The Time to Synchronization for N Coupled Metronomes

Vlad Levenfeld, Luis Jover, Brad Taylor, Jeff Tithof

Nonlinear Dynamics

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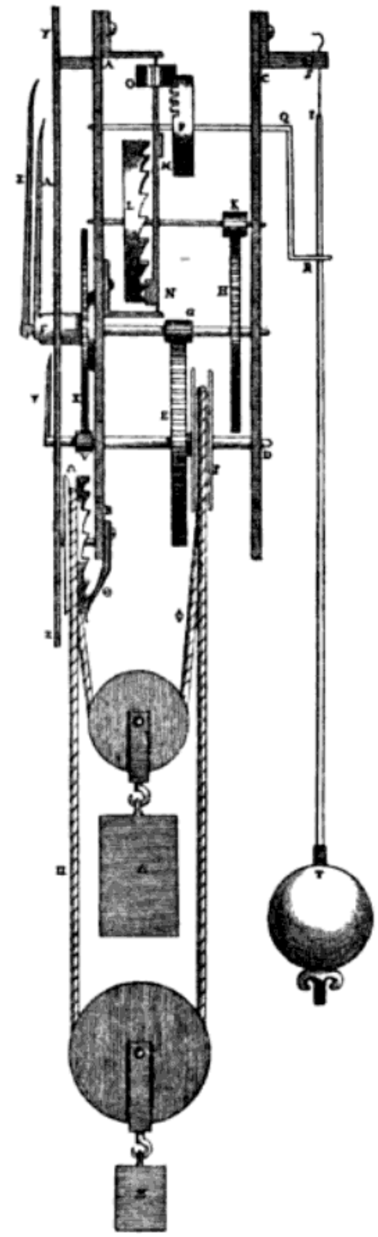
Preview

- Introduction/History
- Experimental
  - Setup
  - Observations
  - Defining Synchronization
  - Results
- Simulation
  - Theory
  - Parameter Estimates
  - Results and Comparison
- Interesting Experimental States
- Future Work

Early Motivations

- Longitude problem
- Christiaan Huygens' pendulum clock
- “An odd sympathy”

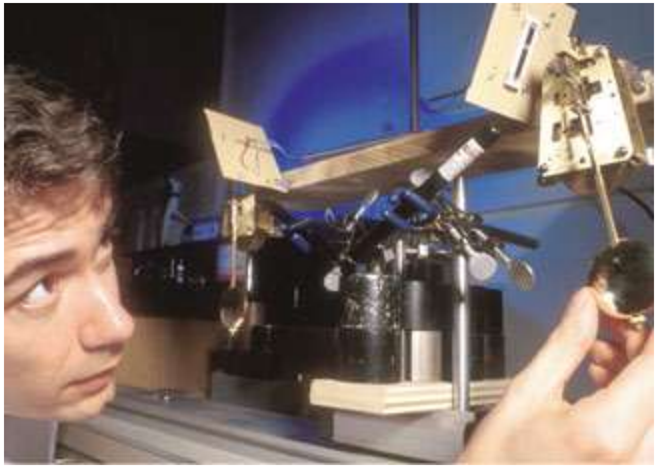


Synchronization in Nature

- “A deep tendency toward order in nature”
- Swarming Behaviors
- Millennium Bridge

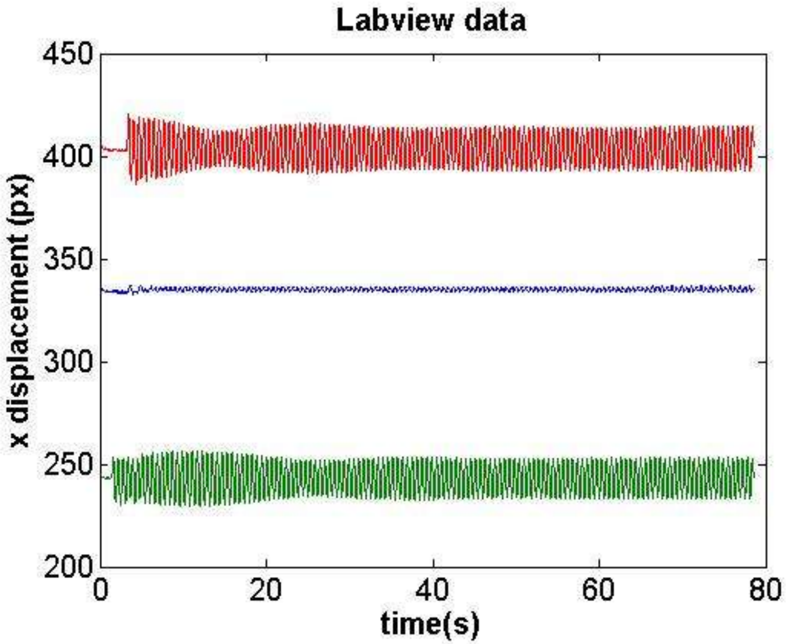
Huygens Revisited

- Korteweg, Blekhman
- Bennett, Schatz, Rockwood, Wiesenfeld
- Common platform, Mass ratio



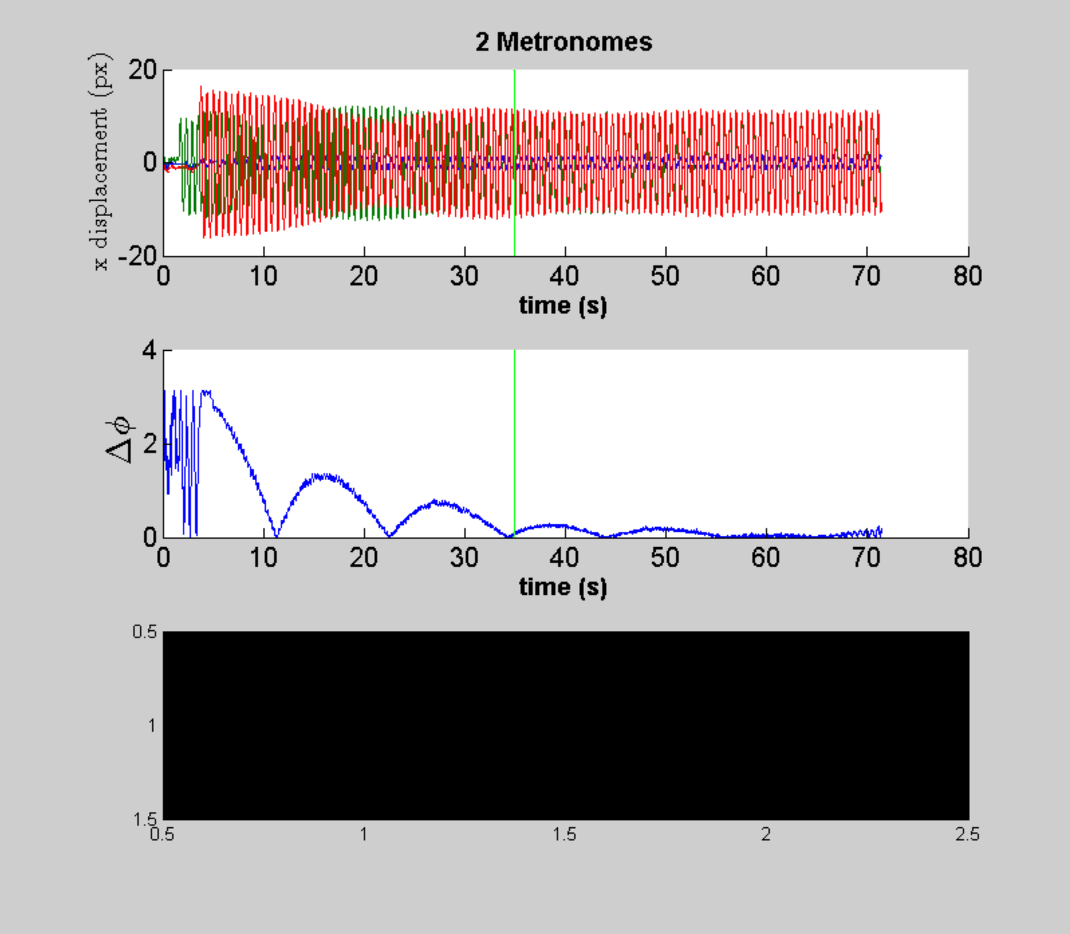
Experimental Setup, N=2

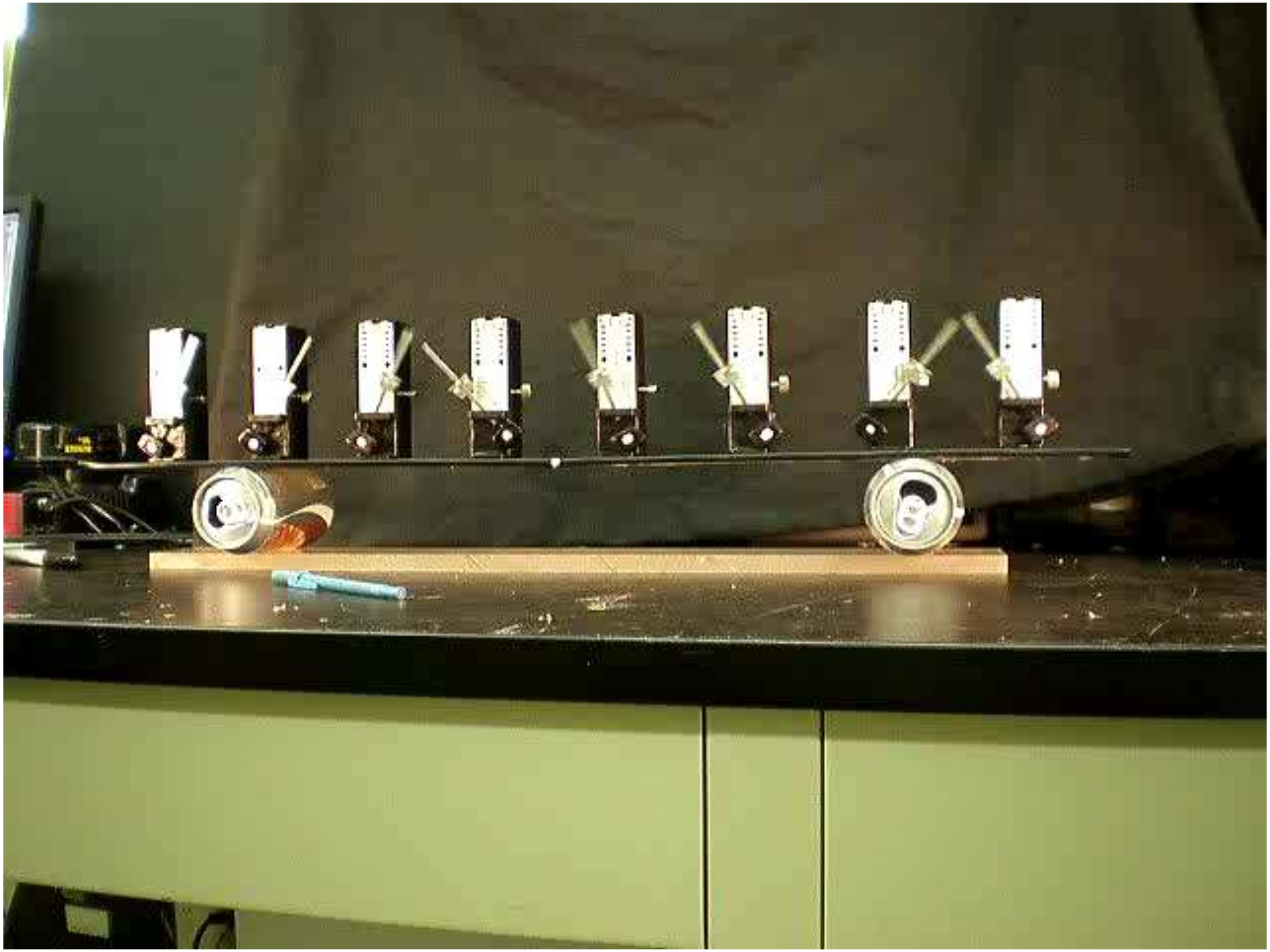
2 metronomes

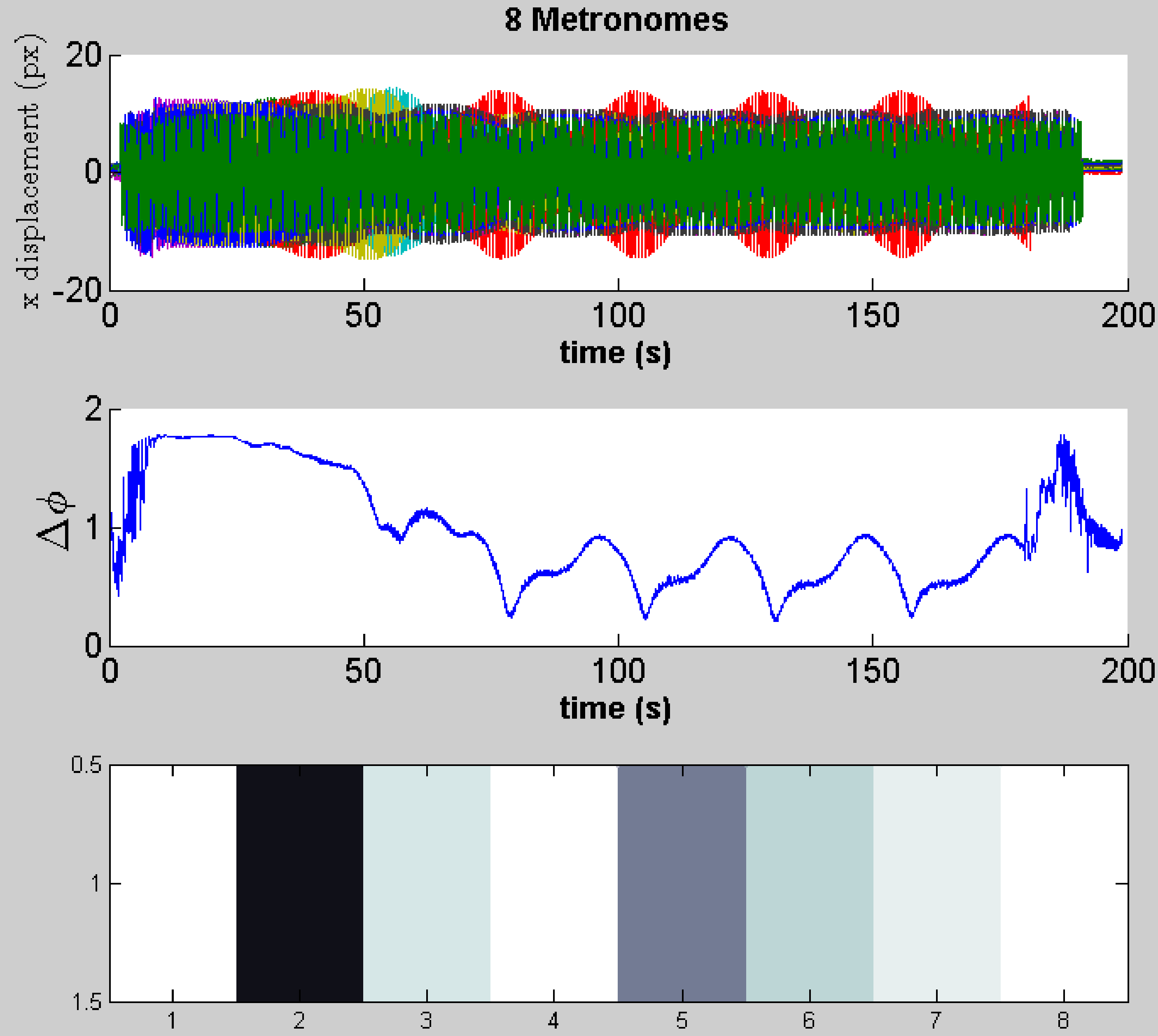


$$\mathbf{H}[y(t)] = \tilde{y}(t) = \frac{1}{\pi t} * y(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{y(\tau)}{t-\tau} d\tau$$

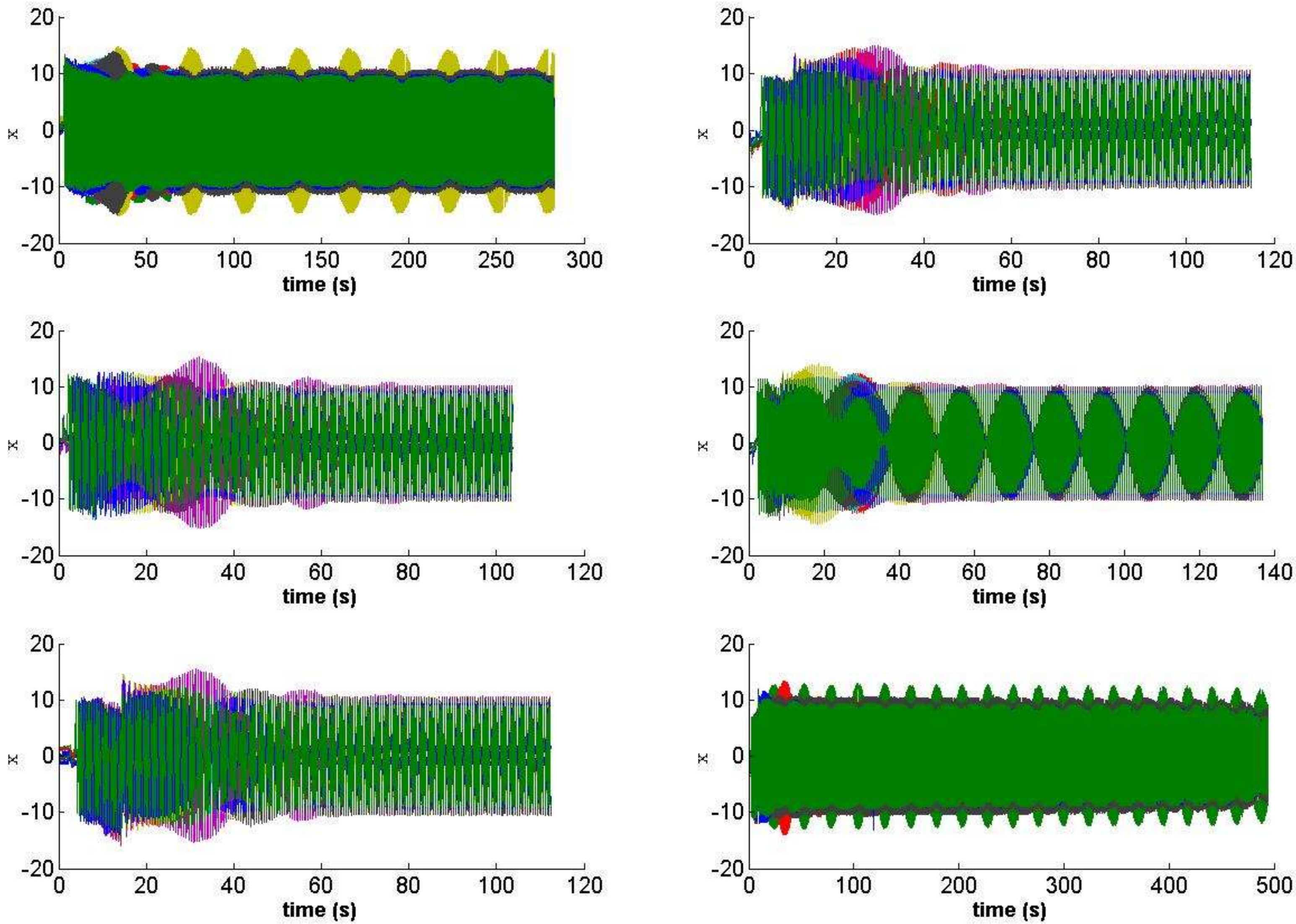
$$Y(t) = y(t) + j\tilde{y}(t) = A(t)\exp(j\psi(t))$$



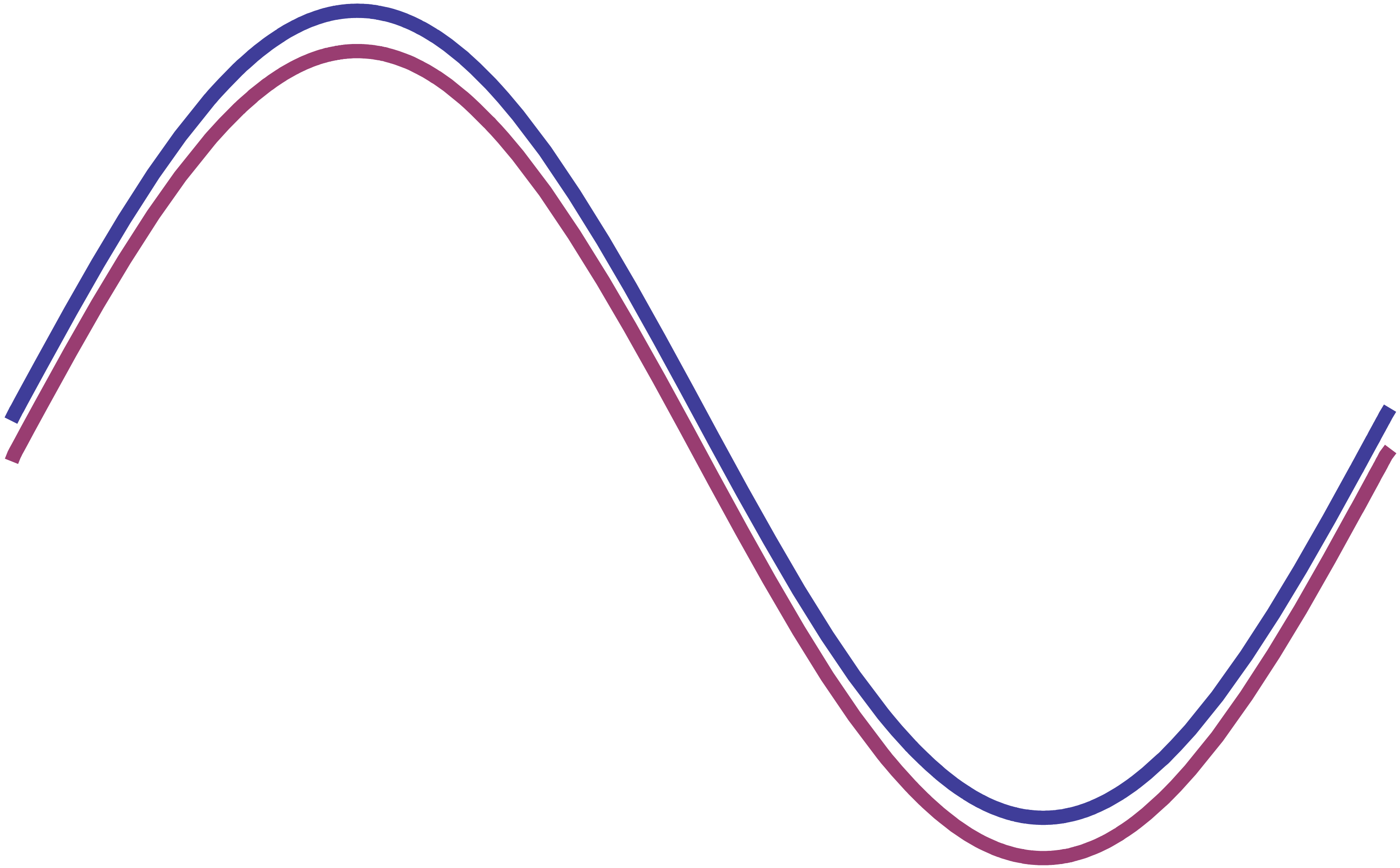



8 Metronomes
x displacement (px)
20
0
-20
0
50
100
150
200
time (s)
$\Delta\phi$
2
1
0
0
50
100
150
200
time (s)
0.5
1
1.5
1
2
3
4
5
6
7
8

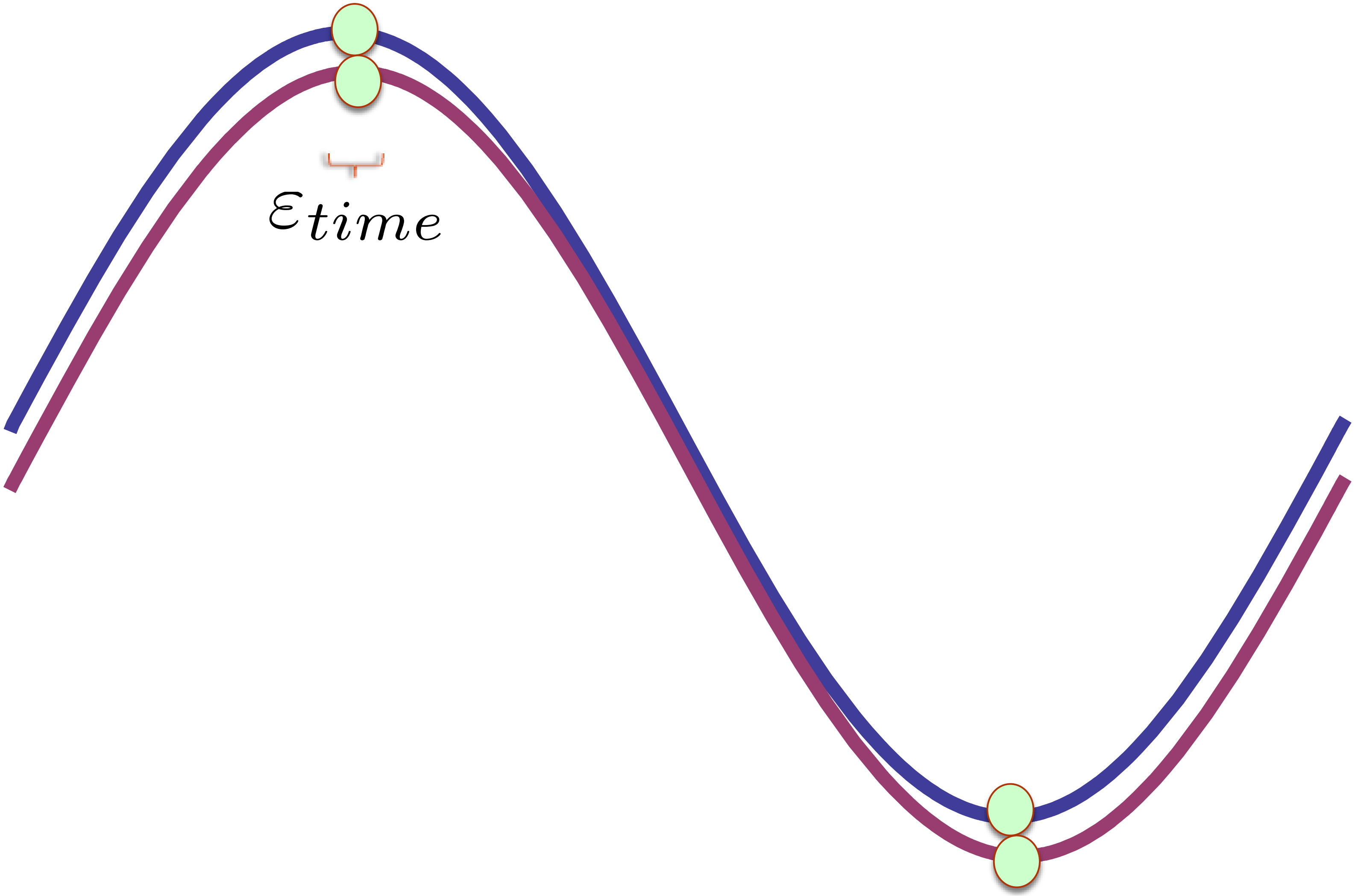
More runs: 8 metronomes

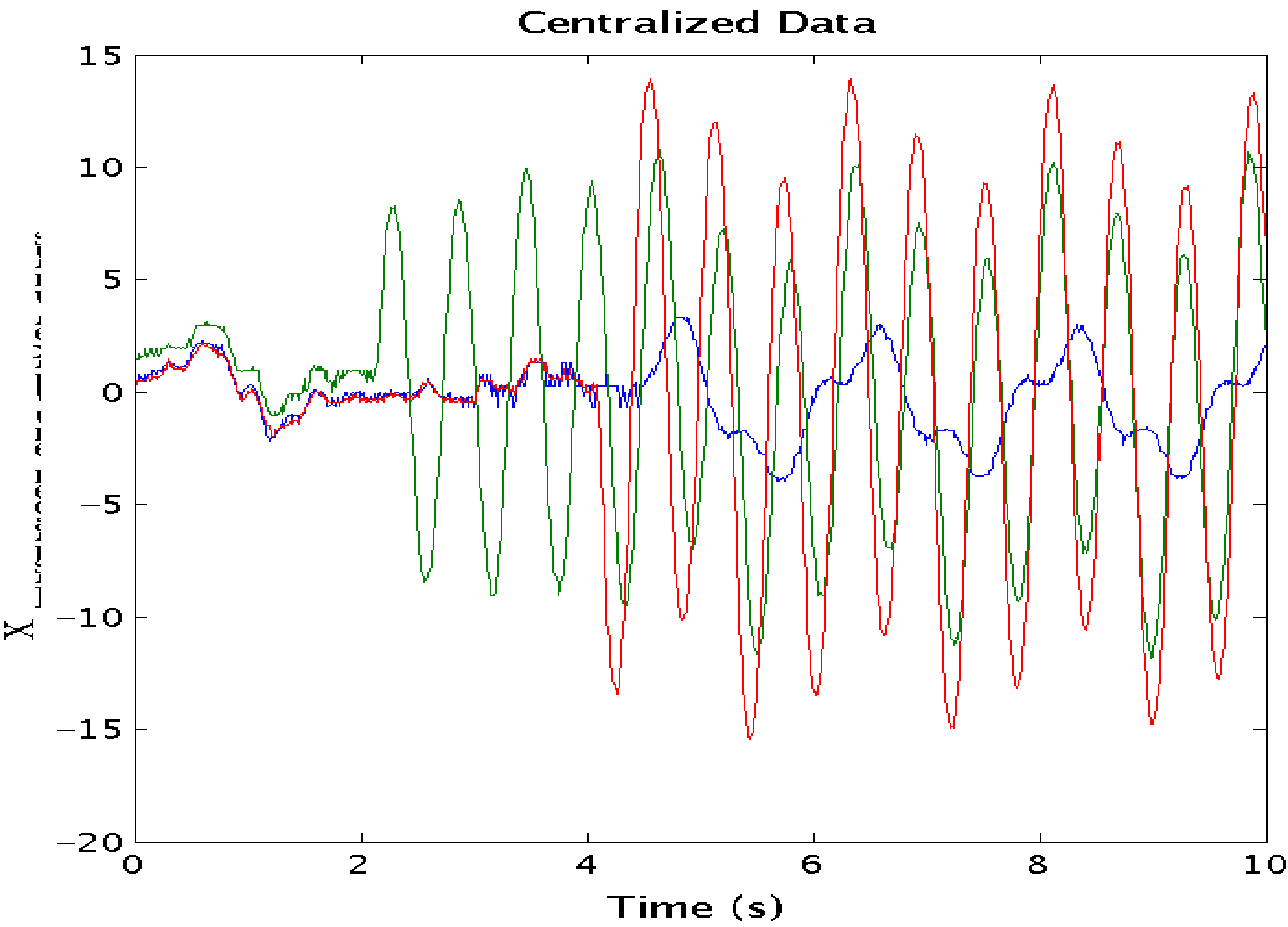


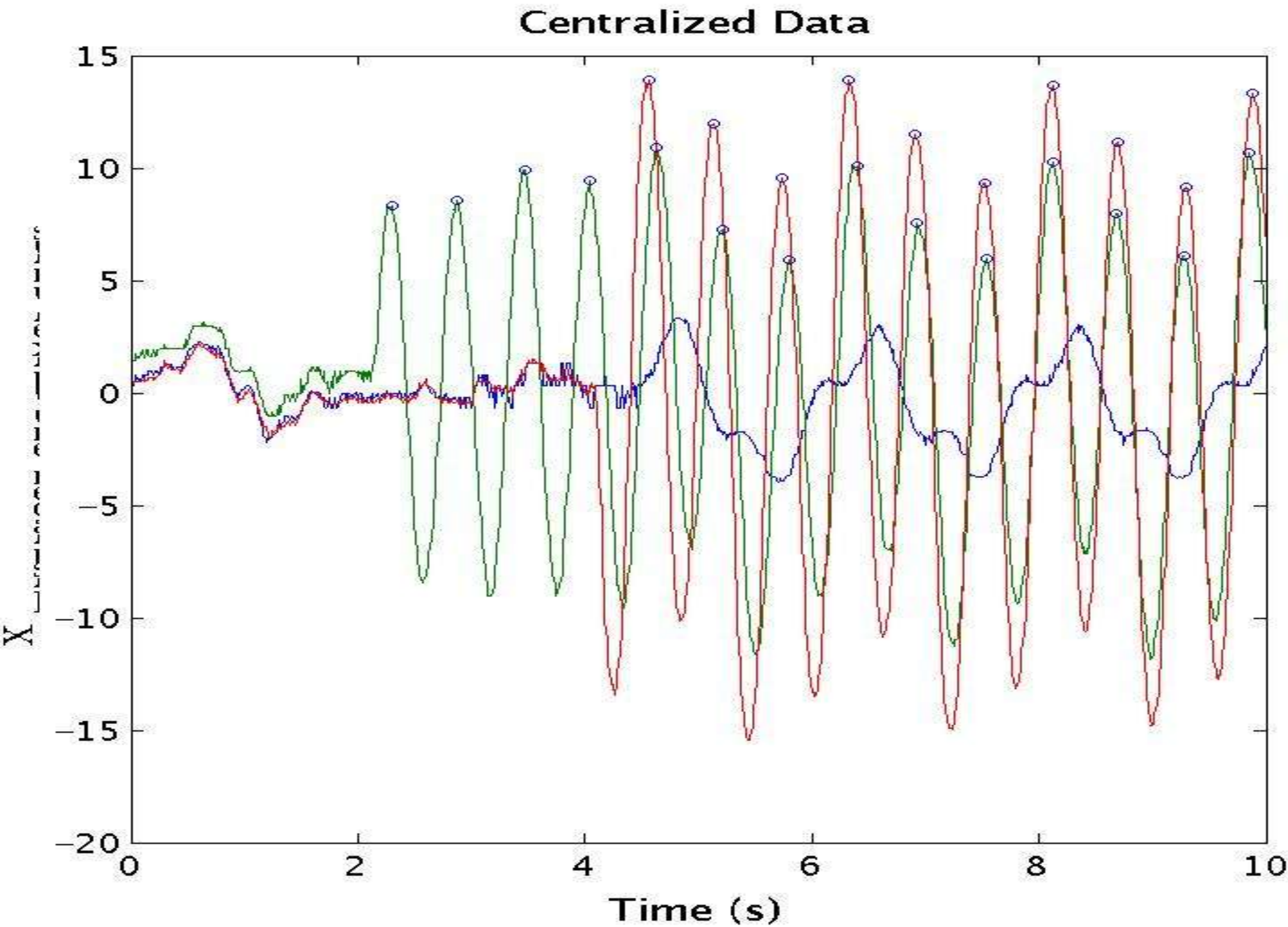
Synchronization in a perfect world…

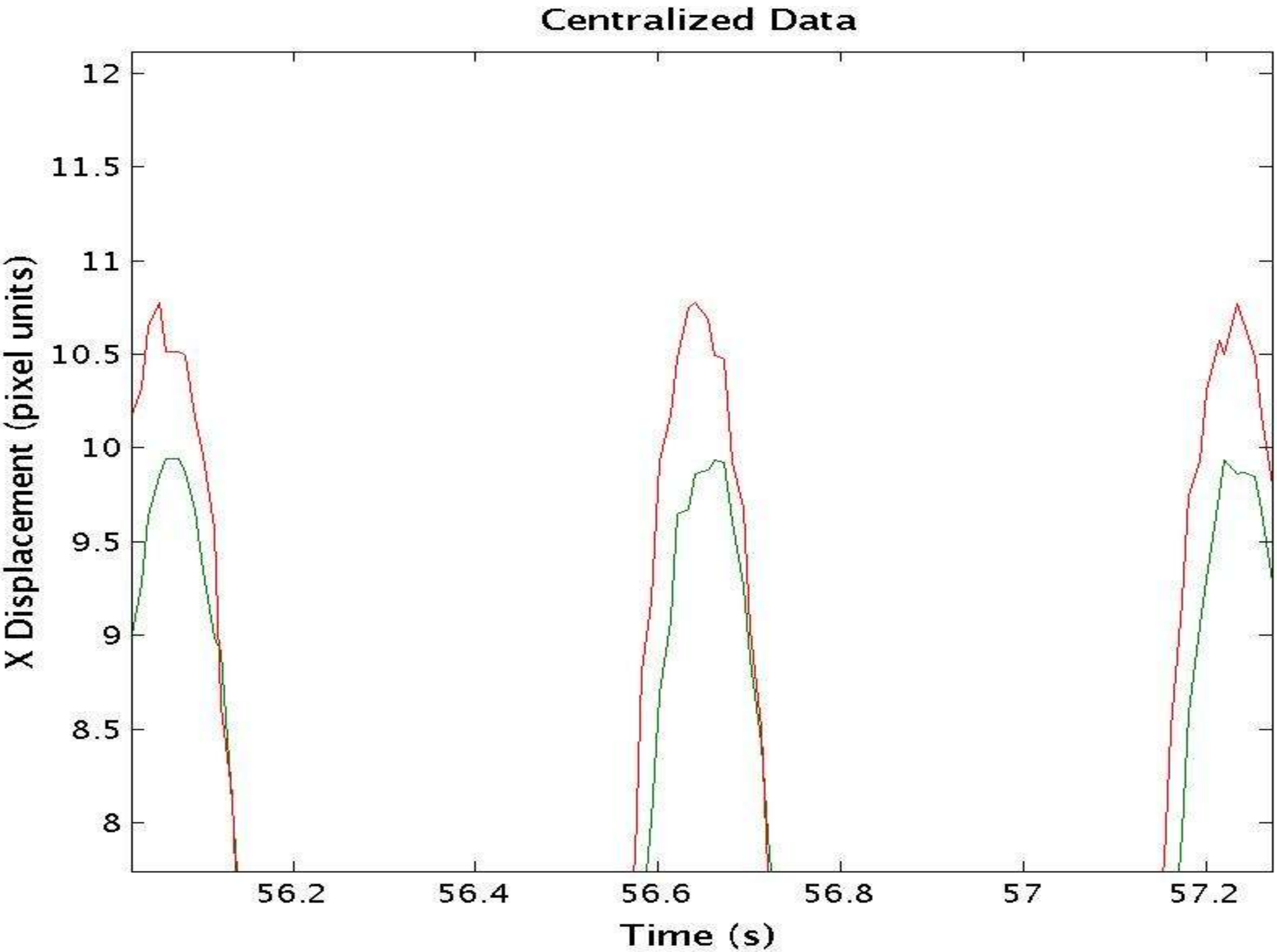


Synchronization in a perfect world…

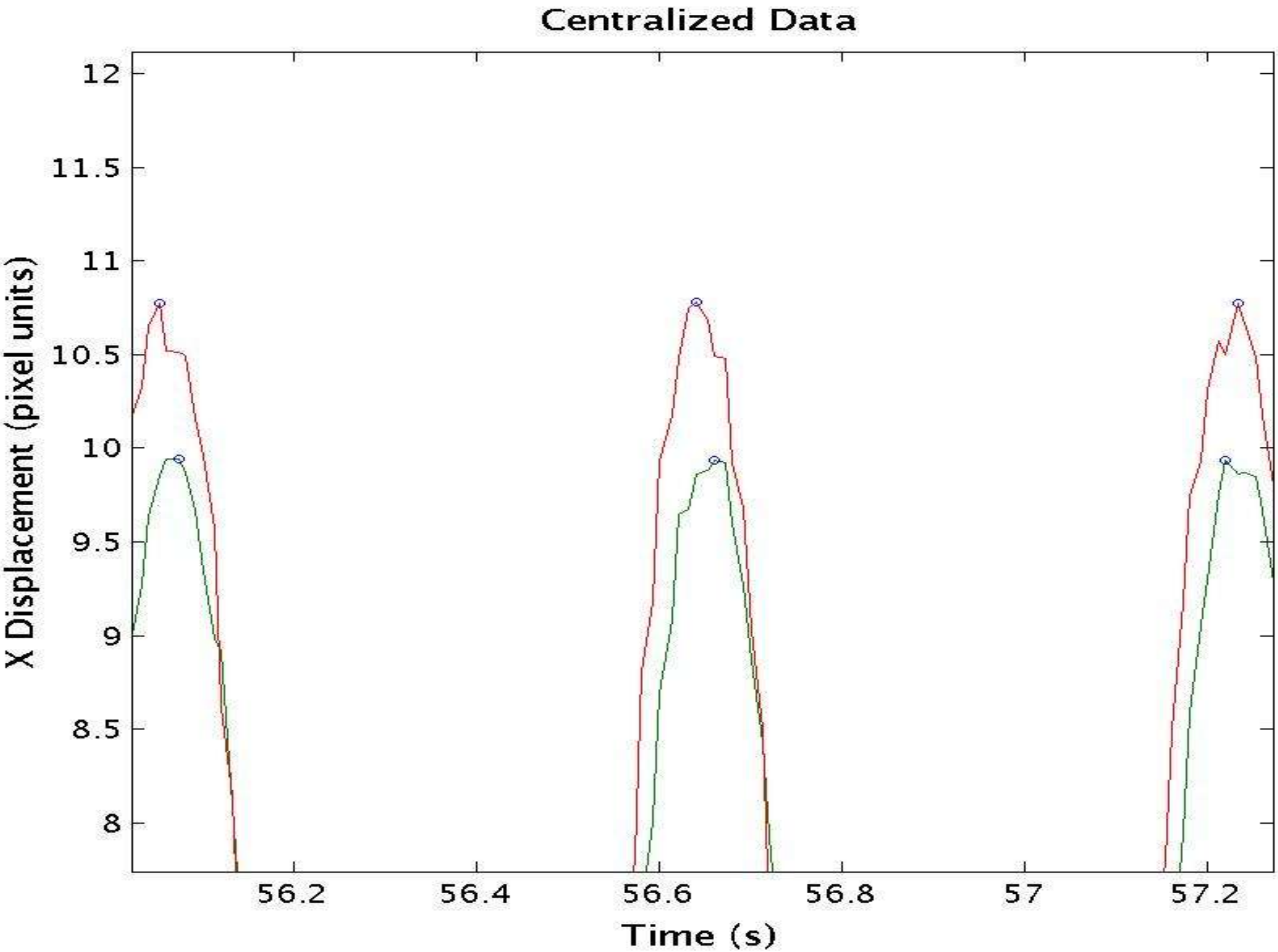



Centralized Data
15
10
5
0
−5
−10
−15
−20
0
2
4
6
8
10
Time (s)
X

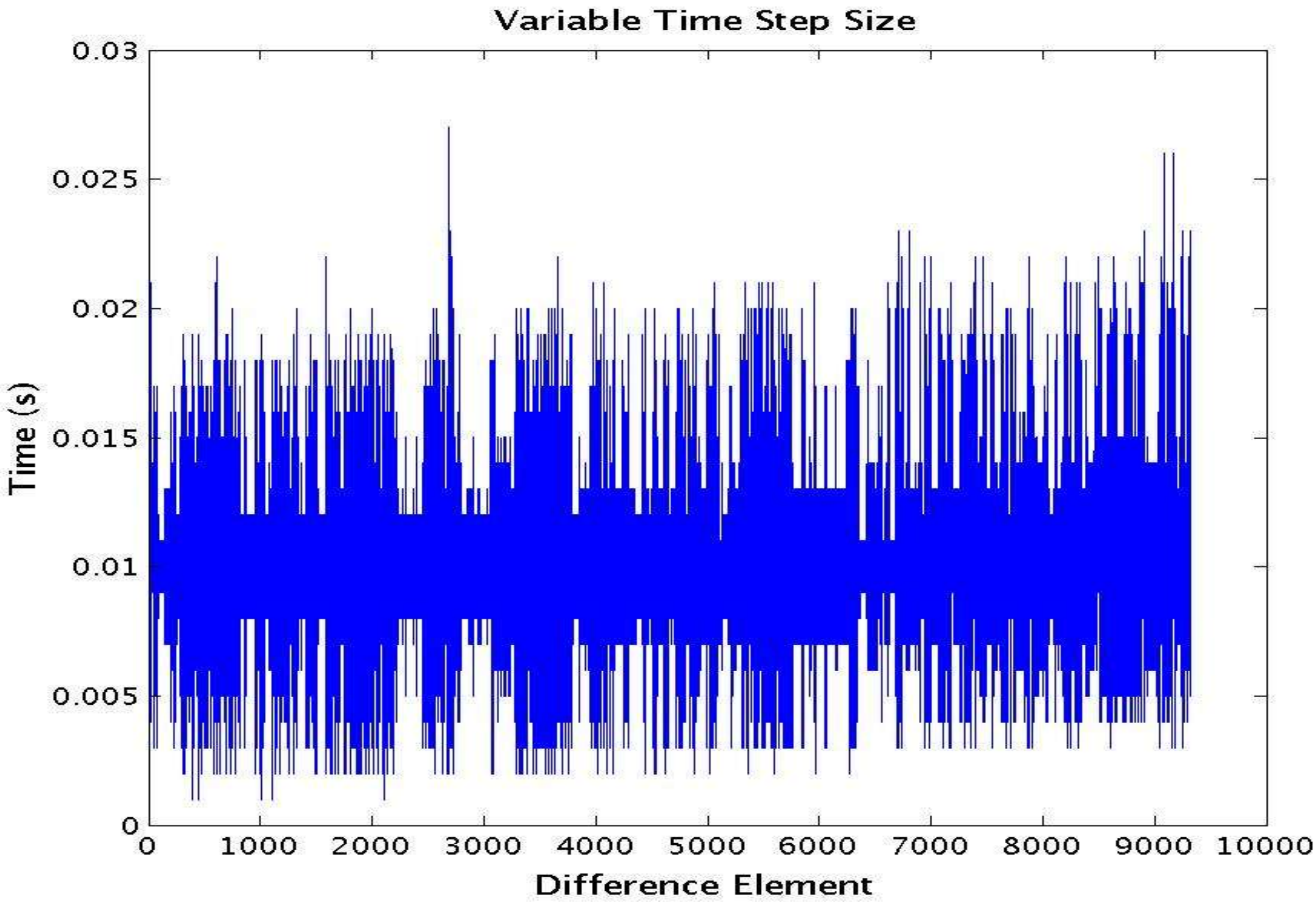

Centralized Data
15
10
5
0
−5
−10
−15
−20
0
2
4
6
8
10
Time (s)



Centralized Data
X Displacement (pixel units)
12
11.5
11
10.5
10
9.5
9
8.5
8
56.2
56.4
56.6
56.8
57
57.2
Time (s)


Centralized Data
X Displacement (pixel units)
Time (s)
12
11.5
11
10.5
10
9.5
9
8.5
8
56.2
56.4
56.6
56.8
57
57.2

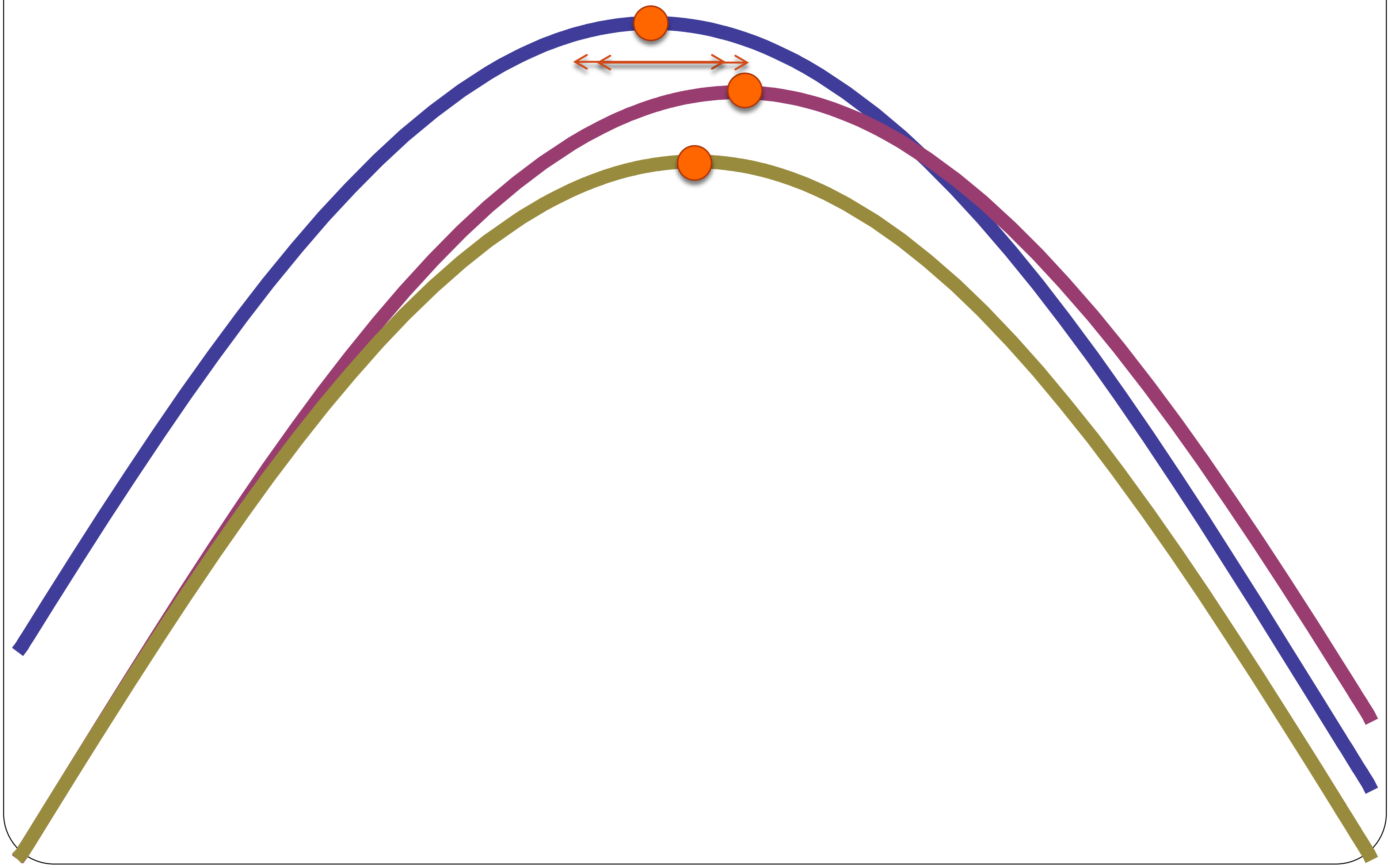
Constraining ε



Synchronization Algorithms

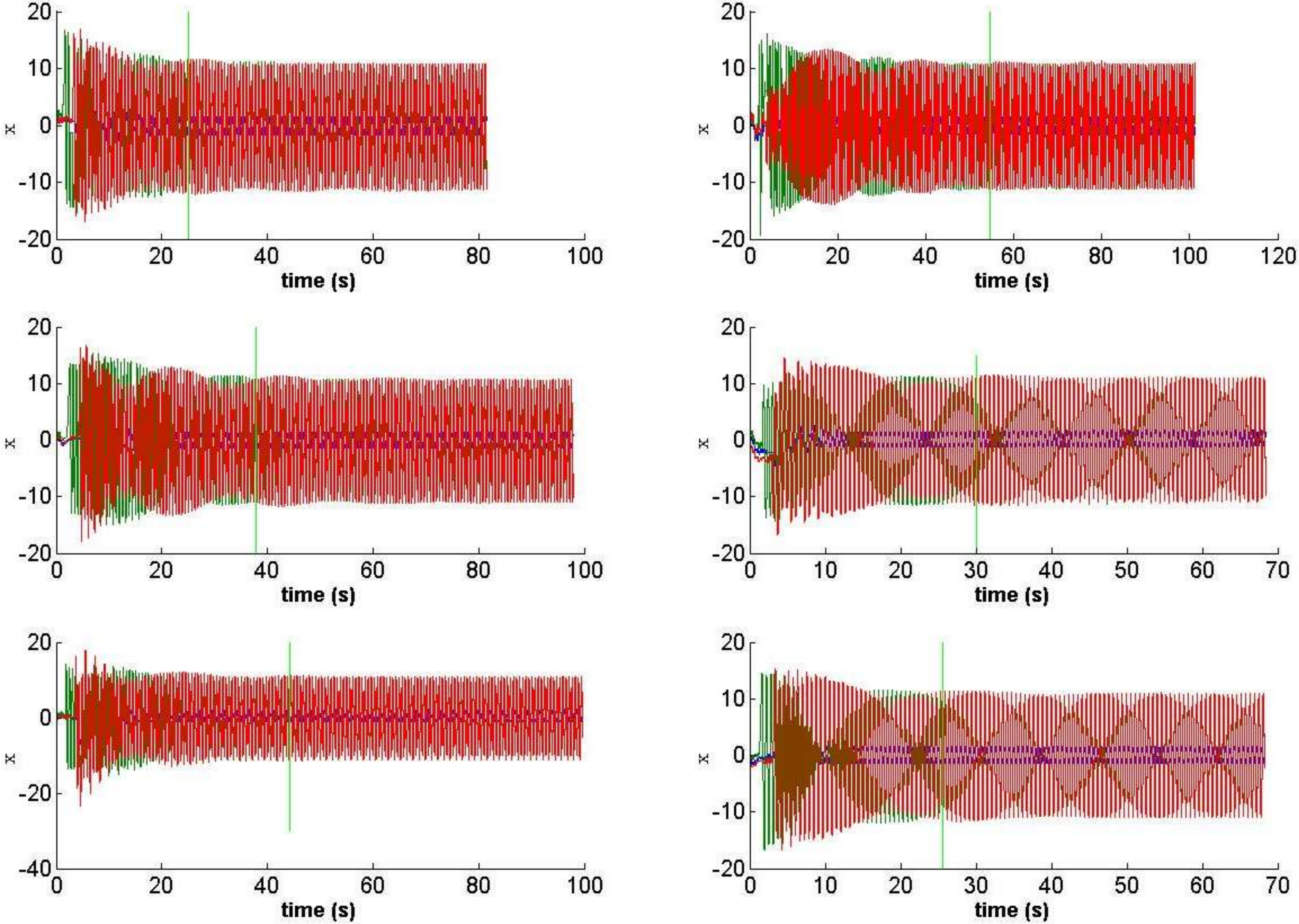
- Computationally feasible
- One Method:
  - Trim Data & Time
  - Determine peak times
  - Choose a metronome (arbitrarily)
  - For each peak find all peaks for other metronomes within ε_{time}-ball
  - Take first time that all metronome peak times are within the ε_{time}-ball for so many peaks in a row.
- Results may depend on choice of metronome. Need a less arbitrary method.

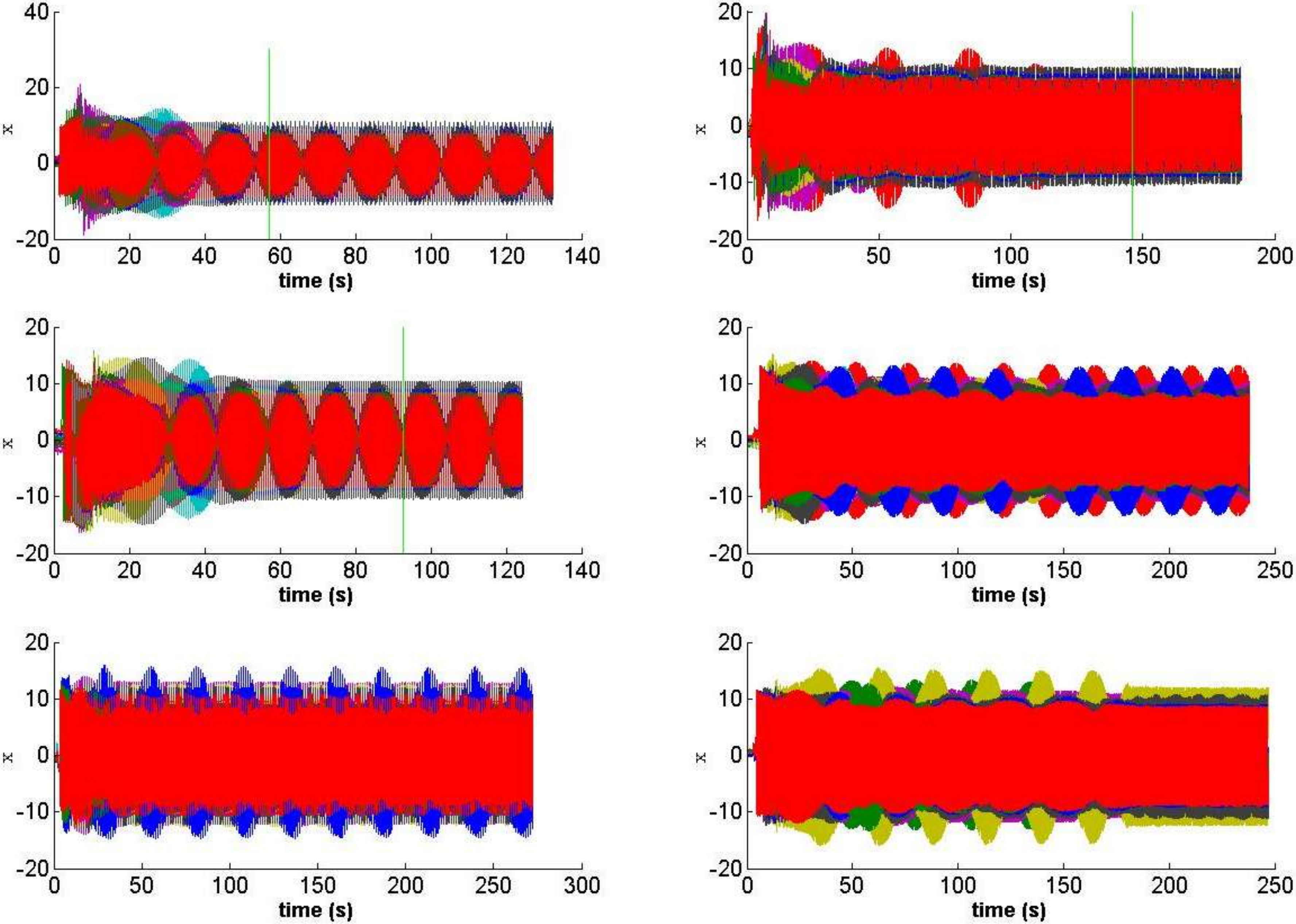
Final Algorithm



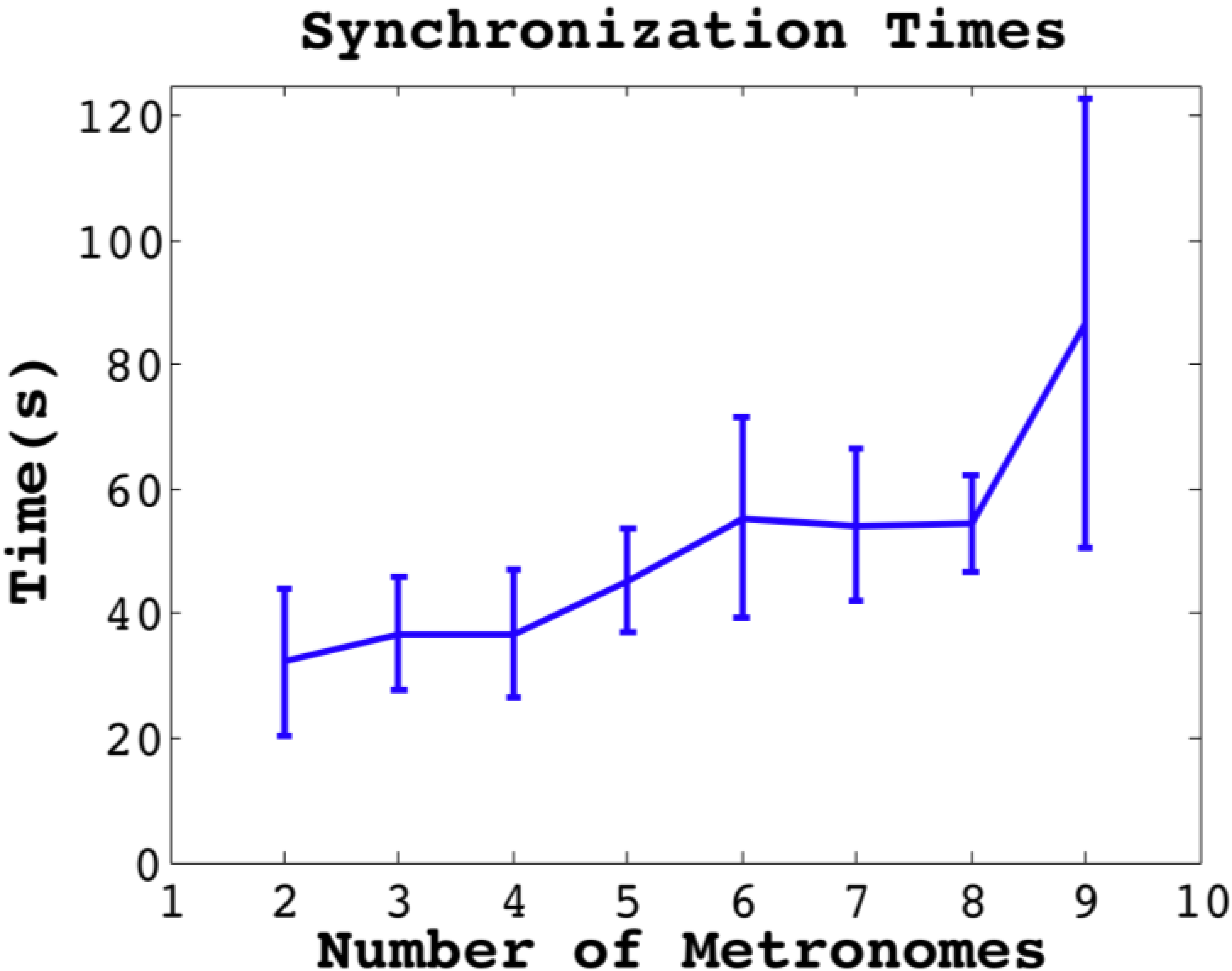
Benefits

- Robust to smaller ε
- Less Arbitrary
- More consistent, not more lenient

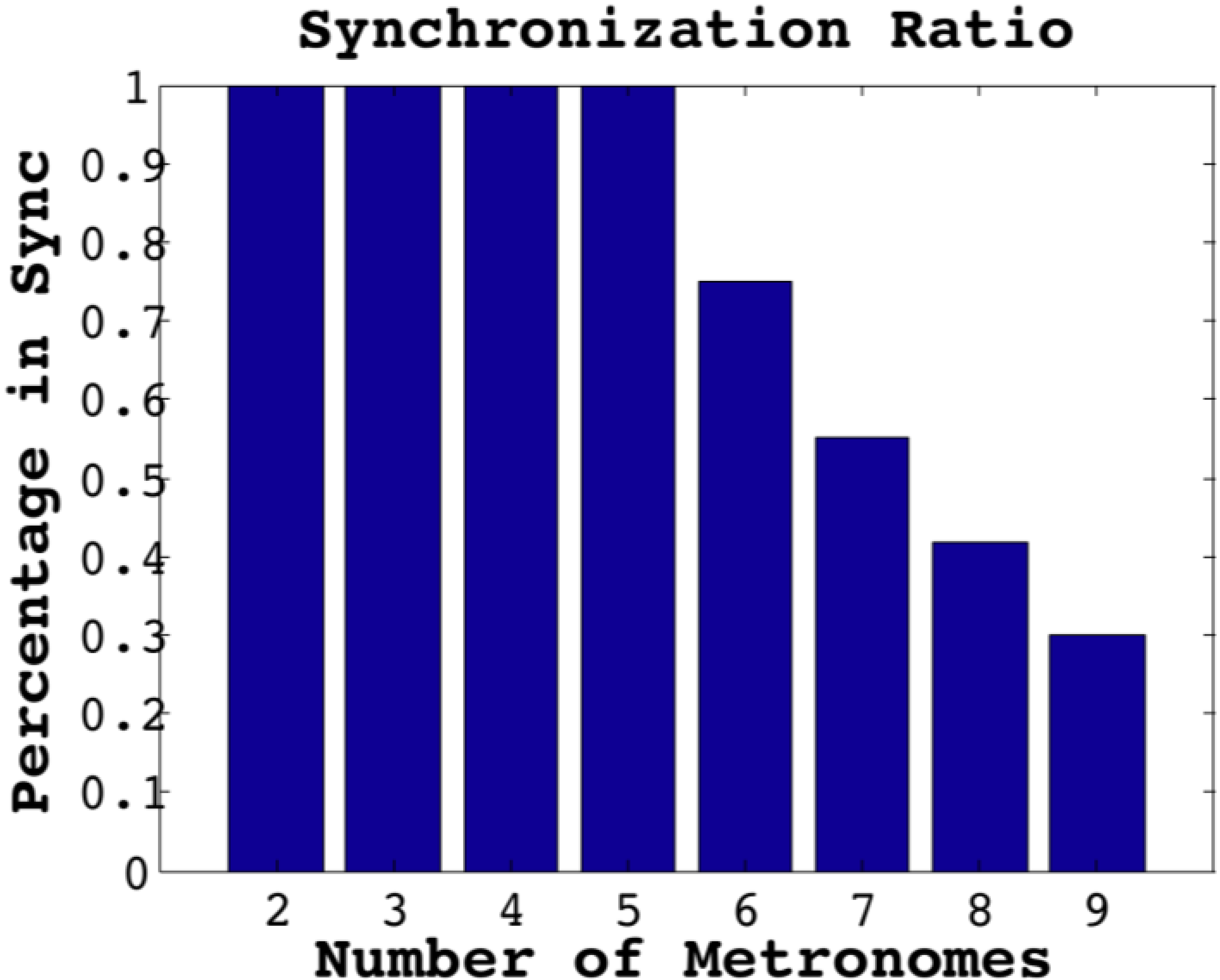

20
10
0
-10
-20
x
0
20
40
60
80
100
120
time (s)
20
10
0
-10
-20
x
0
10
20
30
40
50
60
70
time (s)
20
0
-20
-40
x
0
20
40
60
80
100
time (s)


x
time (s)
x
time (s)
x
time (s)
x
time (s)
x
time (s)
x
time (s)

Final Statistical Analyses



Final Statistical Analyses



Equations of Motion

$$\frac{d^2\phi_j}{dt^2} + b\frac{d\phi_j}{dt} + \frac{g}{l}\sin\phi_j = -\frac{1}{l}\frac{d^2X}{dt^2}\cos\phi_j + F_j$$

$$(M + Nm)\frac{d^2X}{dt^2} + B\frac{dX}{dt} = -ml\frac{d^2}{dt^2}(\sin\phi_1 + \ldots + \sin\phi_N)$$

- φ_j – angular displacement of jth pendulum
- b – pivot damping coefficient
- g – gravitational acceleration
- l – pendulum length
- X – linear displacement of platform
- F_j – impulsive drive
- M – platform mass
- m – pendulum bob mass
- N – number of metronomes
- B – platform damping coefficient

Bennett et al. (2002)

Equations of Motion - Nondimensionalized

$$\frac{d^2\phi_j}{d\tau^2} + 2\tilde{\gamma}\frac{d\phi_j}{d\tau} + \sin\phi_j = -\frac{d^2Y}{d\tau^2}\cos\phi_j + \tilde{F}_j$$

$$\frac{d^2Y}{d\tau^2} + 2\Gamma\frac{dY}{d\tau} = -\mu\frac{d^2}{d\tau^2}(\sin\phi_1 + \mathrm{K} \ + \sin\phi_N)$$

where: $\tau = t\sqrt{\frac{g}{l}}$; $\mu = \frac{m}{M+Nm}$; $\tilde{\gamma} = b\sqrt{\frac{l}{4g}}$; $\Gamma = \frac{B}{(M+Nm)}\sqrt{\frac{l}{4g}}$

- φ_j – angular displacement of jth pendulum
- b – pivot damping coefficient
- g – gravitational acceleration
- l – pendulum length
- X – linear displacement of platform
- F_j – impulsive drive
- M – platform mass
- m – pendulum bob mass
- N – number of metronomes
- B – platform damping coefficient

Escapement Mechanism

- Mimick the metronome "kick" in two parts
  - Bennett et al.; Wiesenfeld & Borrero

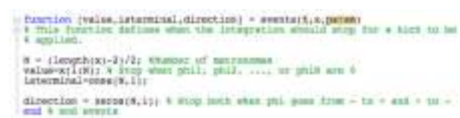
$$\left|\frac{d\phi}{d\tau}\right| \rightarrow \gamma\left|\frac{d\phi}{d\tau}\right| + c$$

Simulation

- Main Function
  - Calls ODE45
  - Calls functions
  - Applies kick
  - Returns time series of all φ and dφ/dτ

- Equations of Motion
  - 2(N+1)
  - Derived in Bennett et al
  - Generalized to N

- Test for Applying Kick
  - Returns index if φcrosses zero



- Parameters
  - List of estimates for parameters

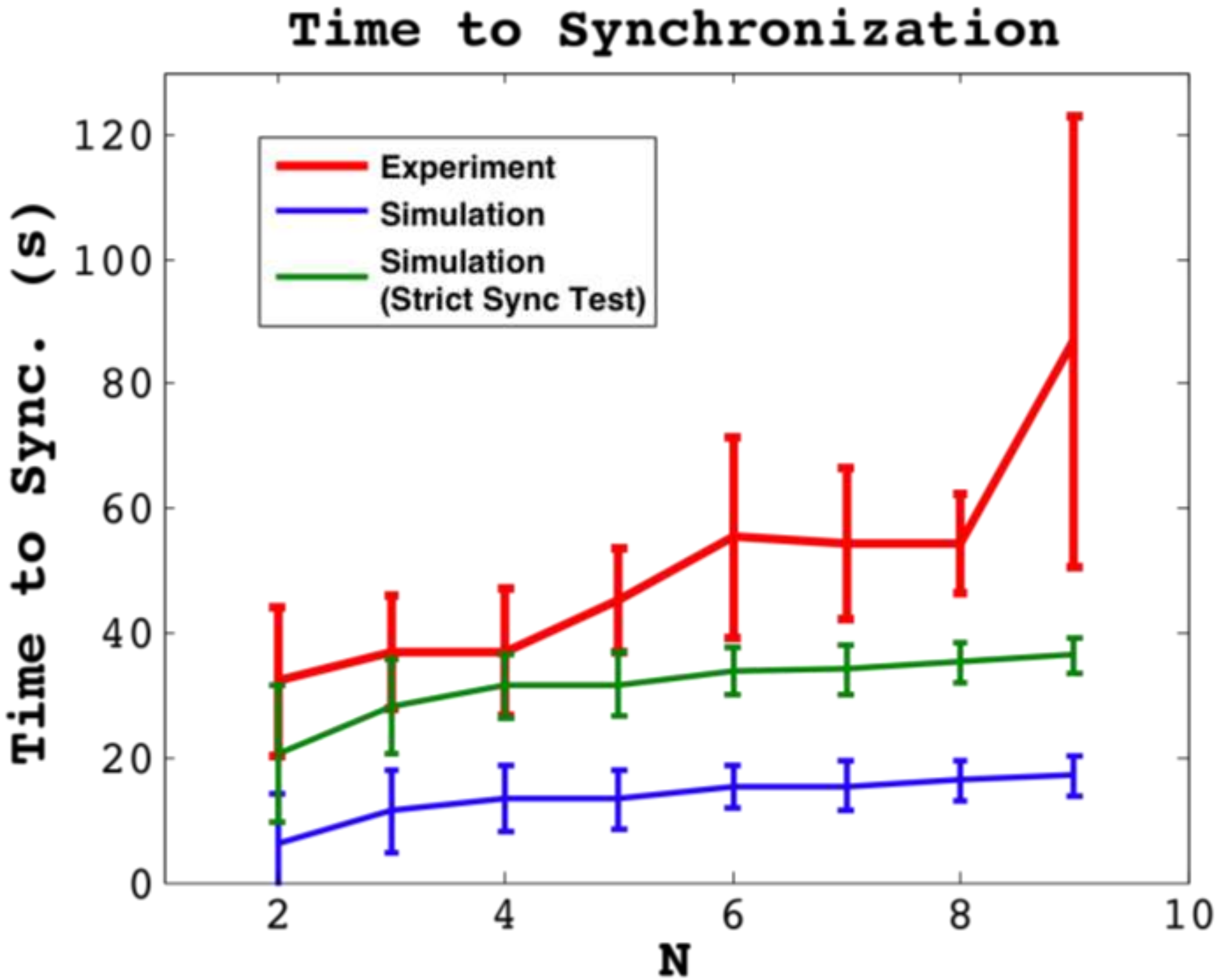
Parameter Estimates

- **Measured / Known**
  - M – platform mass
  - m – pendulum bob mass
  - N – number of metronomes
  - g – gravitational acceleration
- **Estimated / Deduced**
  - b – pivot damping coefficient
  - c – “kick”
  - l – pendulum length
- **Unknown**
  - B – platform damping coefficient
  - γ – factor by which escapement slows velocity

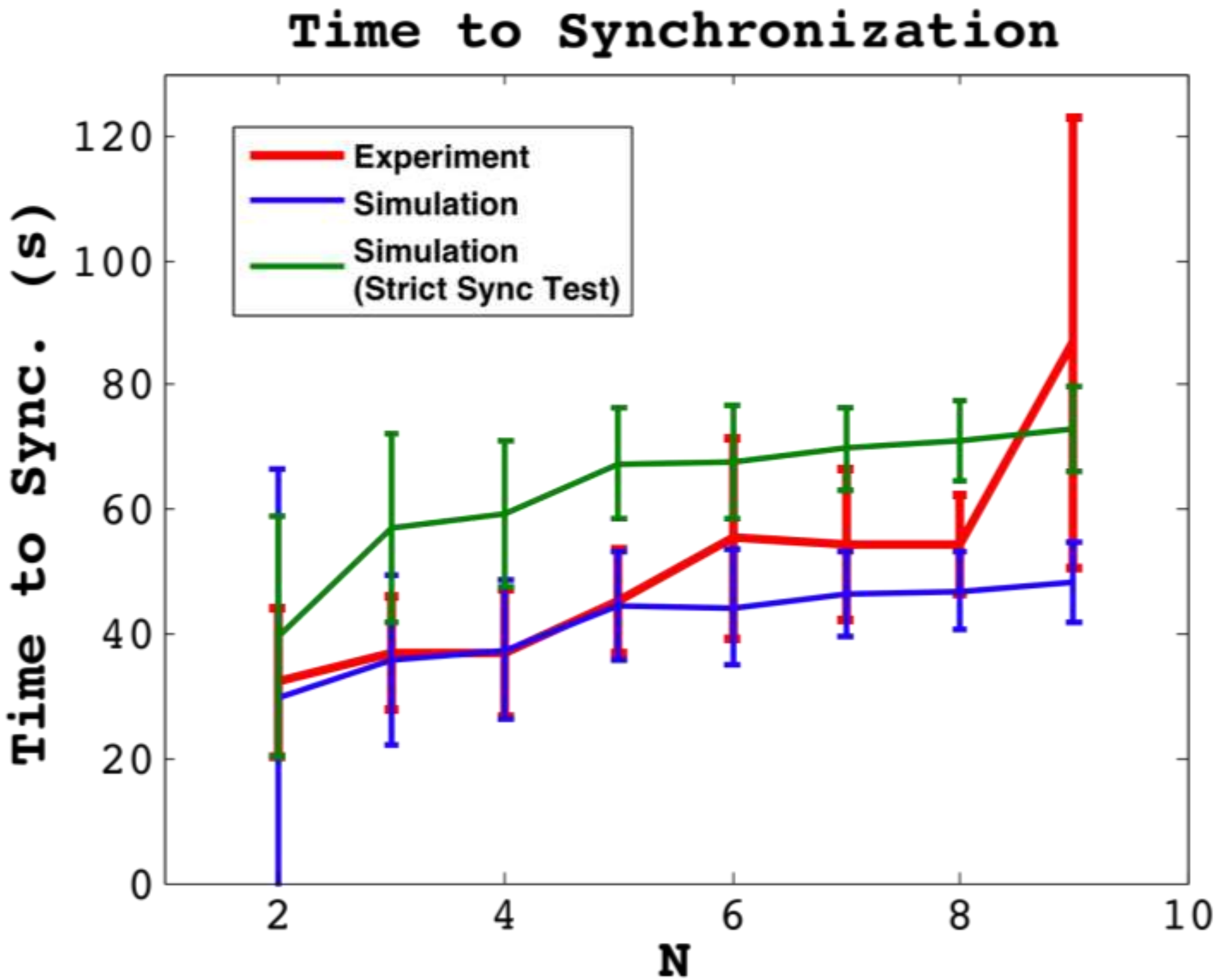
Running the Simulation

- 100 Runs for N=2 to 9
- Store t, φ, φ-dot
- Computation Time: ~6 hours
  - Core 2 Duo, 2.53GHz, 2 GB RAM
- 4.3 GB of Data

“Best Estimate” of Parameters



I Scaled to Match Natural Frequencies



Discrepancy Between Experiment and Simulation

- Simulation always synchronizes for N=3 to 9
  - Did we not wait long enough in the experiment?
- 15% of Simulation N=2 do not synchronize

● In Phase
□ Antiphase
Δ Other

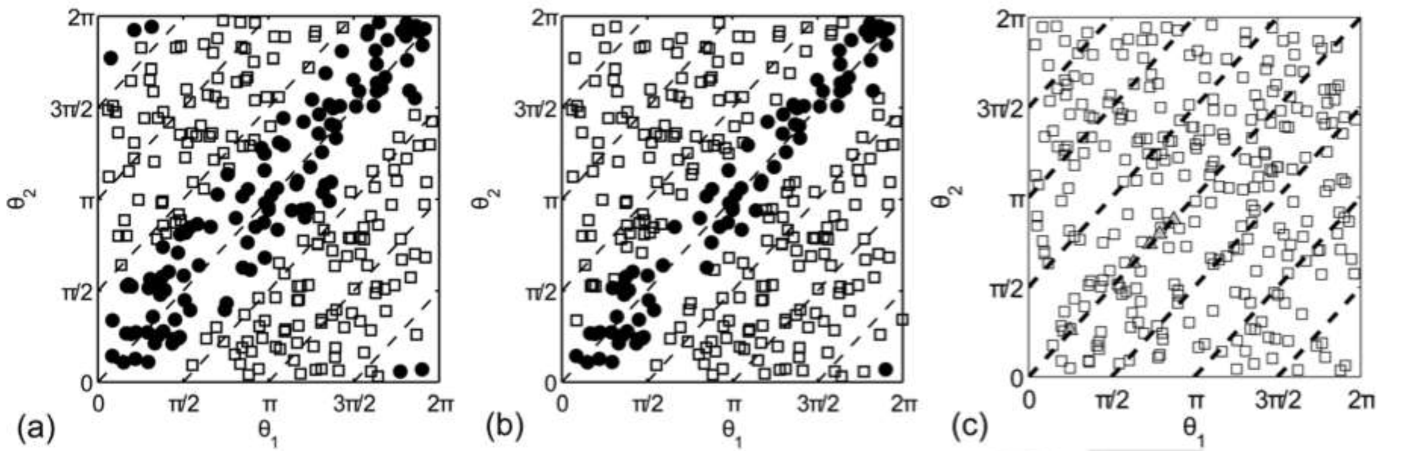


FIG. 6. Results of numerical simulations of Eqs. (13) for different levels of platform damping. For each initial condition, the system evolves into the in-phase state (circles), the antiphase state (squares), or fails to reach either within the allotted integration time (triangles). Parameter values are $\tilde{\gamma} = 1.63 \times 10^{-4}$, $\mu = 7.5 \times 10^{-3}$, $\gamma = 0.97$, $c = 0.0025$, and (a) $\Gamma = 0$, (b) $\Gamma = 0.2$, (c) $\Gamma = 0.5$.

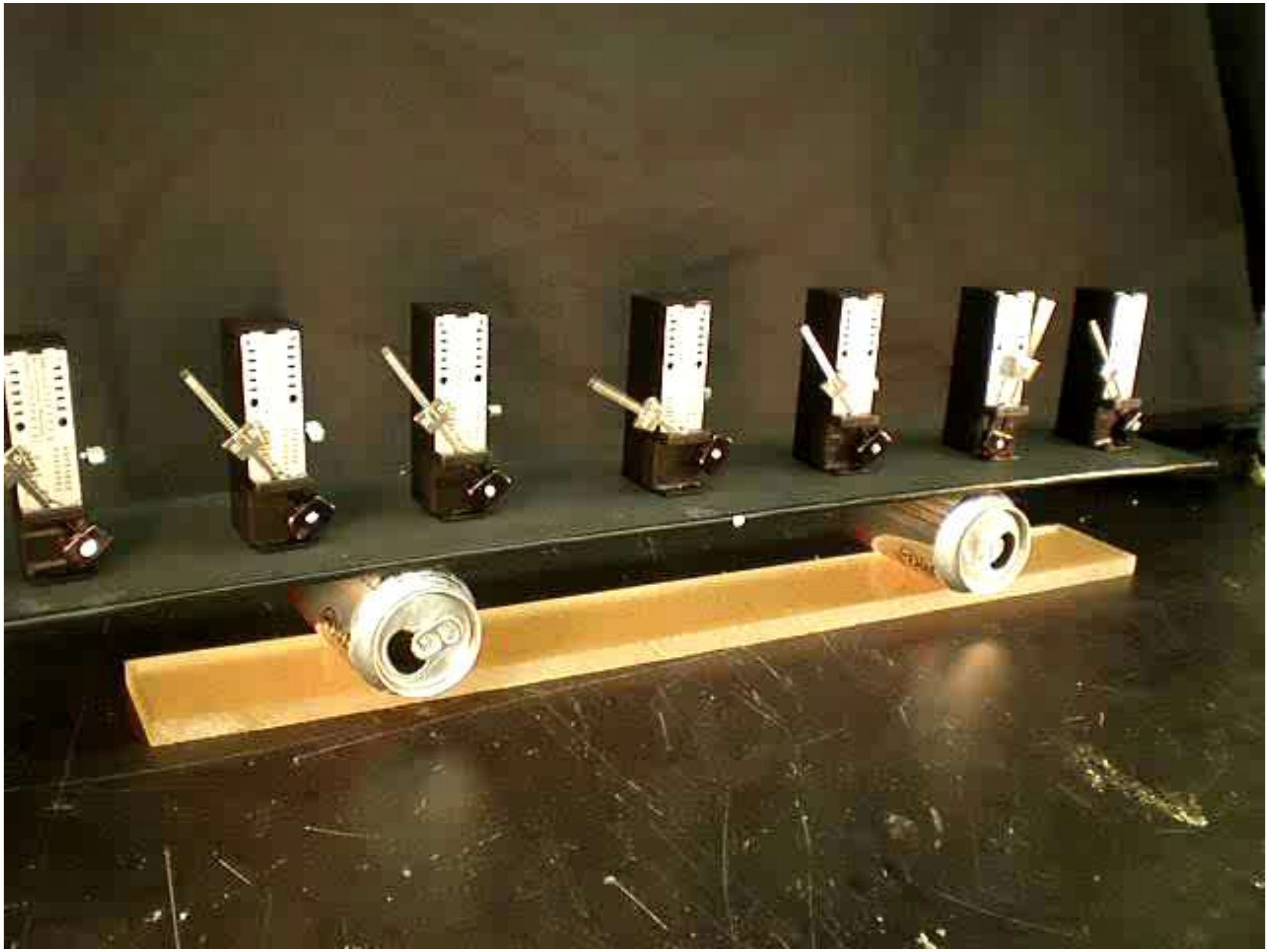
Wiesenfeld & Borrero

$$\Gamma = \frac{B}{(M + Nm)}\sqrt{\frac{l}{4g}}$$

Differences Between Experiment and Simulation

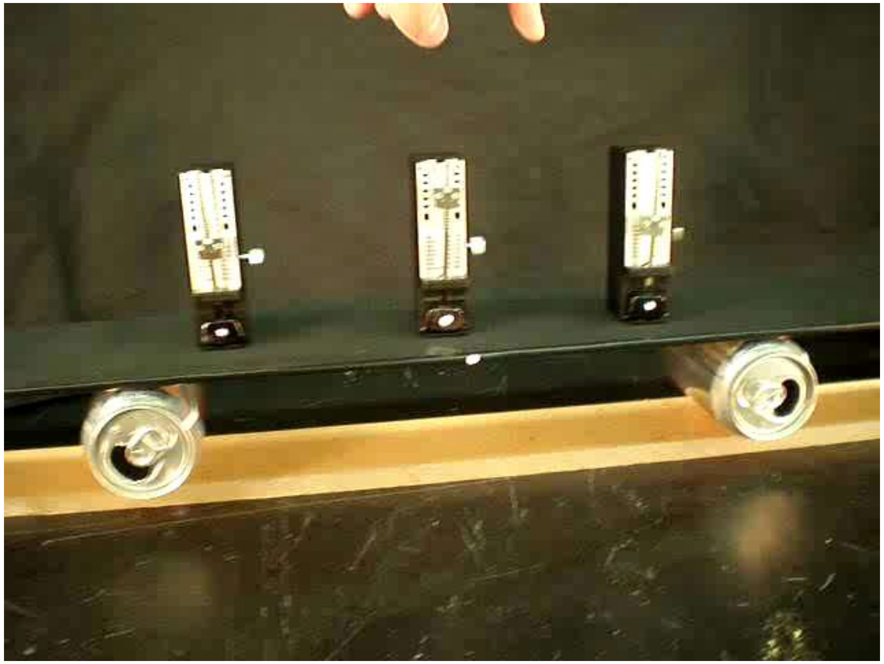
- Simplified version of "kick" mechanism
- Parameter estimates
- Ignoring reaction force on platform (assumption M>>m from Bennett et al.)

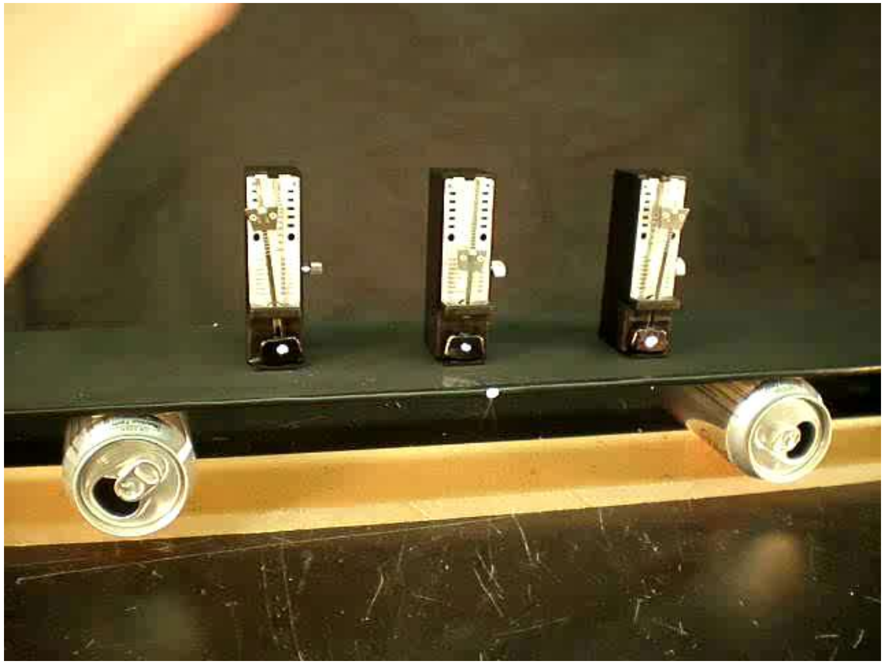
Interesting Experimental States



2 Out of Sync with individual frequencies

Interesting Experimental States

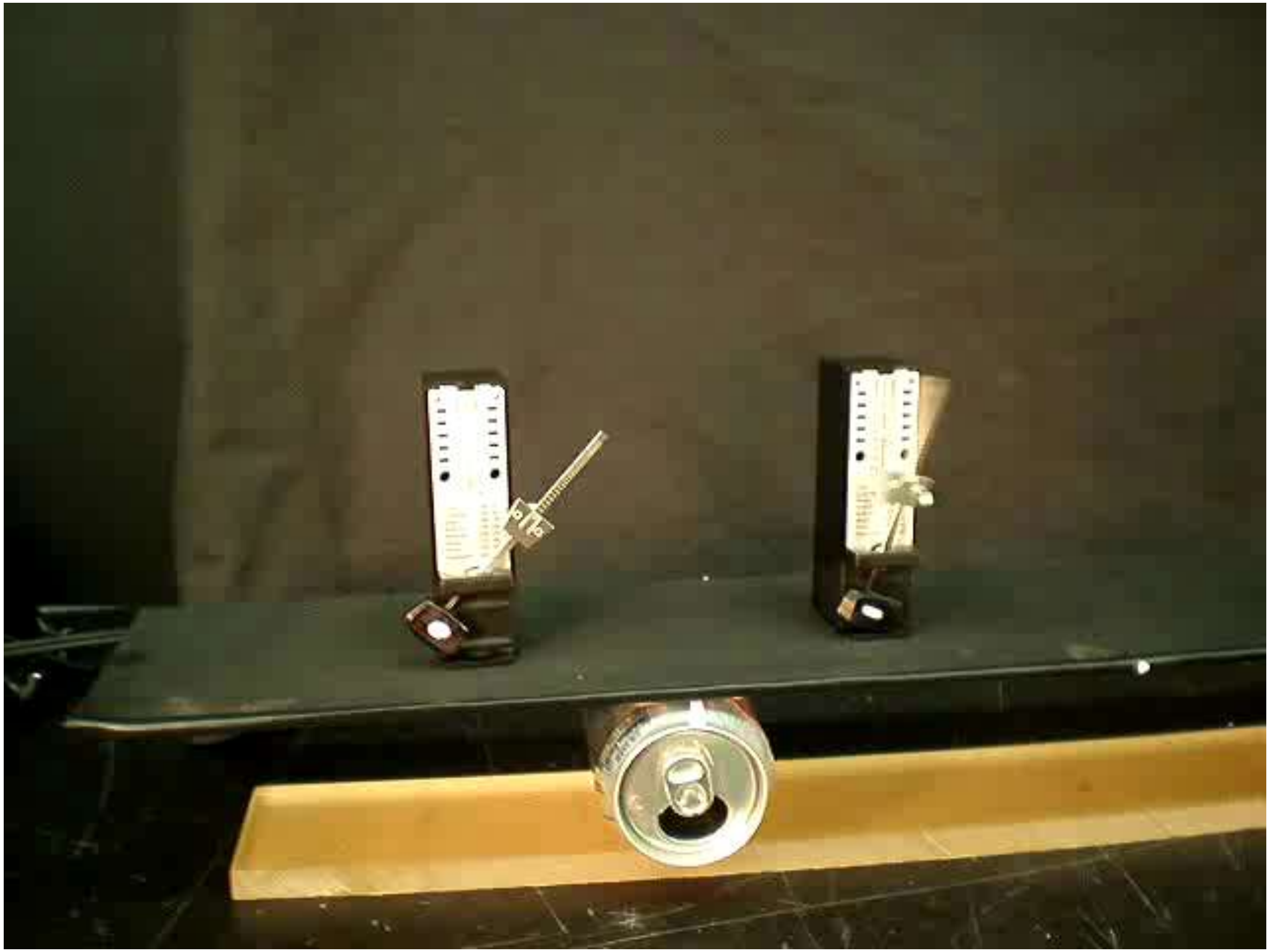




200bpm 100 bpm 200bpm 100bpm 200bpm 100bpm

Didn't seem to sync….

Interesting Experimental States



Increasing platform damping makes antiphase an attractive fixed point!

Future Work

- Finer Experimental Resolution
- More Experimental Data
- Change experimental parameters
  - Natural frequency
  - Platform mass (damping)
- Quantify non-synchronous phase locking

Thank You!

- Acknowledgements
  - Dr. Goldman
  - Nick Gravish
  - Daniel Borrero

- Questions?