The Effect of Platform Mass on Metronome Synchronization

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Abstract

In this project, we examine the way that the mass of a platform has an effect on the coupling between an array of metronomes in a synchronization setting. In particular, we first develop video processing software that enables us to track the trajectories of the metronomes by using color-coded pointers In the sequel, we slowly increase the mass of the platform supporting the metronomes to investigate how the synchronization time alters and make a comparison between our outcomes and the theoretical estimations in a quantitative manner.

I. Introduction

In the foregoing centuries, synchronization phenomena have been thoroughly investigated owing to their emergence in a variety of different disciplines [1]. At first, synchronization was observed by the physicist Christiaan Huygens in 1657, where it was studied mainly in the mechanical sector. In particular, Huygens noted that synchronization happening between two pendulum clocks hanging by an overhead beam in his laboratory. Alleging that this behavior arises from air fluctuations brought about by the motion of the pendulums, Huygens did proceed with his experiments, revealing the stability properties of his pendulum system, although without the mathematical tools that allow him to rigorously describe them [2]. Nevertheless, his initial conjecture on the nature of the coupling mechanism was incorrect.

The need to study synchronization has become obvious by the diversity of different fields where relevant models have been utilized. The rekindling of interest in synchronization was in part owing to their utilization in modeling biological phenomena [3]. One remarkable paradigm is the synchronization of firefly light emission [4], where different cell types have been demonstrated to exhibit oscillatory behaviors that synchronize as needed. As is often the case, some of the most exciting results concern the brain operation. In particular, Norbert Wiener, during his studies of human brain waves, noticed a growth of activity in electroencephalograms in a narrow frequency band around 10Hz. Therefore, he made a hypothesis that oscillating elements in the brain demonstrated collective behavior bringing about this frequency matching [5]. Albeit Wiener did not come up with a theory with regards to those findings, later researchers found out the way that neurons synchronize their spiking behavior with the purpose of creating macroscopic properties such as memory storing [6].

Apart from biological systems, it is worth noting that synchronization has been both noticed and leveraged in several different areas [7]. It is of great importance to refer to synchronization phenomena in the social sciences. The emergent collective behavior of rational agents has been studied adequately from a synchronization-theoretic standpoint because of the influence that personal opinions of individual agents have on their environment. In [8], the authors introduce a Kuramoto-based model of opinion formation that goes beyond the standard consensus-oriented ones that have been extensively studied so far.

It is worth noting that the design of various engineered systems has relied on synchronization principles. In particular, data-mining algorithms have been developed, encoding data vectors – taken from a database of raw data – into vectors of natural frequencies for an oscillators' dynamical model, expecting that an algorithm simulating synchronization dynamics will classify similar data in clusters, therefore revealing patterns in the data sets [9]. In a similar manner, the power production, transmission, dissipation, and consumption of a power grid constitutes a dynamical problem and the power grid can be thought of as a paradigm of a system of oscillators, similar to a second order Kuramoto model [10].

Although the synchronization of mechanical oscillators was one of the first such phenomena to have been investigated, it continues to be an active research topic even nowadays. Great efforts have been focused on the experimental study of metronome synchronization, allowing us to investigate deeper the underlying mechanisms while expanding our understanding of the emerging behaviors. Metronomes have been deployed as a means for simulating the pendulum clocks of Huygens. Pantaleone [11] developed a theoretical framework concerning the synchronization mechanism of metronomes coupled through a platform restricted to move along one direction. Also, he further showed how the derived mathematical expressions could yield a Kuramoto, that is a well-known theoretical system capturing the phase dynamics of coupled oscillators [12].

In the existing literature, the dynamical model introduced by Pantaleone has been studied concerning various of its properties. The nature of the steady-state frequency locking has been studied in numerous publications [13], where it has been noticed that when the number of metronomes is large, it turns out that there are different emerging equilibria. When the phase-locking occurs, the metronomes can converge either to the same phase or in anti-phase locking, wherein the latter case the system converges to a state composed of two clusters with a phase difference of π. On the other hand, in chimera states, a part of the metronome population synchronizes, whereas another oscillates incoherently [14]. In addition, coupled mechanical oscillators have been examined thoroughly on a number of their characteristics. The authors of [15] published a stability analysis of the synchronization of a system of two pendulums, which reveals both the theoretical interest and the difficulty of the metronome system analysis in the research community. The authors of [16] study the behavior of coupled metronomes of different lengths, whereas in [2] the authors investigate bifurcation phenomena in a system of two weakly coupled metronomes. In a similar manner, in [17] the behavior of two metronomes was altered by varying the rolling friction of a PVC rolling basis. Also, bifurcations diagrams were showcased and explained.

Finally, following the above research directions, in this project we shall focus on studying the robustness, stability and convergence properties of a system of three coupled oscillators featuring phase locking, and how the changes in the mass of the platform affect the aforementioned properties. To that end, we deployed a widely-used structure of metronomes operating on a common platform constituting the coupling mechanism, itself resting on two soda cans. The platform mass was changed by the addition of weights in equal discrete steps, whereas visual measurements were obtained by using the camera of a smartphone, thereby enabling us to obtain the trajectories of the metronomes.

II. Experimental Setup

In this section, we shall describe the experimental setup built in the context of this project. Our main components include the metronomes themselves. During the measurement process, we employed three metronomes. Despite the fact that there have been research efforts involving larger numbers of metronomes, we decide to deploy only three of them for two reasons; one regards the fact that the measurement process itself turns out to be more difficult as more metronomes are utilized (and the need for higher resolution camera becomes more obvious), whereas from a theoretical point of view, we have ascertained that the more quantitative experiments have kept the numbers of metronomes at most three. The metronomes utilized were of the brand Wittner, whose frequency was set at 184 beats (half-oscillations) per minute. This corresponds to oscillations with a period equal to $T = 0.652s$. The energy of the metronomes is supplied through a hand-wound spring. The way that this energy is used will be described thoroughly in the below analysis.

The coupling was attained through a board, composed of a simple piece of rigid material. In fact, this rigidity is of great importance with regards to the lossless transmission of inertial forces among the metronomes. In addition, we attempted to keep the weight of the board relatively low, using either foam or a light oven pan. The board and the metronomes, in turn, rest upon two soda cans, which are also rigid as to minimize the deformation and, in a more general sense, the friction of the platform. This mechanism will also become more obvious on the road. The soda cans restrict the system to move along only one direction. As will be further elaborated upon in the sequel, to investigate the parameter space of the system, we increased the mass of the platform by adding weights. The weights utilized were small tomato sauce cans, with masses equal to either 170g or 227g. The structure is depicted in Fig. 1.

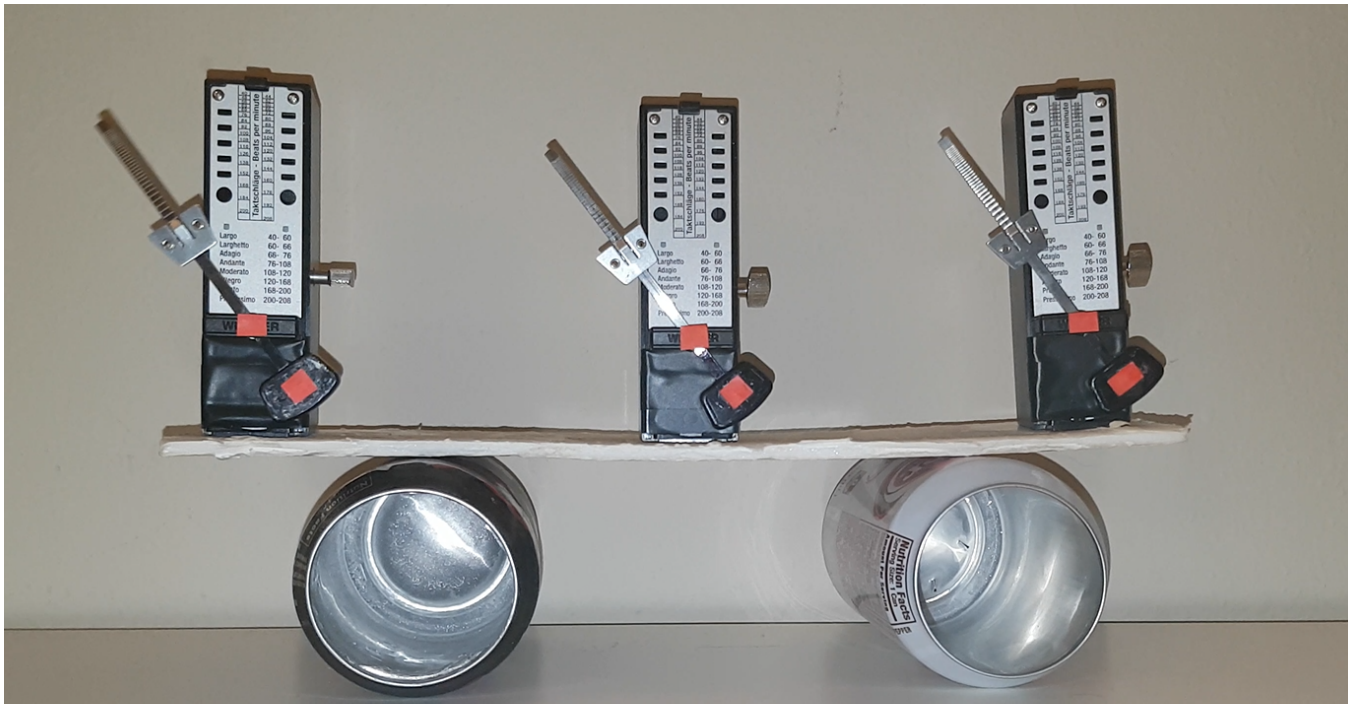


Fig.1: The experimental setup employed for the project.

The most challenging part of the project was the one concerning collecting and processing of the measurements. To that end, we select to acquire visual measurements of the system oscillations. The video acquisition system is composed of a phone camera, a number of paper wafers and a computer. The camera resolution is 3840×2160, recording at 30 Hz. The paper wafers employed are made of brightly colored paper, and are leveraged to enable the tracking of the metronome bobs. In particular, we attached the pointers in the bottom end of the bobs. By doing so, the movement of the metronome is captured more easily owing to the smaller frame needed for the camera to keep all the metronomes in view while being sufficiently close for the video processing software to track the positions of the wafers. Also, special care was taken in choosing a background that was sufficiently white to facilitate the software to discriminate the colored pointers.

To capture the motion of the oscillating metronomes, we employed Matlab code, utilizing both the video processing and image recognition software incorporated therein. After recording the video of the experiment, the file was processed by Matlab and the image was split into three parts, each corresponding to only one metronome. In the sequel, a code was created that looked for the colored wafer on the metronome bob in each frame of the video. Moreover, to enhance the efficiency of the proposed algorithm, we partitioned the frame into different areas and look for the colored wafers only in those that included the metronome. Because in the preliminary processing stage of the trial we had saved the RGB color code corresponding to each wafer, the code was used to determining pixels in the image with the suitable RGB code. Once the wafers were found, that point was indicated as the required point. By locating both the end of the bob and the pivot point of metronome in the same manner, the angle of the metronome was computed by $\phi_i = \text{arc}\tan \delta y_p / \delta x_p$.

III. Theoretical model

In the literature, we have found two different manners of theoretically describing coupled metronome systems. The first is a low-level second order model obtained by using the Euler-Lagrange equations

[18], [19], given, for all $i \in \{1, \ldots, N\}$ metronomes, as

$$\ddot{\phi}_i + b\dot{\phi}_i + \frac{g}{l}\sin\phi_i + \frac{1}{l}\ddot{x}\cos\phi_i + \bar{F}_i = 0, \tag{1}$$

$$(M + Nm)\ddot{x} + B\dot{x} + Kx + ml\sum_{j=1}^{N}\sin\phi_i = 0,$$

where ϕ_i denotes the metronome bob's angle with the horizontal axis, x the platform position, l the length of the bob, and m, M the mass of the metronomes (supposed to be identical) and the platform, respectively. The mass M will be the parameter altered throughout our experimental trials, and we shall experimentally examine its effect on the coupling strength. It can be readily found out that the system is composed of $N + 1$ second-order equations of motion, corresponding to the N metronomes and moving platform. This model is quite helpful in demonstrating the coupling mechanism in a more physical level. Each metronome has an internal dissipative force because of friction, $b\dot{\phi}_i$, which would cease the oscillations in a free metronome. To preserve the oscillation amplitude, metronomes are fitted with a mechanism that attenuates the dissipation, termed as an "escapement" mechanism. The force of the escapement mechanism is denoted by $\bar{F}_i$, and is usually modeled as a van der Pol term. It can also be noticed that each metronome is steered by the inertial force of the moving platform owing to the term $1/l\ddot{x}\cos\phi_i$. The platform equation displays the coupling mechanism among metronomes. It can be noticed that the oscillation of the platform is affected by a friction force $B\dot{x}$ kept at a minimum by the utilization of rigid soda cans. Also, the platform is steered by the motion of the metronomes. This "feedback" coupling among metronomes and platform is the mechanism by which the metronomes control their phase difference.

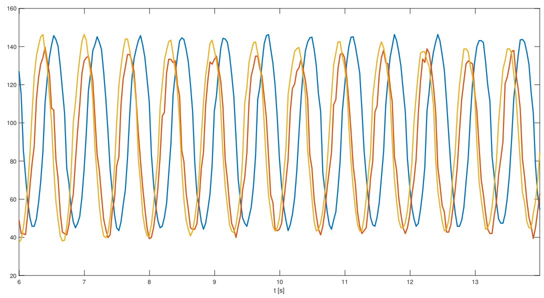
In fact, Kuramoto models are simplified first-order models that have been displayed to capture a diversity of synchronization phenomena. In their normal form, the Kuramoto oscillator model is described by

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_i - \theta_j), \ \forall i \in \{1, \ldots, N\}, \tag{2}$$

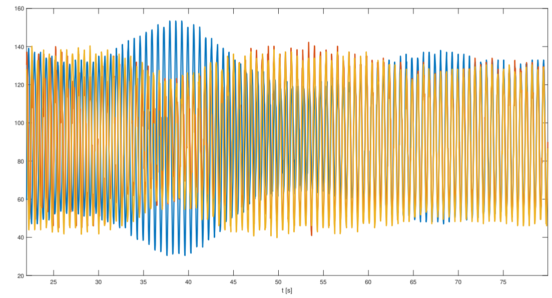
where θ_i, ω_i denote the phase and angular velocity of the i-th metronome, respectively. It is worth noting that θ_i is not a physical angle such as ϕ_i, but instead it describes the phase of the oscillation of the bob. It has been displayed that, through the application of averaging techniques [11], the behavior of the coupled metronome system can be in fact examined by employing a Kuramoto model. The coupling parameter K depends on the mechanical properties of the system and will alter as the mass of the platform increases.

IV. Experimental Results

In the beginning, we carried out a simple trial that displayed a three stage synchronization/desynchronization process and was employed to test and debug the developed code for tracking the trajectories of the metronomes. Before being placed metronomes on the platform, they were wound by hand. In the sequel, the platform with the oscillating metronomes was placed on the soda cans, effectively activating the coupling forces between them. In this stage of the run, the platform began oscillating while perturbing the phases of the metronomes, thereby forcing them to converge to a phase-locked state. In the third stage of the execution the platform was moved off the soda cans. We observed that after removing the coupling, it led to the desynchronization of the metronomes. Albeit the ideal dynamics would not predict desynchronization to occur, both the movement of the platform and the disturbances because of air disturbances introduced forcing that brought about this phenomenon to happen. From Fig.2a, one can observe that after the coupling among the metronomes is activated, there is a transient period during which the metronomes are not synchronized. After an amount of time has passed, the metronomes get

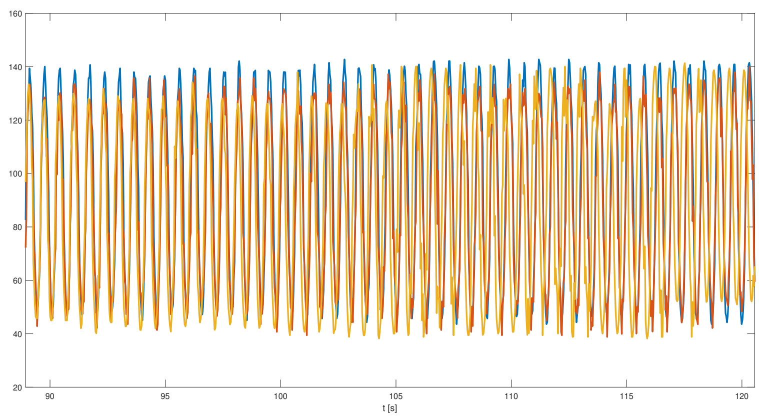


(a) Metronome trajectories before the coupling is activated.



(b) Metronome trajectories after the coupling occurs: transience and steady-state synchronization.

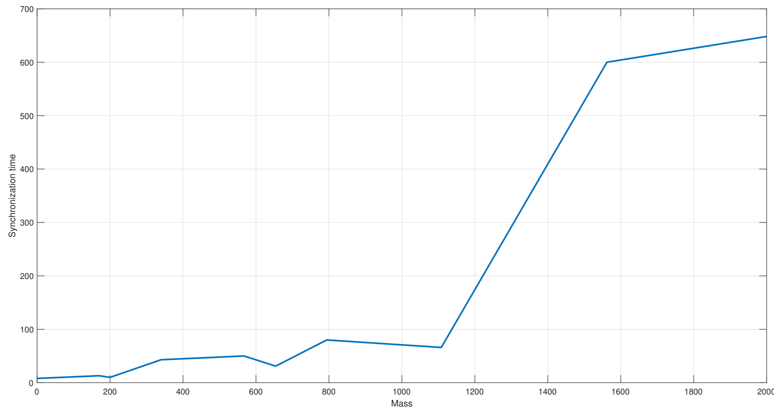
Fig.2: Synchronization of metronomes.



Metronome trajectories after the decoupling happens, and subsequent desynchronization.

synchronized and continue to be so. This happens since it has been demonstrated that for a system of coupled Kuramoto oscillators, the state where the phase difference is an even multiple of 2π is asymptotically stable. On the other hand, in Fig.3, the metronomes are removed from the platform. Looking at the Kuramoto oscillator, in the ideal case, it should hold that $\dot{\theta}_i = \omega_i$, entailing that the phases should remain synchronized (because the metronomes have the same eigenfrequency). Nevertheless, because of disturbances arising from the movement of the platform, the metronomes desynchronize in the long run.

Next, we carried out several experimental runs that were designed to demonstrate the variation in synchronization speed as the mass of the platform increased. The most challenging part of this problem was decoupling the several parameters in each problem. Ceasing the metronomes, changing the mass and then re-executing the experiment would imply that we would have no control over the initial conditions of the system, in particular during the placement of the platform on top of the cans. Rather, the variation of the mass was made once the metronomes had synchronized. Hence, on the fully synchronized system, we putted a single weight on the platform, resulting in changing the coupling strength among the metronomes. In the sequel, to examine the synchronization time of the system systematically, we putted an obstacle on the path of one of the metronomes, in particular when $\phi_i = 90$, for a specific time period. Because the frequency of the metronomes does not change, the phase difference is made during this time period will be identical in all runs of the system. The metronomes were then capable of achieving synchronization again, and the time of synchronization was recorded. The data collected are displayed in the following table.



Time to accomplish synchronization in terms of the platform mass.

Synchronization time with different platform mass			
Foam platform		Tin platform	
Mass (kg)	Time (sec)	Mass (kg)	Time (sec)
~ 0	8	0.200	10
0.170	13	0.654	31
0.340	43	1.108	66
0.567	50	1.562	600
0.794	80	2.016	∞

It is of interest to observe the resulting behavior owing to the increase of platform mass, specifically, we shall figure out that the synchronization rate decreases. Looking at equation (2), we utilize the proposed results of [20], wherein the following Theorem is stated:

Theorem: The Kuramoto model given by (2) with $K > 0$, will synchronize to phase differences of an even multiple of 2π. The rate of approach to synchronization is no worse that $(2K/\pi N)\lambda_2(NI - \mathbf{1}_N\mathbf{1}_N^T/N)$, where I is the identity matrix of appropriate dimension, $\mathbf{1}_N$ a vector of ones and $\lambda_2(A)$ is the Fiedler eigenvalue of a matrix A.

Therefore, the fact that the synchronization rate and the coupling parameter K are linearly dependent entails that the growth in the mass has reduced K. To ascertain the explicit dependence of K to M, we notice that, owing to [11], the coupling parameter is given by

$$K = \sqrt{\left(3\frac{\beta}{\mu}\right)^2 + \left(\frac{\beta}{\gamma}\right)^2}\left(\frac{\gamma}{2N}\right),$$

where the only term connected with the platform mass is $\beta = \left(\frac{m^2 r^2}{(M+2m)I}\right)$. Consequently, growing the platform mass, β is reduced and, in a more general sense, decreases the coupling strength K. In Fig. 4 is depicted the change of synchronization time as the mass varies.

V. Conclusion

In this project, we investigated the behavior of a system of three metronomes, mimicking the first executed experiments by Huygens, and carried out experiments to determine the connection between the mass of the moving platform and the synchronization time. First, we developed video-processing software allowing us to determine the time evolution of the angle of the metronomes by utilizing visual measurements. In particular, the code disintegrates the video into several partitions at each frame, depending on the position of the platform, and locally looks for color-coded pointers to determine the position of the metronome bob. The angle is determined trivially as a consequence. After noticing the importance of coupling strength via the platform's behavior, we deemed a manner that would enable us to measure the synchronization time from nearly identical initial conditions as the platform mass was slowly increased. The theoretical relationship between platform mass and synchronization time matches from a qualitative standpoint with our outcomes, but a more quantitative analysis would need a deliberate consideration and calculation of the several parameters that have an effect on the synchronization time. This research line would experimentally support the association between the abstract Kuramoto oscillator model and the Euler-Lagrange based second-order mechanical model of metronome synchronization.

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