Forcing Effects on Synchronized Rijke Tube Dynamics

1 INTRODUCTION & MOTIVATION

Thermoacoustic combustion instabilities have plagued aerospace engineers for many years, driving the accumulation of a large body of literature focused on both understanding such instabilities and learning how to design around them. Simply put, thermoacoustic instabilities refer to the, typically unwanted, self-excited oscillations induced by the coupling between pressure and heat release rate fluctuations. In aerospace applications, such as a rocket or gas turbine engine, these oscillations have a significant detrimental impact to the operational lifespan [1].

There have been a multitude of different approaches taken to learn how these instabilities work [1]. Some have applied advanced diagnostics to full-scale combustor rigs, while others have attempted to isolate the fundamental physics involved by stripping down models to their most basic elements. The Rijke tube is an example of the latter approach, which removes the complexity of combustion reactions and isolates pure thermoacoustic coupling. A Rijke tube functions via the heat transfer from a heated mesh or coil mounted inside a tube away from the midpoint. A schematic of a Rijke tube is presented in Figure 2 below. The heat transfer from the heated mesh to the surrounding gas increases the local pressure and drives a convective flow through the tube. Heat losses and the asymmetry in the distance between the mesh and each end of the tube results in the formation of a standing acoustic wave along the length of the tube. For a more detailed description of Rijke tubes, see Refs. [2, 3, 4]. Overall, Rijke tubes serve as the simplest model for an actual combustor.

Unfortunately, a complete understanding of the fundamental mechanisms that govern the initiation and maintenance of thermoacoustic instabilities in a Rijke tube continue to elude researchers. There is, however, interest in understanding how such instabilities can interact with one another. In particular, there has been recent interest in how the coupling between multiple combustors influences the observed dynamics [2, 3, 4]. Such research has made use of concepts from nonlinear dynamics and, more specifically, synchronization theory to analyze such phenomena.

It is hypothesized that external forcing can be used to shift the coupled dynamics between regions of the synchronization tongue. As described in Ref. [5], transitions between the various regions of the synchronization tongue are demarked by specific bifurcations in the phase space of the system. These bifurcations include torus-birth and torus-death, saddle-node and homoclinic bifurcations [5]. Thus, it should be possible to detect such bifurcations as the forcing frequency and amplitude are varied. The analysis of power spectra, Poincaré maps or stroboscopic sections and phase portraits can be used for this purpose [5].

This paper describes an experiment that attempted to analyze the effects of external forcing applied to two coupled Rijke tubes. The goal of this experiment was to assess how such forcing could be useful for active control or suppression of thermoacoustic instabilities. Due to unforeseen technical difficulties, however, the scope of the experiment had to be reduced to exploring the nature of the synchronized dynamics of two Rijke tubes. Regardless, the universality of synchronization theory [5] allows the Rijke tubes to be treated as "black-box self-oscillators", vastly simplifying the analysis of the coupled system. In fact, the black-box treatment demonstrates the robustness of synchronization theory. The choice of Rijke tubes for the system is important as an aerospace-specific and practical demonstration of the relevant phenomena.

2 EXPERIMENTAL SETUP

2.1 Test Cases

Each test case is defined by three characteristics: harmonic ratio, coupling strength and forcing configuration. The harmonic ratio, or winding number, denoted by w, is the ratio of the natural frequencies of the Rijke tubes, ω_{0i} (where i differentiates between tubes), i.e.,

$$w \equiv \frac{\omega_{0A}}{\omega_{0B}} = \frac{m}{n}, \qquad m, n \in \mathbb{N} \tag{1}$$

One can consider a generic coupling strength, ϵ, that determines how strongly one Rijke tube affects the other, as described above. Together, the harmonic ratio and coupling strength define the synchronization tongue [5] as illustrated in Figure 1. The numbers in Figure 1 indicate proposed test cases. Note that, due to the power requirements and limitations on the coupling strength, it may not be possible to achieve complete synchronization using the proposed experimental apparatus (see Section 2.2). The remaining test cases are still sufficiently interesting to warrant investigation.

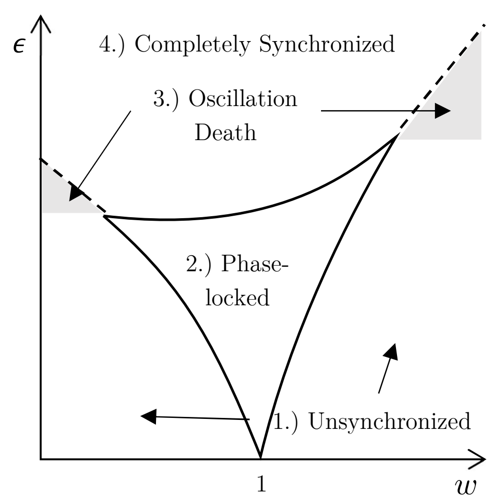

Figure 1 - Schematic of Synchronization Tongue with Proposed Test Cases (numbers)

The forcing configuration refers to whether one or both Rijke tubes are subjected to external forcing. For each (w, ϵ) (corresponding to cases 1-4 in Figure 1), first only tube A will be forced and then the run will be repeated with both tubes forced. Since the coupling is assumed to be symmetric, forcing tube B only should yield similar results as forcing tube A only [5]. This can also be verified via additional test cases, as necessary.

The remainder of this section describes the complete experimental apparatus and design as originally proposed. Due to some design flaws and technical limitations, however, not all test cases were realizable and so some components of the apparatus were not used. Section 5 provides a more detailed discussion of these issues, but the remainder of this section indicates where and how these issues impacted the experimental setup. Specifically, only two synchronized test points were achieved and only without forcing.

2.2 Experimental Apparatus

Figure 2 depicts a simplified schematic of the experimental apparatus, while a detailed image of the final setup is shown in Figure 3. The apparatus consists of two Rijke tubes, of potentially different natural frequencies (controlled by varying the tube lengths), coupled together via a short tube and a loudspeaker to provide forcing to one tube. The coupling tube, or coupler, transmits the pressure fluctuations from one Rijke tube to the other, yielding mutually-coupled dynamics. The loudspeaker was mounted beneath one of the Rijke tubes to apply forcing to that tube. One microphone was suspended below each of the Rijke tubes to measure the sound waves emitted from the Rijke tubes and thereby capture the coupled dynamics of the system.

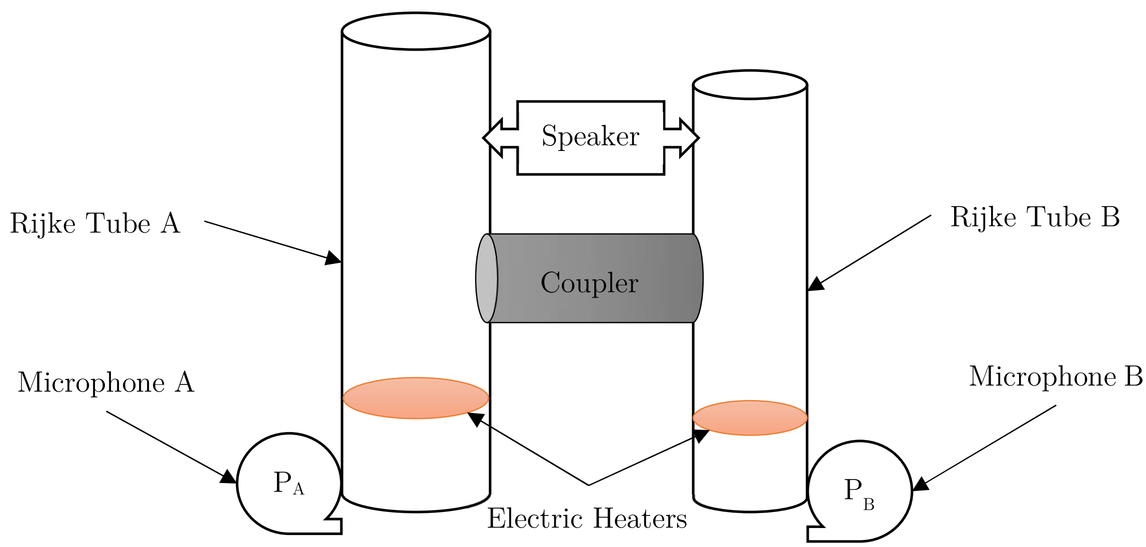

Figure 2 - Schematic of Experimental Apparatus

The Rijke tubes had an inner diameter of approximately 35 mm and a wall thickness of 3.56 mm (NPS 1 ¼, sch. 40). To reduce cost and mass, the Rijke tubes were constructed in two segments joined end-to-end via a custom 3D printed linkage. The upper segment was made of PVC for reduced mass and because it is easy to machine with. Due to the very high temperatures of the heaters, the lower segment was made of steel for its high melting temperature and poor thermal conductivity compared to other metals. To control the natural frequencies of the Rijke tubes, the lengths of the upper segments were varied. Rijke tube A had a length of approximately 0.61 m in both test cases while Rijke tube B was either the same length (0.61 m) or shorter, at 0.58 m, depending on the test case. The two cases were designated the long tube case and the short tube case accordingly.

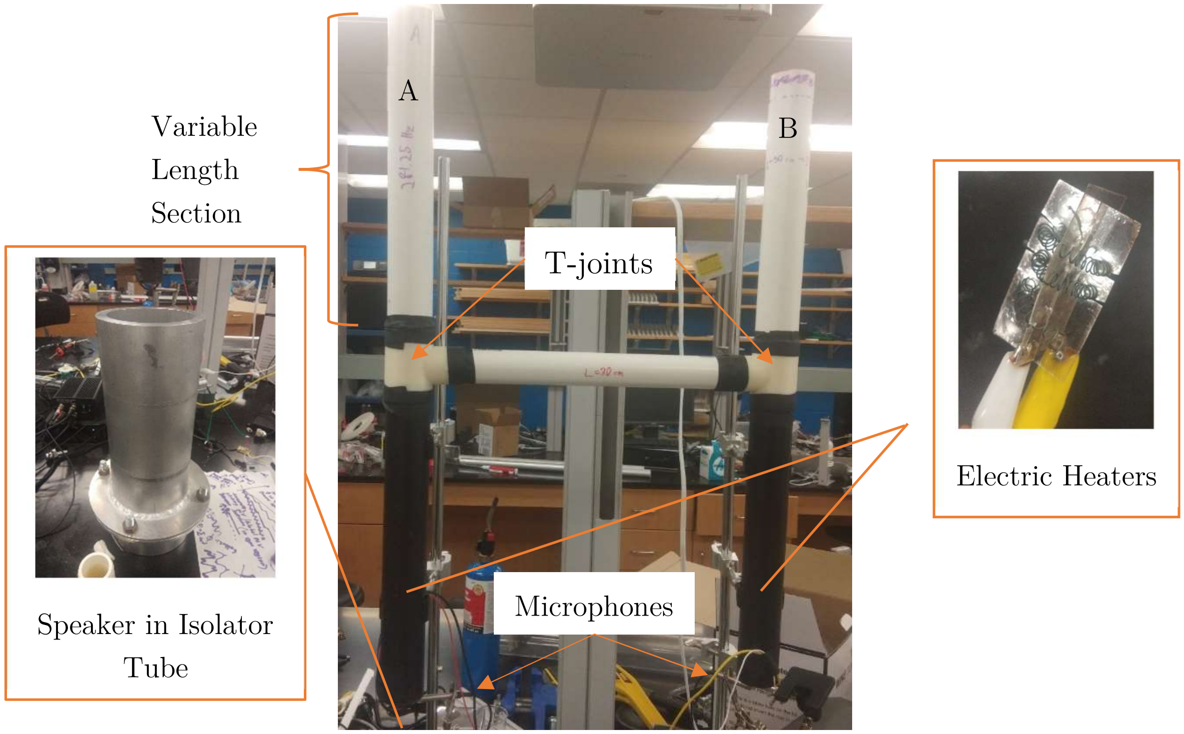

Figure 3 - Image of Experimental Apparatus and Components

The coupler consisted of a 30 cm length of PVC pipe with an inner diameter of approximately 19.05 mm and wall thickness 2.87 mm (NPS ¾, sch. 40). The dimensions of the coupler were based on those used in Ref. [3], which describes how to choose coupler length to avoid (or achieve) oscillation death. Although the complex physics involved make exact evaluation of the coupling strength (ϵ) difficult, linear acoustic theory suggests that, by designing the coupler to avoid resonance, ϵ_{max} (i.e., max transmission) occurs when the coupler is empty [6]. Furthermore, the coupling strength between the tubes should be symmetric (i.e., the effect of tube A on tube B is the same as tube B on tube A). The coupling strength was varied by inserting cotton balls and packing peanuts into the coupler to reduce the transmissivity in the coupler. Despite the coarse control offered by this procedure, the various coupled dynamics have sufficiently different characteristics to determine which region of the synchronization diagram the system lies in for a given configuration [5]. Unfortunately, after inserting any amount of damping material into the coupler, the Rijke tubes stopped working and would not produce any sound, which was part of what reduced the scope of the final experiment.

The electric heaters were constructed from 24-gauge nichrome wire coiled around a mica tile support (see Figure 3) and connected to the power supply from a hair dryer. The power delivered to the wires was controlled using a variable voltage transformer (variac). Ring stands and clamps supported the apparatus above the workbench, with the microphones (G.R.A.S. 40BF with 26AC preamplifier and 12AR power supply) suspended beneath each of the Rijke tubes. The data was recorded in MATLAB using a data acquisition unit (NI DAQ USB-6216) sampling at 10 kHz.

To force only one of the coupled tubes, a speaker was enclosed within an isolation tube (152.4 mm diameter, 304.8 mm length) and positioned directly beneath Rijke tube A. The original aim of the experiment was to vary the forcing frequency and amplitude for each test case to determine how the forcing alters the coupled dynamics. Unfortunately, when the speaker was positioned beneath the Rijke tube, the thermoacoustic oscillations stopped. As such, the speaker was not used for the final experiment.

3 ANALYSIS TECHNIQUES

There are several ways to determine the synchronization state of coupled self-oscillators. For this experiment, three different methods were used. For sake of simplicity, this experiment analyzes one-to-one or same-frequency coupling (i.e., $m = n = 1$) which has the largest synchronization region for a given coupling strength. For completeness, the harmonic ratio is retained in the definitions below. The most basic indicator of synchronization is the frequency difference or detuning, denoted by $\Delta\omega$, between the oscillators:

$$\Delta\omega \equiv m\omega_B - n\omega_A \tag{2}$$

where m and n come from the harmonic ratio of the natural frequencies and A and B denote the different oscillators. Note that $\Delta\omega$ uses the observed frequencies of the oscillators and not the natural frequencies. For both phase-locking and complete synchronization, $\Delta\omega = 0$.

Alone, frequency coincidence (i.e., $\Delta\omega = 0$) does not necessarily imply synchronized dynamics. A better indicator of synchronization is the generalized phase difference,

$$\Delta\phi = n\phi_A - m\phi_B \tag{3}$$

When $\Delta\phi$ is constant in time, then the oscillators are phase-locked [5]. The presence of random noise and other factors can complicate the temporal evolution of $\Delta\phi$, requiring visual inspection of $\Delta\phi$ VS time to identify the synchronized state. For example, $\Delta\phi$ can occasionally jump or slip by a multiple of π over a short time without desynchronization occurring [5]. As such, a more robust and quantitative indicator of phase-locking is the phase-locking value (PLV),

$$PLV \equiv \frac{1}{T}\left|\sum_{t=1}^{T} e^{i\Delta\phi}\right| \tag{4}$$

where T is the time interval over which the PLV is being evaluated, $i \equiv \sqrt{-1}$ and the summation is over all discrete time stamps in a data set. PLV is normalized quantity that $PLV \approx 1$ implies phase-locking, $PLV \approx 0$ implies no phase-locking and intermediate values are indeterminate [7].

In this experiment, the data was processed as follows. First, a low-pass filter with a cut-off frequency of 100 Hz was applied to the data to remove the DC content and any bulk pressure modes present in the room. Based on data collected while the Rijke tubes were not operating, the room had a pressure oscillation at about 3.33 Hz, likely caused by the ventilation system, which needed to be removed from the data. Next, the phases of the oscillations were extracted as the phase angle of the Hilbert transform of the data, as in Ref. [8], and used to compute $\Delta\phi$. Finally, the PLV was calculated in three ways: using the entire timeseries, over the time intervals before and after the thermoacoustic oscillations started and using a sliding window of 500 samples over the full timeseries.

4 RESULTS & DISCUSSION

Before the Rijke tubes were coupled together, the natural frequencies of the tubes in isolation were measured. For a length of 0.61 m, the natural frequency was approximately 281.25 Hz, while for the shorter length of 0.58 m, the natural frequency was approximately 304.50 Hz. Thus, the theoretical winding numbers for the two cases were $w_{long} = 1$ and $w_{short} = 0.92$. Note that the exact natural frequencies of the Rijke tubes are technically unimportant, as synchronization can occur for any pair of frequencies [5].

Figure 4 displays the preprocessed timeseries from both Rijke tubes for each of the test cases. Both data sets began recording before the oscillations started to have a way to differentiate background noise from thermoacoustic oscillations (see the lower plots in Figure 4). Note that the heaters were run at approximately 54 V. For the short tube case, this voltage was exceeded slightly while ramping up and resulted in the overshoot indicated in the timeseries.

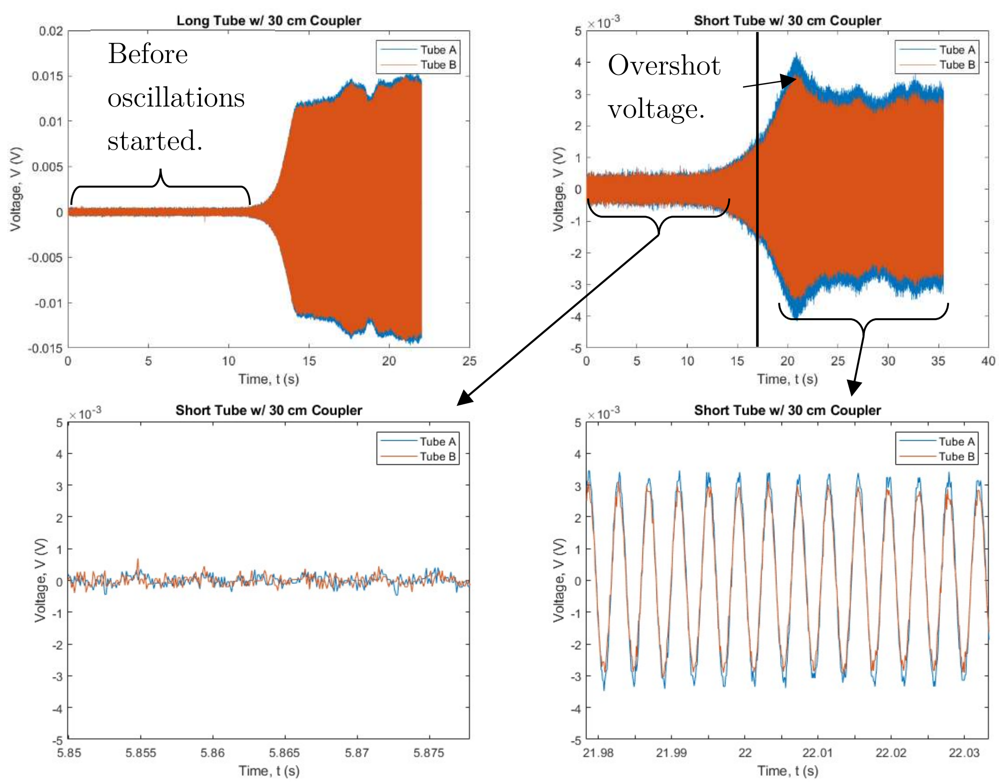

Figure 4 - Timeseries of Long Tube Case (top left) and Short Tube Case (top right) with Zoomed-In Segments Before Oscillation (bottom left) and During Oscillation (bottom right)

From Figure 4, one can see that the thermoacoustic oscillations have a distinct sinusoidal character (bottom right plot of Figure 4), while the ambient data (lower left plot of Figure 4) appears to be background noise. Additionally, the waveforms for both tubes appear to have the same frequency and both of their amplitudes vary in a similar manner. Figure 5 shows the Fourier frequency spectra for both test cases, confirming that both tubes are oscillating at the same frequency (240.46 Hz in the long tube case and 244.20 Hz in the short tube case).

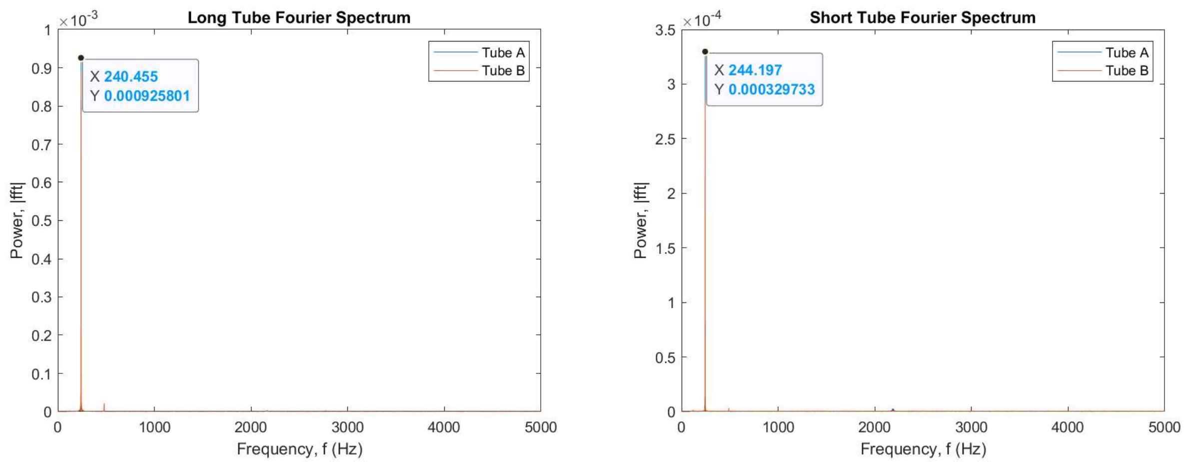

Figure 5 - Fourier Spectra of Long Tube Case (left) and Short Tube Case (right) with Peak Frequencies Indicated

The observed increase in the frequency between the long and short tube cases is expected, since increasing the frequency of one tube should result in the synchronized frequency also increasing. Interestingly, the synchronized frequency in both cases is lower than the natural frequencies of either tube. Basic mutual coupling should result in the synchronized frequency being either between the two natural frequencies or slightly higher than both [5]. The low observed frequency could have been a resonant bulk mode for the coupled system, instead of a synchronized thermoacoustic tone. In this case, the increase in frequency between cases is also expected, as the resonant frequency would be inversely proportional to the internal volume of the system.

To determine if the observed tone was a thermoacoustic mode, two tests were performed. First, one of the heaters was disconnected and one tube was run in isolation. If the observed frequency is a synchronized frequency, then the Rijke tubes should return to their natural frequencies when the other tube is not operating. Unfortunately, the single heater was not able to excite the Rijke tube in the coupled configuration, rendering the test inconclusive.

For the second test, the coupler length was halved to 15 cm to further reduce the total acoustic volume of the apparatus and both Rijke tubes were powered. For a resonant tone, the reduction in acoustic volume should increase the observed frequency, but for a synchronized tone, this should have no effect [5]. In this test, the Rijke tubes emitted the same frequency tone as for the 30 cm coupler, indicating that the observed tone was indeed a synchronized thermoacoustic mode.

For further confirmation, the phase difference and PLV were analyzed, shown in Figure 6. The PLV values in each plot correspond to the segment of the data separated by the black vertical line, which roughly marks the time where the thermoacoustic oscillations began in the timeseries.

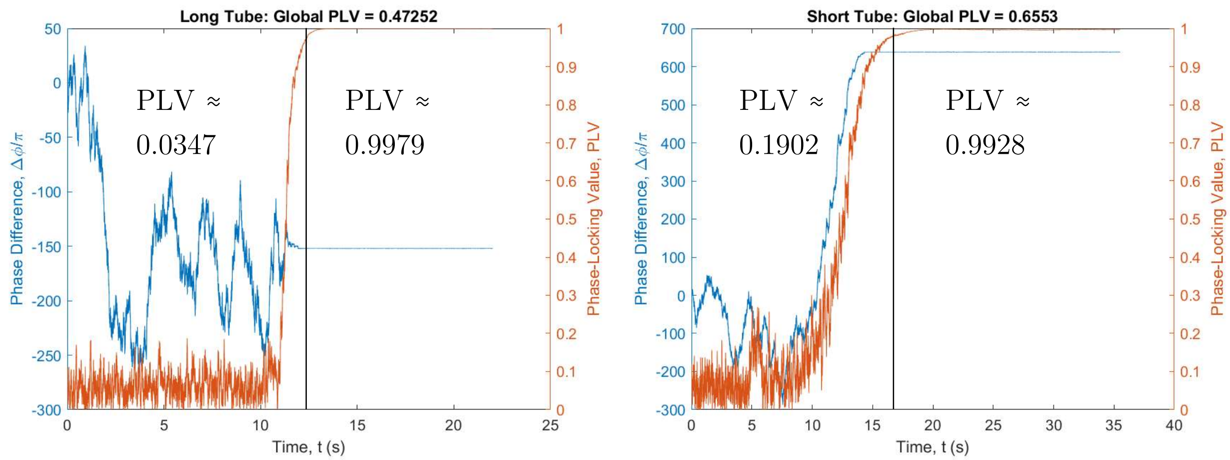

Figure 6 - Phase Difference and Sliding PLV for Long Tube Case (left) and Short Tube Case (right)

From Figure 6, it is obvious that the two Rijke tubes become phase-locked very quickly as the thermoacoustic mode increases in amplitude. Note that $\Delta\phi$ 'locks-on' faster than the PLV likely because of the size of the sliding window. The large variation in $\Delta\phi$ before synchronization in both cases is likely because the oscillation phase is an ill-defined quantity for noise and thus does not vary linearly until the thermoacoustic mode dominates the data.

It is not clear from the data whether the Rijke tubes start at different frequencies and then rapidly lock onto each other, or if the entire system starts up at the synchronized frequency. The phase-locking occurs so rapidly that no temporal variation in frequency is visible in the power spectrograms of the data (not shown). Furthermore, while the temporal variation of the amplitudes of the Rijke tubes appears related (see Figure 4), it is not clear whether the system is completely synchronized or only phase-locked. Regardless, the analysis presented here clearly indicates that the coupled Rijke tubes synchronize during operation and that the synchronized frequency is related to the natural frequencies of the individual Rijke tubes.

5 CONCLUSIONS & FUTURE WORK

This paper reports the analysis of a coupled Rijke tube system and its synchronized dynamics. Specifically, by linking two electrically heated Rijke tubes via a length of pipe, it was possible to observe both tubes oscillate at a frequency lower than either of the individual natural frequencies. The resulting oscillations were clearly phase-locked and exhibited some amplitude relationship, which suggests the system may be completely synchronized.

The results of this experiment have several implications. The black-box treatment of such a complex system demonstrates the robustness and universality of synchronization theory. Empirical evidence of this universality in a system more complex than those traditionally analyzed suggests not only that there are universal mechanisms governing every physical system, but also that there is some ‘language’ capable of describing these fundamental processes.

There were several issues with the experimental design that limited the scope compared to the original proposed experiment. These issues are listed below along with a discussion on how the experiment could be improved to allow for completing the original goal or for enabling future work with regards to other aspects of the experiment:

- Could not use full voltage range of variac, since the coils, wires or power supply would burn out. As a result, the amplitude of the thermoacoustic oscillations was relatively quiet. By designing a more robust power supply and heaters, it could be possible to operate the Rijke tubes at a wider range of steady state conditions, even permitting an exploration of the triggering threshold of the instabilities.
- Could not run the experiment for very long since the tubes would get too hot, which limited the length of data sets and restricted how many runs could be done in succession. An improved design could utilize a ceramic material for at least the lower segment of the Rijke tubes, permitting the heaters to be run at higher temperatures or for longer.
- It was very difficult to seal the joints, which frequently resulted in leaks after modifying the setup between runs. These leaks prevented the thermoacoustic mode from being excited altogether, making the apparatus rather delicate during use. This issue can be remedied by utilizing standardized and self-sealing fittings to join the various tube segments, which were not employed here due to time and cost.
- Inserting any damping material into the coupler silenced the tubes, disabling the variation of coupling strength. Control over the coupling strength is critical to any exploration of the synchronization region. This issue could be resolved by properly designing the damping material to fit in the coupler tube without leaking and designing variants to yield specific values of transmissivity in the coupler. It is important to have a more quantitative knowledge of how the damping material affects the coupler’s transmissivity to investigate more accurately the different synchronization states.
- Attempting to use the speaker silenced the Rijke tubes, preventing any experiments involving forced response. The ability to force the Rijke tubes is critical to the original aim of this experiment. It is likely that the speaker’s isolation tube interfered with the natural convection driven by the electric heaters, thus preventing the Rijke tubes from operating. In addition, the acoustics of the isolation tube could have acted as a Helmholtz resonator or other acoustic filter, which could have silenced the thermoacoustic tone. An improved design could replace the isolation tube with a barrier between the Rijke tubes to prevent the forcing from the speaker from affecting both tubes.

Of special importance for improving this experiment is the possibility to induce oscillation death for a pair of phase-locked self-oscillators. Since multi-mode systems can also be treated as a collection of coupled unimodal oscillators [5], this also implies that multi-mode oscillations can be suppressed via active control at a single forcing frequency. These results would provide valuable insight for the development of new active control strategies in, for example, aerospace engine design.

6 REFERENCES

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