2 Microlab Project Paper - Study of Locally Induced Waves

Introduction

An interesting phenomenon at the intersection of fluid mechanics and nonlinear dynamics is Faraday waves. Faraday waves are standing waves that occur when a fluid comes into contact with a source of vibration at a certain frequency. At the specific frequency for the waves, there are visible patterns that can be seen on the surface of the fluid due to the vibration. Researchers who have explored the dynamics of Faraday waves experimentally most often gather data using a specific setup, that being a fluid in a container that experiences vibration globally from below. In Figure 3[2] it is possible to see the resulting patterns caused by the global vibrations in this specific experimental setup. Various parameters of the experimental setup are varied in order to yield different results and patterns. Prior experimental work, including that of Faraday himself, showed that that the variable parameters are the viscosity of the liquid and the frequency and amplitude of the driving waves which effect the appearance of patterns on the water's surface. At some combinations of these parameters there are no notable patterns in the water, while at other combinations there are chaotic pattern formations. This quality of Faraday waves, that indicates a possible bifurcation, has been explored previously. In addition to frequency, another parameter that is frequently varied is the type of fluid, or more specifically, the viscosity of the fluid used. This project proposes to build upon the prior research completed on Faraday waves by considering a different experimental setup with more variable parameters. The main difference between the proposed experiment and the traditional experiment is the location and generation of the driving frequency. In the proposed setup this driving force is local and above the container of fluid rather than below. This results in more variables that can be altered for each experiment, such as the depth of the source of local vibrations and the size of the source, and the shape of the source. The proposed experiments are aimed at gathering results regarding the affect that different local positions for the source of the vibrations have on the generation of these waves. Other than the fact that this way of inducing Faraday waves has not been explored as thoroughly, another benefit to the exploration of locally forced waves rather than globally forced waves is the fact that in nature, many water sources are much larger than disturbances within them. The results of experimentation with local vibrations may help to better understand real world examples.

Literature Review

Faraday waves were first discovered by Faraday in the 1800s, and since then, there have been multiple experimental and theoretical studies further exploring the behavior of these standing waves. For example, physicists at Haverford College studied "Localized spatiotemporal chaos in surface waves" and created a traditional experimental setup using a circular container with a single driving frequency induced globally from below[2]. By varying the driving frequency and amplitude induced, researchers in this study measure the spatial inhomogeneity of each experimental setup and observe the development of spatiotemporal chaos (STC). Another team of physicists from Simon Fraser University used a similar setup, shown in Figure 4, to

[2] Localized spatiotemporal chaos in surface waves, A. Kudrolli and J. P. Gollub, Phys. Rev. E 54, R1052(R) – Published 1 August 1996

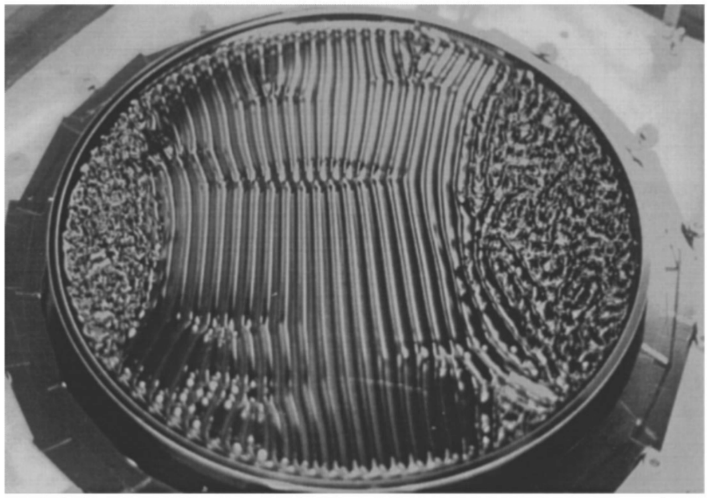

Figure 3: An example of Faraday Waves

induce Faraday waves, and they discover what conditions are necessary for the effect of the sidewalls of the container the fluid is in to be negligible and for the final results of experiments completed in a finite container to match with the theoretical results assuming an infinite container with a finite depth [3]. There are examples of researchers considering different experimental setups, such as in the study of standing waves waves caused by vibrations from the mating calls of alligators, where researchers vibrated a 3-D printed alligator back within water[4], however the majority of studies completed with Faraday waves use the traditional setup. These previous experiments also determine general characteristics that Faraday waves possess that differentiate them from other waves that form in water. Faraday found that the surface waves that were generated through vibrations oscillated at half of the frequency of the vibrations [5]. Researchers Sheldrake and Sheldrake, who viewed various patterns of Faraday waves in water, also observed that the amplitude of the driving force did not effect the wave pattern formation[5]. These prior characterizations are relevant to characterizing the observations of the experimental setup presented in this study and determining the question of if it is possible to excite Faraday waves without a global forcing.

Theory

The derivation of equations that model the behavior of Faraday waves has been completed in multiple different ways. Some physicists consider the waves to be analogous to parametric resonance, and model the amplitudes of the induced waves using Matheiu equations for harmonic oscillators. [3]. A more recent derivation of amplitudes and patterns in Faraday waves begins the investigation with a derivation of Navier-Stokes to find an equation for the amplitude of standing waves[6]. These researchers discover a resulting Lyapunov function

[3]Bechhoefer, J., Ego, V., Manneville, S., and Johnson, B. (1995). An experimental study of the onset of parametrically pumped surface waves in viscous fluids. Journal of Fluid Mechanics, 288, 325-350. doi:10.1017/S0022112095001169

[4]Faraday waves produced by periodic substrates: Mimicking the alligator water dance. Peter Moriarty and R. Glynn Holt. The Journal of the Acoustical Society of America 129:4, 2411-2411

[5]Sheldrake, Merlin Sheldrake, Rupert. (2017). Determinants of Faraday Wave-Patterns in Water Samples Oscillated Vertically at a Range of Frequencies from 50-200 Hz. Water. DOI: 10.14294/WATER.2017.6. 10.14294/WATER.2017.6.

[6]Amplitude equation and pattern selection in Faraday waves. Peilong Chen and Jorge Viñals. Phys. Rev. E 60, 559 – Published 1 July 1999

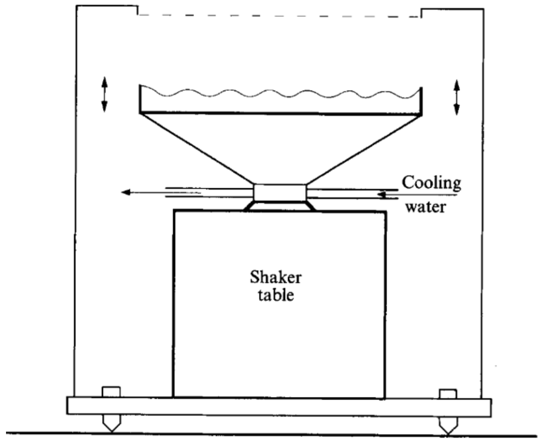

Figure 4: Traditional Setup of the Faraday Wave Experiment

that predicts the pattern of standing waves. They consider a fluid under a vertical vibration $fcos(\omega t)$ to be described by the equation:

$$\delta_t \mathbf{u} + (\mathbf{u} \cdot \Delta)\mathbf{u} = -\frac{1}{\rho}\Delta p + \nu \Delta^2 \mathbf{u} + G(t)\hat{e}_z$$

where **u** is the velocity field of the fluid, $G(t)$ is the effective gravity of the setup which is a combination of the gravitational force on the fluid and the force imparted by the vibrating plate, and the other parameters are dependent on the fluid properties and the environment of the fluid. The effective gravity $G(t) = -g - \frac{1}{2}f(e^{i\omega t} + e^{-i\omega t})$ is dependent on the frequency of the forcing. The basic physical phenomenon behind the creation of these waves is the balance between gravity pulling the fluid downwards and the driving plate pushing the fluid upwards, as well as interactions within the fluid and between the fluid and the boundary, resulting in various patterns. After deriving this equation further, Chen et al. are able to categorize the frequencies and viscosities at which different patterns occur. They divide the possible patterns into stripes, hexagons, squares, and a mix of different patterns. The frequency of the upwards forcing will be an important determinant of if patterns will appear in the fluid at all. A similar pattern forming phenomenon can be viewed in the same experimental setup with sand, or thousands of small solid objects, as well as in convection rolls where fluid is trapped between a hot and a cold plate. However, each of these examples can be decomposed into an upwards force opposing a downwards force. When considering a local forcing case, the impulse to the fluid surface is now approaching from above, and the resulting waves are a consequence of the surface interactions in the fluid. The result of the following experimental setup provides more insight into the physical properties of the locally forced standing waves.

Experimental Setup

The experimental setup for this investigation consisted of a few main parts. This was the fluid container, the generator of vibrations, and the apparatus used to adjust the height of the forcing piston. The fluid container remained constant throughout most of the experiments, and it was a plastic container with an approximate diameter of fifteen centimeters. The fluid used in each experiment was water. The vibrations

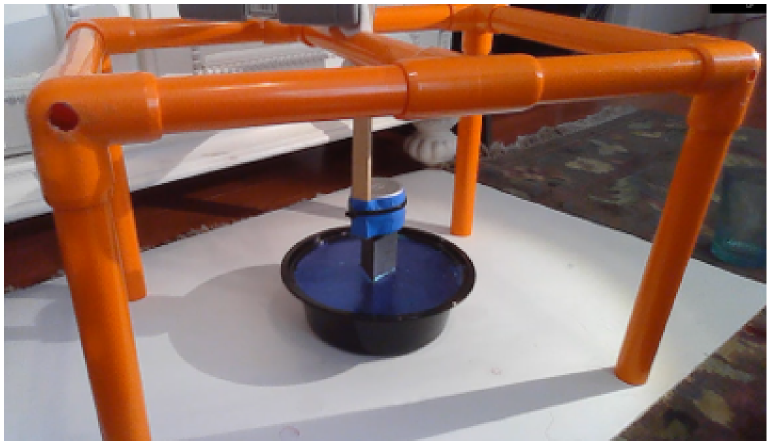

Figure 5: Experimental Setup

were generated by a small Bluetooth speaker that could output a wide range of frequencies, and the vibrations were transferred to the water through a plastic piston. An apparatus for adjusting the height of the speaker and piston combination relative to the water surface was also included to allow for change of depth to be an explored parameter in the experiment. A picture of the experimental setup is shown in Figure 5. In addition to varying the traditional variables of the frequency and amplitude of the vibrations, this experimental setup allows for the variation of the depth of the source of the local vibrations. Another source of variation is the shape of the piston that is interacting with the fluid to induce the surface waves. There are multiple diameters and geometries of pistons available to attach to the Bluetooth speaker that will be emitting the frequencies. The experiment was completed with three geometries and four sizes of each of those geometries. The three geometries were circular, square, and triangular, and the sizes, or the diameters of the circles that the shapes were inscribed in, was 5 mm, 10 mm, 15 mm, and 20 mm. There were a total of twelve plastic pistons created using additive manufacturing that were used in the experiment with the standing waves. With this setup in mind, the experimental procedures and expected results can now be discussed. Ultimately the procedure for this experiment involved varying the height of the piston depth in the fluid in 10 mm increments from $0 - 40$ mm, where 0 indicated that the end of the piston was tangent to the surface of the water and 40 was an almost fully submerged piston. At each height, a frequency of 20 Hz, 40 Hz, 80 Hz, and 120 Hz was played through the speaker to impart a driving force onto the water. The same procedure was completed for every piston shape considered.

Results

After completing the experimental procedure as described above, a few different surface wave patterns were observed that can be seen in Figure 6. The most often observed pattern was that of concentric circles emanating from the piston. There was also a similar state of waves generating from the piston that were not concentric circles but an intersection of concentric waves. The most interesting pattern observed was a ”bubbling” pattern, that resembled the chaotic state pictured in Figure 3. These were most relevant to determining if Faraday waves were generated through the locally-forced experimental setup. Finally there were waves that resembled

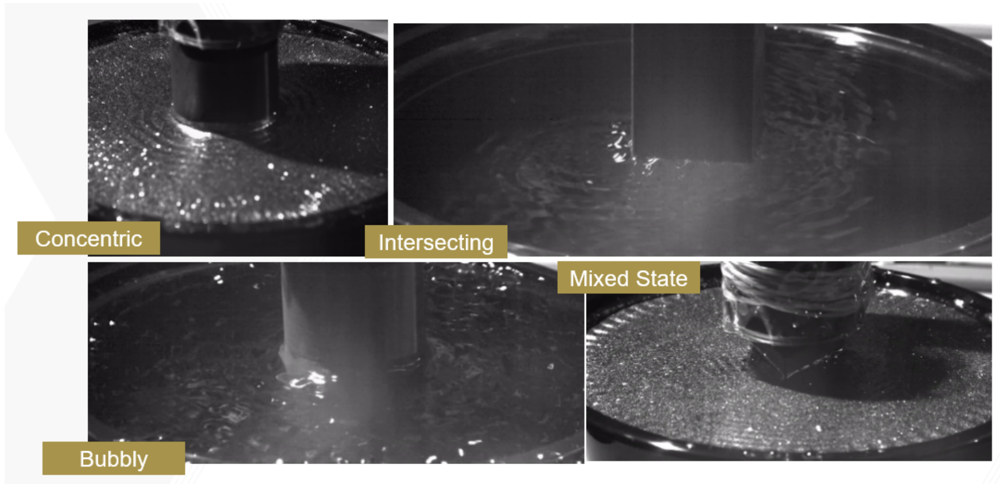

Figure 6: Classification of different wave patterns

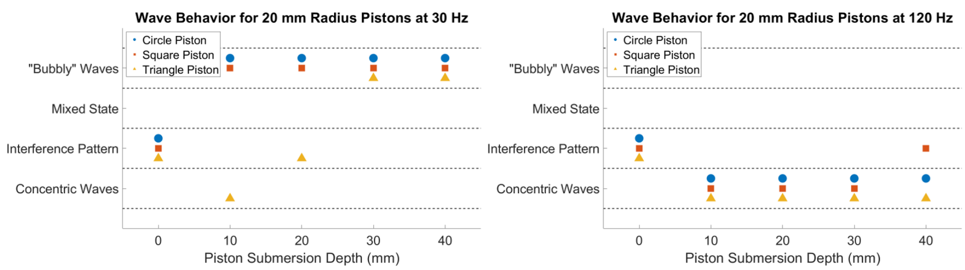

Figure 7: Wave complexity compared to piston shape and depth

a "mixed state" where the outer edge of the container exhibited a striped wave pattern, while the center of the container appeared as concentric waves. These patterns were ranked by complexity, where concentric waves were the least complex, the intersecting concentric waves were next, the mixed states were the next most complex, and the "bubbling" patterns were the most complex. Using this complexity ranking, some trends between the complexity of the surface wave pattern and the parameters, such as piston size, shape, and frequency of forcing, can be made. One observation was that the largest pistons resulted in more interesting surface patterns as compared to smaller pistons. In addition, complex patterns were more likely to appear around 30 to 40 Hz frequencies of the forcing. Figure 7 shows a graph of piston depth compared to the complexity of the waves for two separate frequencies and the same size piston, and it is visible that at the higher frequency of 120 Hz, there was very little to no observation of complex behavior.

In addition to the experimental setup initially described, some more exploration was completed by including different factors. A few experiments were completed with the addition of glitter to the surface of the water for a better visualization of the wave patterns. While some glitter provided information regarding the speed and direction of flows on the surface of the water, a thick layer of glitter suppressed complex behaviour that would have been viewed previously for the same parameters without glitter. Another experiment that decreased complex behavior was dampening the container itself. Upon viewing experimental videos, it became evident that the plastic container that was holding the liquid began to oscillate. In an attempt to prevent the behavior of the container from influencing the experiment, a different container dampened by weights was chosen for a

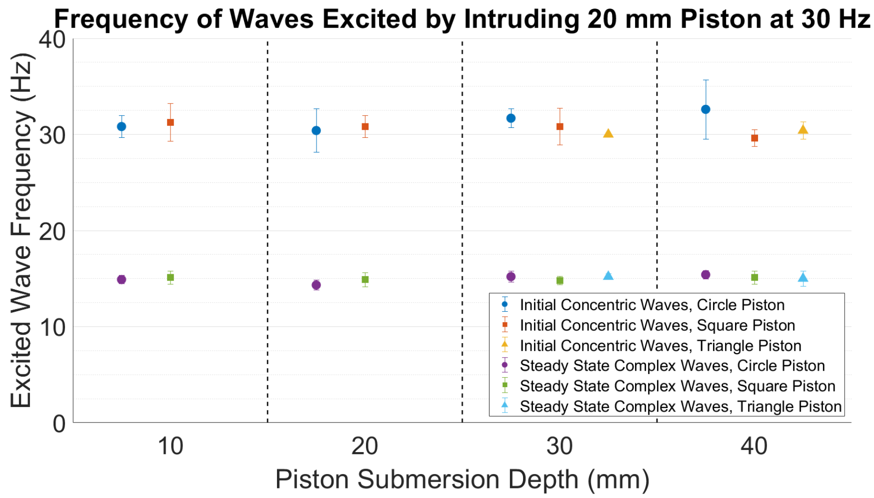

Figure 8: Frequency of wave oscillation

few experiments. Finally, a short exploration of differences in amplitude was completed, and it was observed that the amplitude changed the size of the waves but not the pattern. The influence of these parameters that were not initially considered provides some insight into the underlying dynamics of the experimental system which will be outlined in the discussion section.

Discussion

Using the resulting data, the first question to be answered is if the resulting waves are Faraday waves or another form of complex waves. As discussed in the Theory section, Faraday waves on the surface of liquids were found to oscillate at half of the frequency of the forcing. Using the video data taken from the experiments, it is possible to determine what the frequency of oscillations of the surface waves is. Figure 8 shows the oscillation frequency of concentric and more complex waves at 30Hz. The concentric waves oscillate at a frequency close to the forcing frequency, while the more complex waves oscillate at half of the forcing frequency. This provides more confidence that some of the complex wave patterns that were appearing during the locally forced experiments were Faraday waves, due to the fact that they resembled Faraday waves and they also met a key characteristic of Faraday waves. A physical explanation for some of the other phenomena observed in the experiments was also considered. While the complex waves may be categorized as Faraday waves, the physics of the concentric wave patterns may be explained with simple wave physics. When viewed at high speeds, it is evident that the concentric waves begin at the piston and travel to the edge of the container where the static concentric waves begin. This may be explained by considering the edge of the container as a boundary that the wave can reflect from. Ultimately, the concentric waves may be described as interference waves created by the combination of a simple sinusoidal waves and its reflection.

In general, the local forcing of standing waves is much different from a theoretical standpoint. Rather than considering the gravitational force and its opposition as the vertical vibration of the fluid, the forcing is pushing down onto the fluid along with gravity. The other considerations for the creation of Faraday waves is surface tension and boundary conditions, and these may be more important actors in the locally forced case. The fact that the glitter dampened the complexity of the waves may indicate the role that the surface properties of the water plays in the local generation of Faraday waves. In addition, the properties of the boundary were tested when the container is changed and dampened. A dampened container produced less complex patterns. The results of the dampened container may also indicate that some global forcing was involved in the results of the experiment. If the entire container is oscillating, then the water is undergoing a vertical vibration as well as the local vibration. The size of the piston and the depth of the piston were also varied in the experiments, and it showed that larger pistons provided more complex behavior. A physical consideration could be the fact that a larger piston and a piston that is deeper in the water is oscillating more of the water and is therefore able to apply the vibrations more effectively.

Conclusions and Future Work

The goal of this project was to explore a novel experimental setup for the generation of complex standing waves or Faraday waves, the main difference being the fact that the waves are generated locally rather than globally. The main research questions of this study were if it was possible to generate Faraday waves locally, and how did the various parameters associated with local wave generation change the shape of the waves. In order to answer these questions, an experiment was completed that varied the size, shape, and depth of the local forcing along with the frequency of the forcing, and the results were analyzed. The complex wave patterns showed characteristics of Faraday waves, and the correlation between the key parameters regarding locally generated vibrations and the complexity of the resulting standing wave provided insight into the direction to a theoretical model for locally-induced Faraday waves.

Some future directions to continue the exploration of this topic could include a determination of how local was the driving force. This would include further experiments with a dampened container to determine if any complex wave patterns would appear for a completely fixed container, as well as completing experiments using a global forcing with the same container and placing the unforced pistons at various points in the water. The second experiment could be compared to the results of the experiment completed for this project to determine if the vibration of the container may have given the results of a globally forced experimental setup. In addition, a further exploration into the physics of surface tension and the surface behavior of liquids would be useful to explore the physical phenomenon behind locally forced waves.

Code for Problems

```
%% a = 0;
v0 = [-0.25; 0; 0.5];
a = 0;
[t,v] = ode45(@(t,v) rossler(t,v,a),[0 100], v0);
string = '; a = 0';
plotter(t,v,string)

%% a = 0.05;
v0 = [-0.25; 0; 0.5];
a = 0.05;
[t,v] = ode45(@(t,v) rossler(t,v,a),[0 100], v0);
string = '; a = 0.05';
plotter(t,v,string)

%% a = 0.1;
v0 = [-0.25; 0; 0.5];
```

```
a = 0.1;
[t,v] = ode45(@(t,v) rossler(t,v,a),[0 100], v0);
string = '; a = 0.1';
plotter(t,v,string)

%% a = 0.15;
v0 = [-0.25; 0; 0.5];
a = 0.15;
[t,v] = ode45(@(t,v) rossler(t,v,a),[0 100], v0);
string = '; a = 0.15';
plotter(t,v,string)

%% a = 0.2;
v0 = [-0.25; 0; 0.5];
a = 0.2;
[t,v] = ode45(@(t,v) rossler(t,v,a),[0 100], v0);
string = '; a = 0.2';
plotter(t,v,string)

%% a = 0.25;
v0 = [-0.25; 0; 0.5];
a = 0.25;
[t,v] = ode45(@(t,v) rossler(t,v,a),[0 100], v0);
string = '; a = 0.25';
plotter(t,v,string)

%% a = 0.3;
v0 = [-0.25; 0; 0.5];
a = 0.3;
[t,v] = ode45(@(t,v) rossler(t,v,a),[0 100], v0);
string = '; a = 0.3';
plotter(t,v,string)

%% a = 0.35;
v0 = [-0.25; 0; 0.5];
a = 0.35;
[t,v] = ode45(@(t,v) rossler(t,v,a),[0 100], v0);
string = '; a = 0.35';
plotter(t,v,string)

%% a = 0.4;
v0 = [-0.25; 0; 0.5];
a = 0.4;
[t,v] = ode45(@(t,v) rossler(t,v,a),[0 100], v0);
string = '; a = 0.4';
plotter(t,v,string)

%% 12.4.3;
v0 = [-0.25; 0; 0.5];
a = 0.4;
[t,v] = ode45(@(t,v) rossler(t,v,a),[0 500], v0);
string = '; a = 0.4';
tau = 1.5;
new_t = t+tau.*ones(length(t),1);
new_x = ones(length(t),1);
closestIndex = 0;
i = 1;

while closestIndex ~= length(t)
[minValue, closestIndex] = min(abs(t - new_t(i).*ones(length(t),1)));
new_x(i) = v(closestIndex,1);
i = i + 1;
end

tiledlayout(1,2)
nexttile
plot(new_x, v(:,1))
xlabel('x(t)')
ylabel('x(t+tau)')
nexttile
plot3(v(:,1),v(:,2),v(:,3))
title(strcat('Rossler System',string))

function derivative = rossler(t, v, a)
    b = 2;
    c = 4;
    derivative = [-v(2)-v(3); v(1) + a*v(2); b + v(3)*(v(1) - c)];
end

function plotter(t,v,string)
tiledlayout(1,2)
nexttile
plot(t, v(:,3))
title(strcat('z(t)',string))
xlabel('t')
nexttile
plot3(v(:,1),v(:,2),v(:,3))
title(strcat('Rossler System',string))
end
```