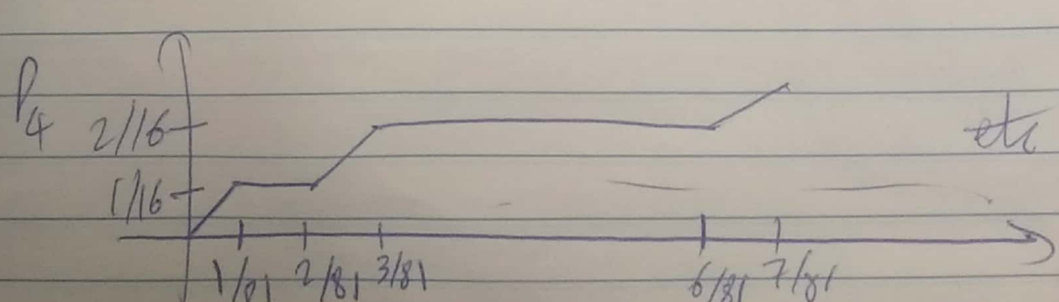
b) $P_1(x) = \begin{cases} x/(2/3) & x \le \frac{1}{3} \\ 1/2 & \frac{1}{3} \le x \le \frac{2}{3} \\ \frac{\frac{1}{3} + \left(x - \frac{2}{3}\right)}{2/3} = \frac{3x-1}{2} & x \ge \frac{2}{3} \end{cases}$



c) n = 2







d) slopes → $\left(\frac{3}{2}\right)^n$ & $\boxed{0}$ derivative is 0 almost everywhere

1/4 - it is continuous

11.3.9

$(27) - (6+1) = \boxed{20}$ copies at each iteration.
$= m$

Scaling $r = 3$

$\therefore d = \frac{\ln(m)}{\ln(r)} = \frac{\ln(20)}{\ln(3)} = 2.73$

Hypersponge :-

~~some number~~

$$\boxed{d = \frac{\ln\left(\underbrace{N \cdot 2^{N-1} + 2^N}_{m}\right)}{\underbrace{\ln(3)}_{r}}}$$

$\cancel{{}^N C_0 (N-2)^N + {}^N C_1 (N-2)^{N-1} \ldots}$

11.4.3

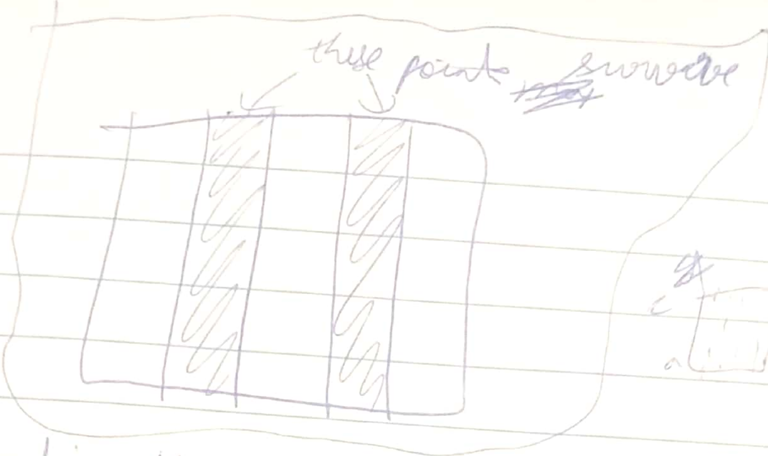
$\epsilon = \left(\frac{1}{3}\right)^n$

$N(\epsilon) = 20^n$

$d = \lim_{\epsilon \to 0} \frac{\ln(N(\epsilon))}{\ln\left(\frac{1}{\epsilon}\right)} = \lim_{n \to \infty} \frac{\ln(20)}{\ln(3)}$

$= \frac{\ln(20)}{\ln(3)} = 2.73$

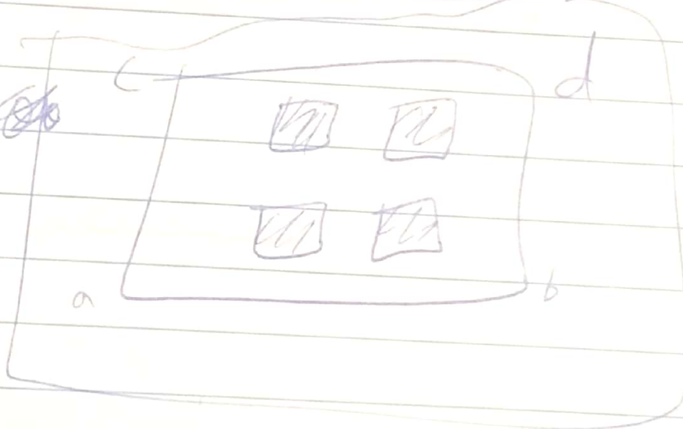
12.17 a)



b) Applying the map again, ~~by symmetry~~

of map 1

survive.



$\therefore$

these survive

c) On continuing to apply the transformation, we get the Cantor dust

The cross-sections of the structure are the Cantor set

3.1

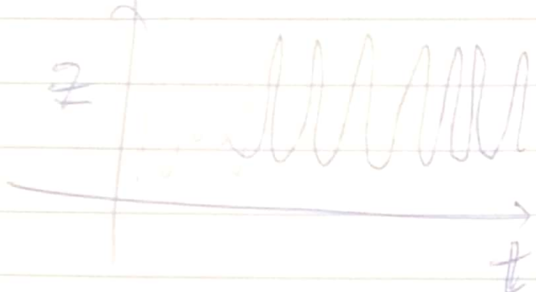
$$\dot{x} = -y - z$$
$$\dot{y} = x + ay$$
$$\dot{z} = b + z(x - c).$$

$\boxed{b = 2,\ c = 4}$

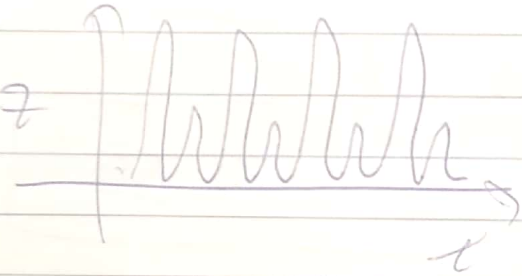
$a = 0$ to 0.4

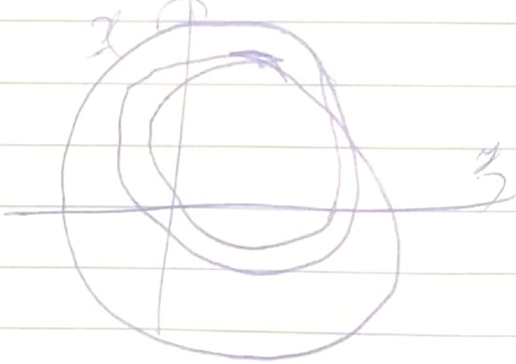
There is a period-doubling route to chaos.

b) At $a = 0.33$

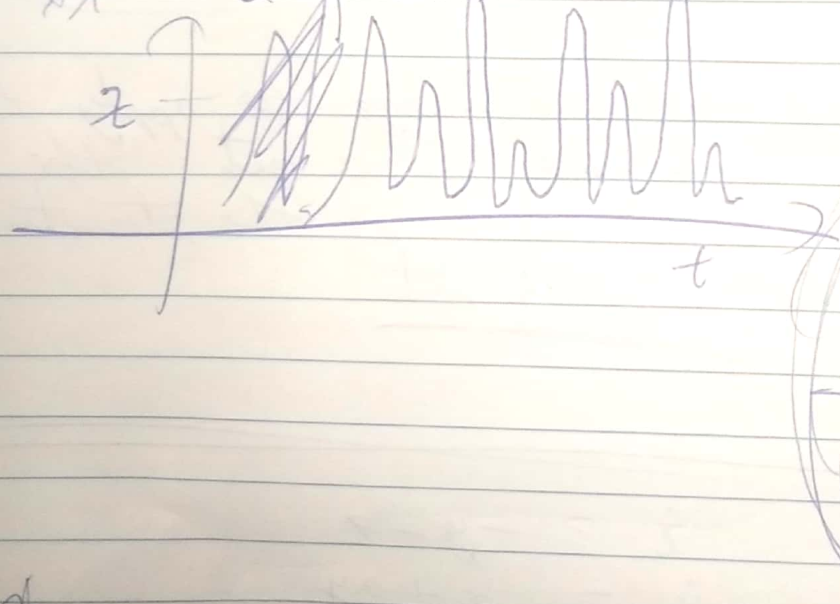


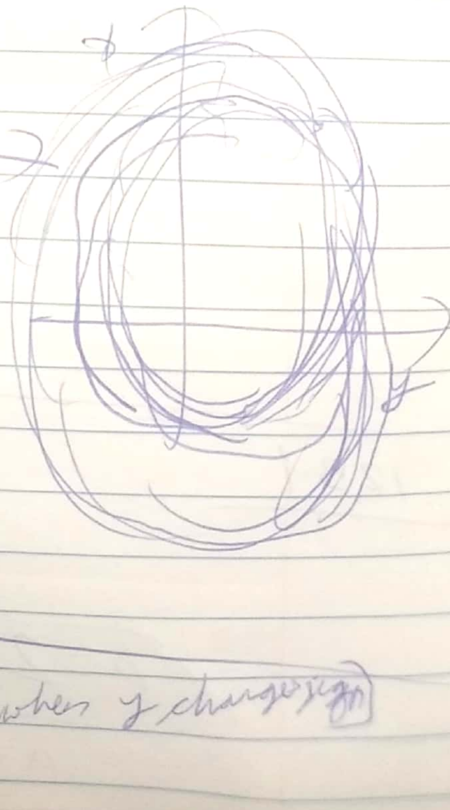
At ~~...~~ $a =$ ~~...~~ 0.36



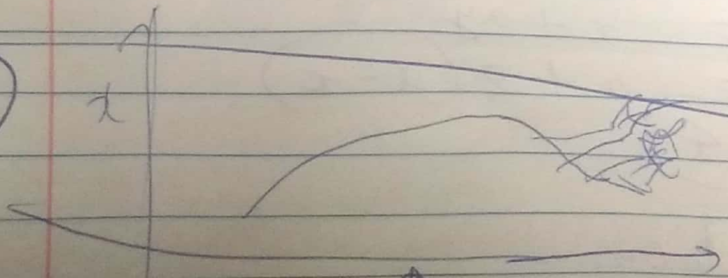


At $a = 0.38$





c)



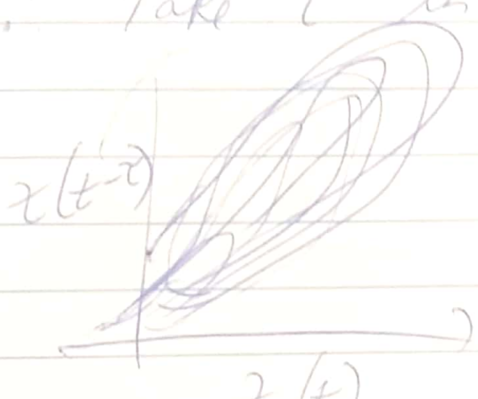
[x when y changes sign]

~~1~~2.4.3

$a = 0.4$
$b = 2$
$c = 4$

Oscillation period is about 6 s.

$\therefore$ Take τ in range $\left(\frac{6}{2} s\right)$ to $\left(\frac{6}{10} s\right)$



Values ~~betw~~ of τ between (1.5 s) & (3 s) give shapes close to the simulation (for $x(t)$ vs $z(t)$).

[i.e. $\left(\frac{1}{2}\right)$ to $\left(\frac{1}{4}\right)$th]

C & G

1.3 (x_i, y_i) 4000 points

One way to tell would be to take a ~~f~~ fourier transform (fft2)

Non-noisy data will have a structured 2d Fourier representation, while white-noise will have no structure