“Georgia Institute of Technology”

Nonlinear Dynamics & Chaos
Physics 4267/6268

Faraday Waves

One Dimensional Study

Juan Orphee,
Paul Cardenas,
Michael Lane,

Dec 8, 2011

Presentation Outline

1) Introduction
2) Theoretical Background
3) Experimental Setup
4) Visualization Of Faraday Waves
5) Results & Discussion
6) Conclusions
7) Acknowledgements
8) Other Captured Phenomena (Videos)

1 - Introduction

Experimental Goals

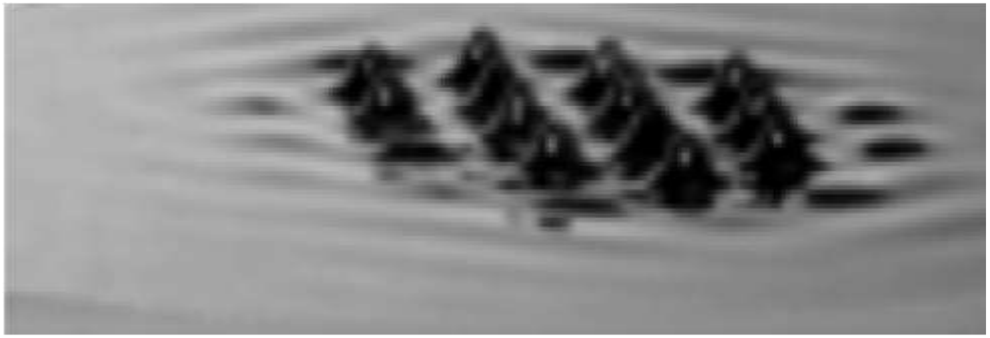
Explore how 1-D patterns change for the following:

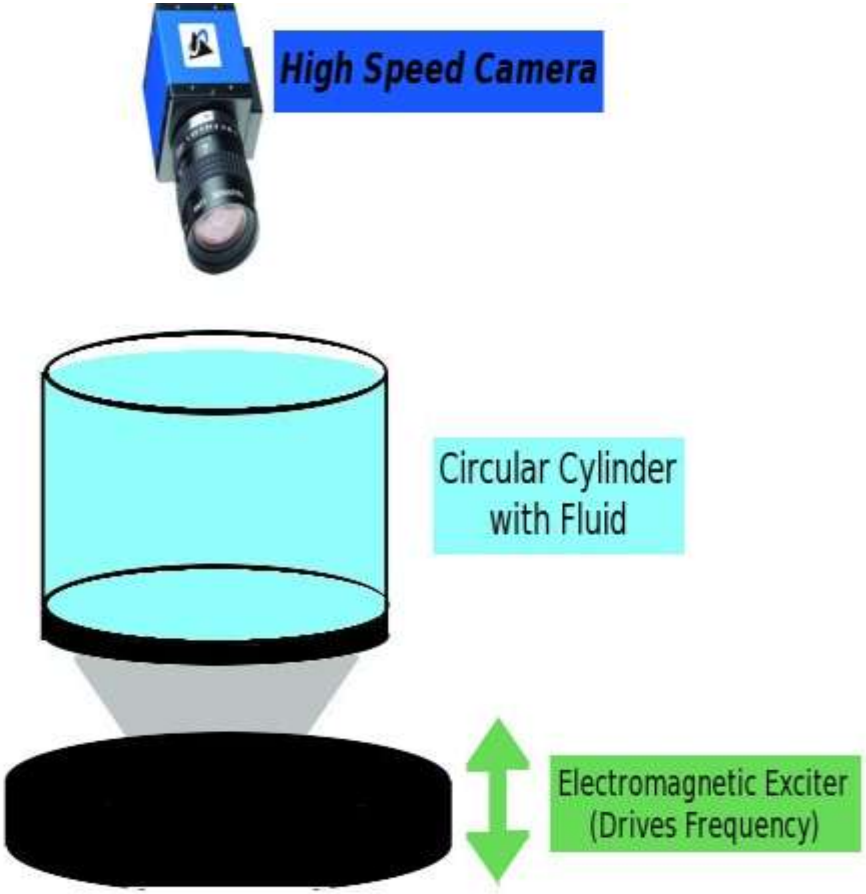
- Change in boundary conditions
- Different working fluids: Water, Oil, Non-Newtonian
- Change in amplitude of the faraday pattern vs. frequency, for increasing and decreasing input amplitude to capture hysteresis
- Change in input wave functions: sinusoidal, triangular and square wave

Faraday First Instability

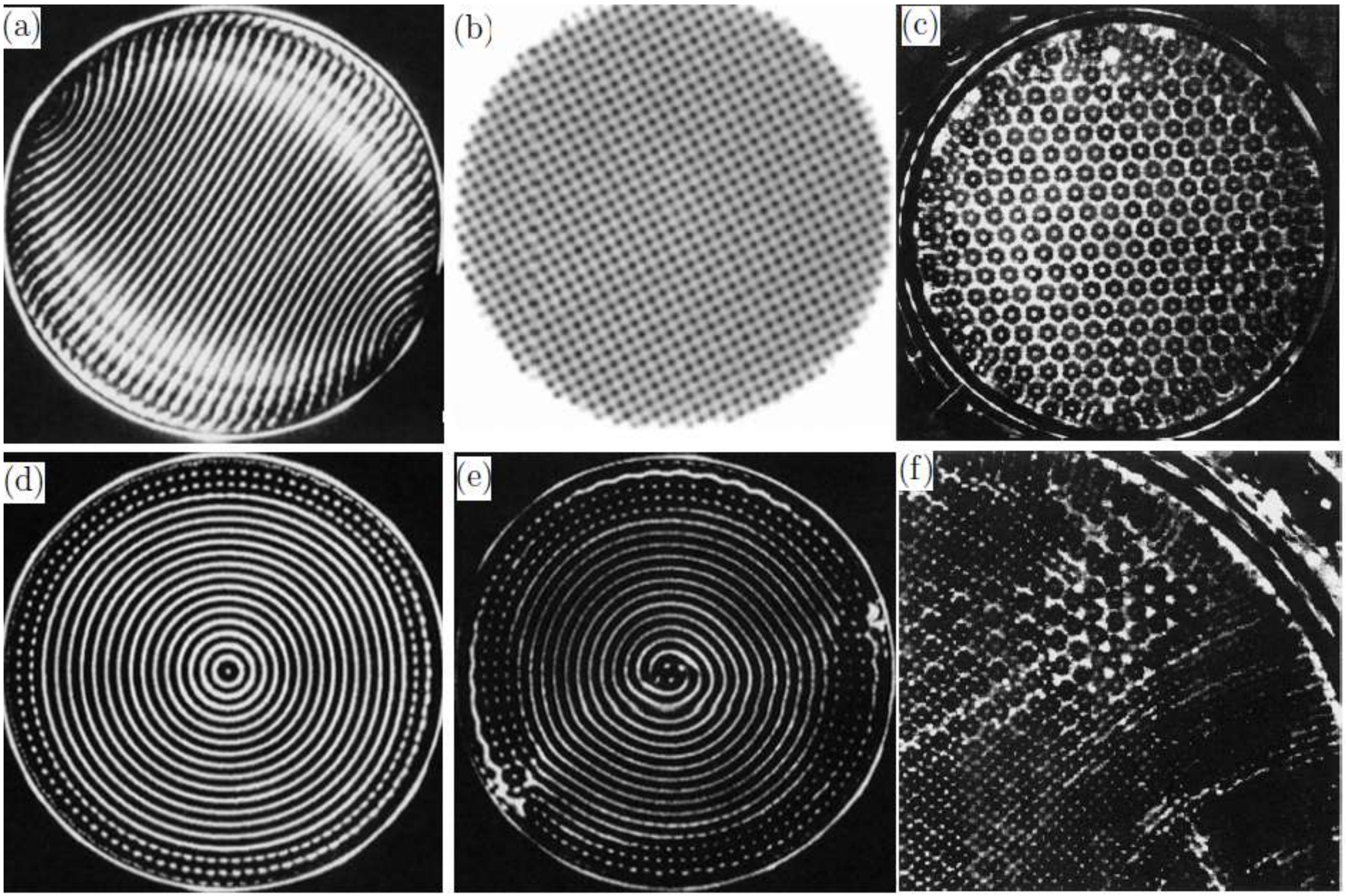
The objects of study are the surface standing waves in a viscous liquid. Liquid is placed on a vertically vibrating container, where a variety of patterns on the surface of a the fluid are observed.

VIDEO-1 Circular Boundary





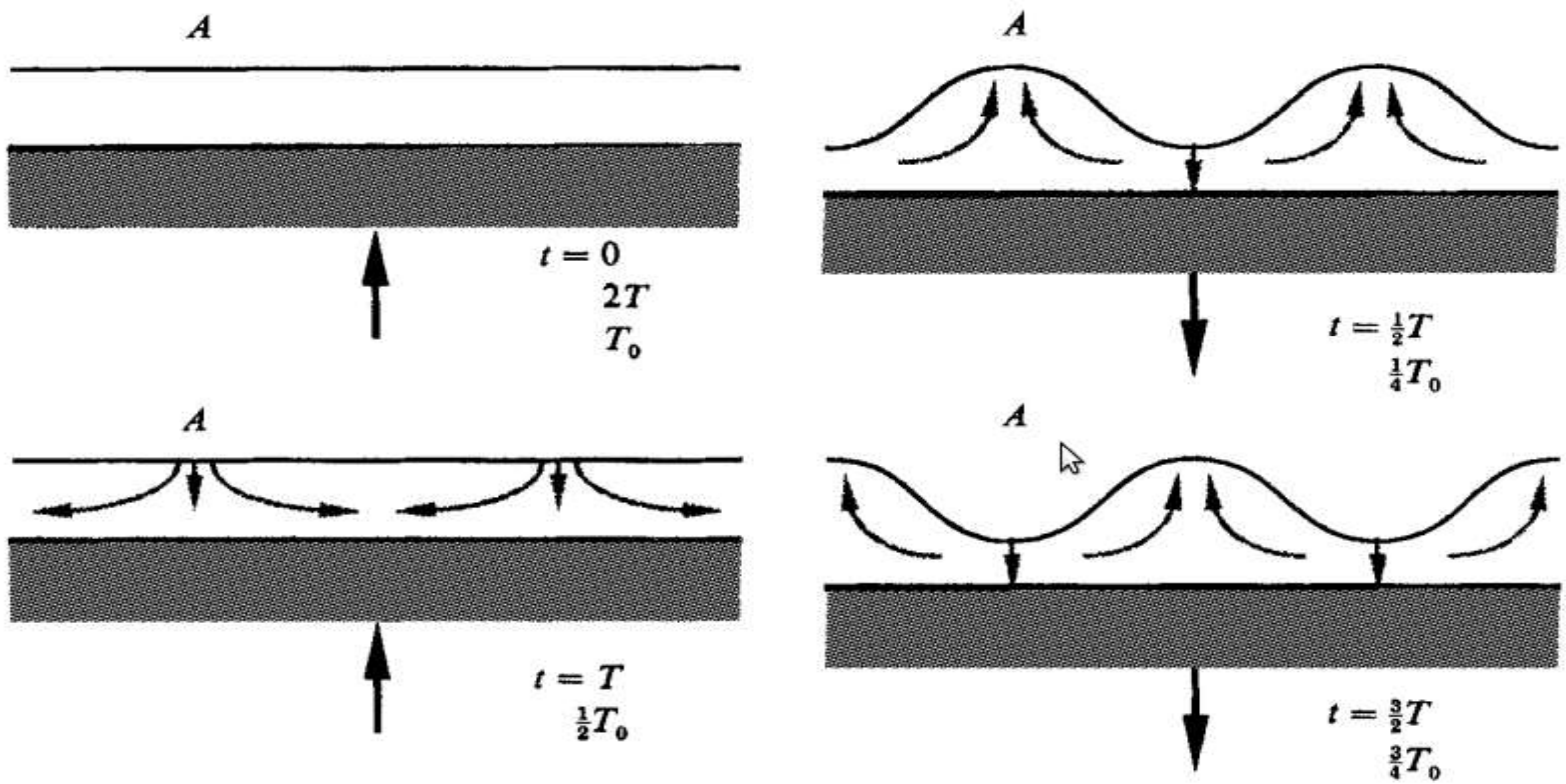
Faraday Waves



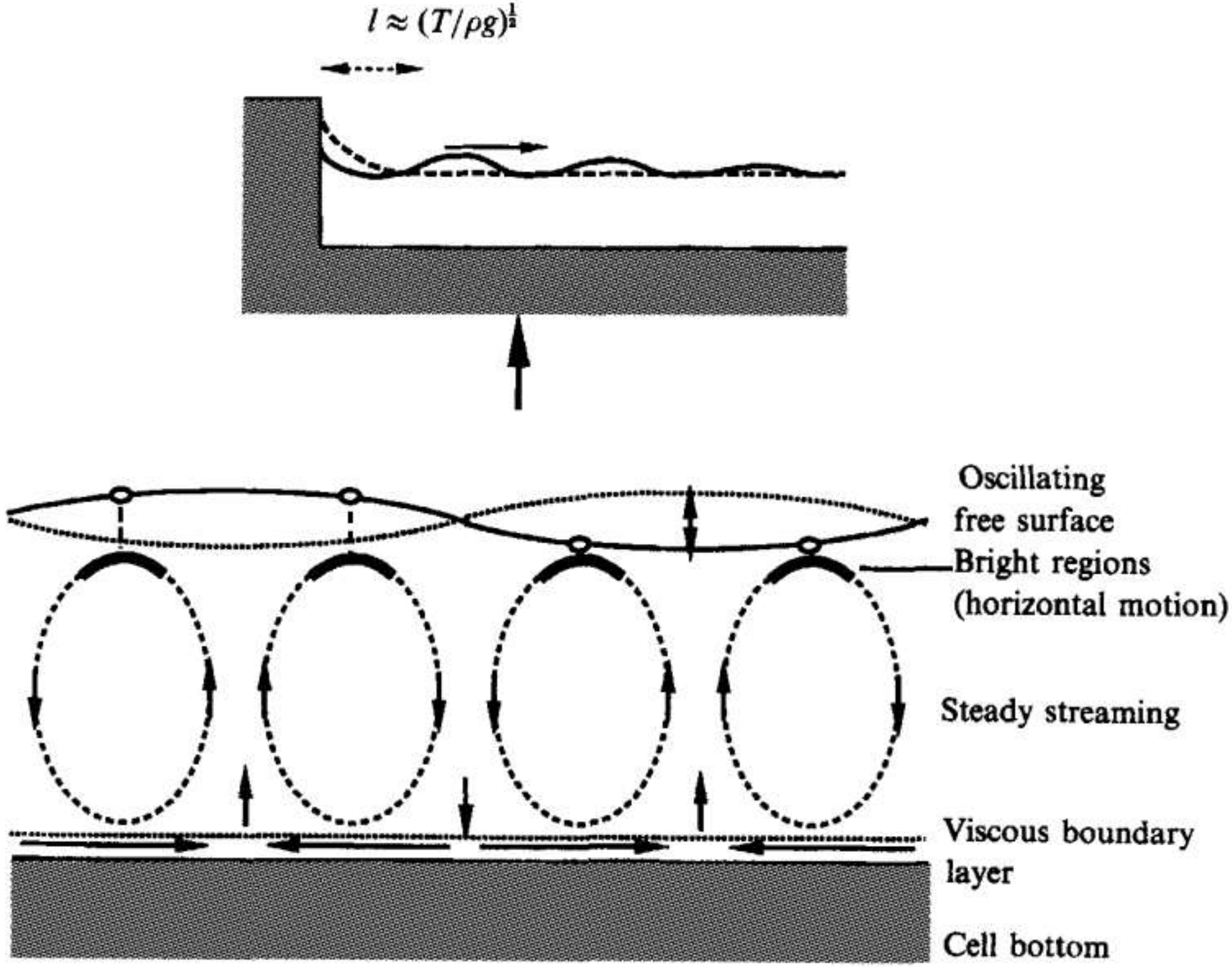
(a) Stripe pattern from. (b) Square pattern from. (c) Hexagonal pattern from. (d) Target pattern from. (e) Spiral pattern from. (f) Region of coexisting squares and hexagons from.

Faraday Instability

When the vessel goes down, the fluid inertia tends to create a surface deformation. This deformation disappears when the vessel comes back up, in a time equal to a quarter-period of the corresponding wave (T). The decay of this deformation creates a flow which induces, for the following excitation period T, exchange of the maxima and the minima. Thus one obtains 2T behavior.

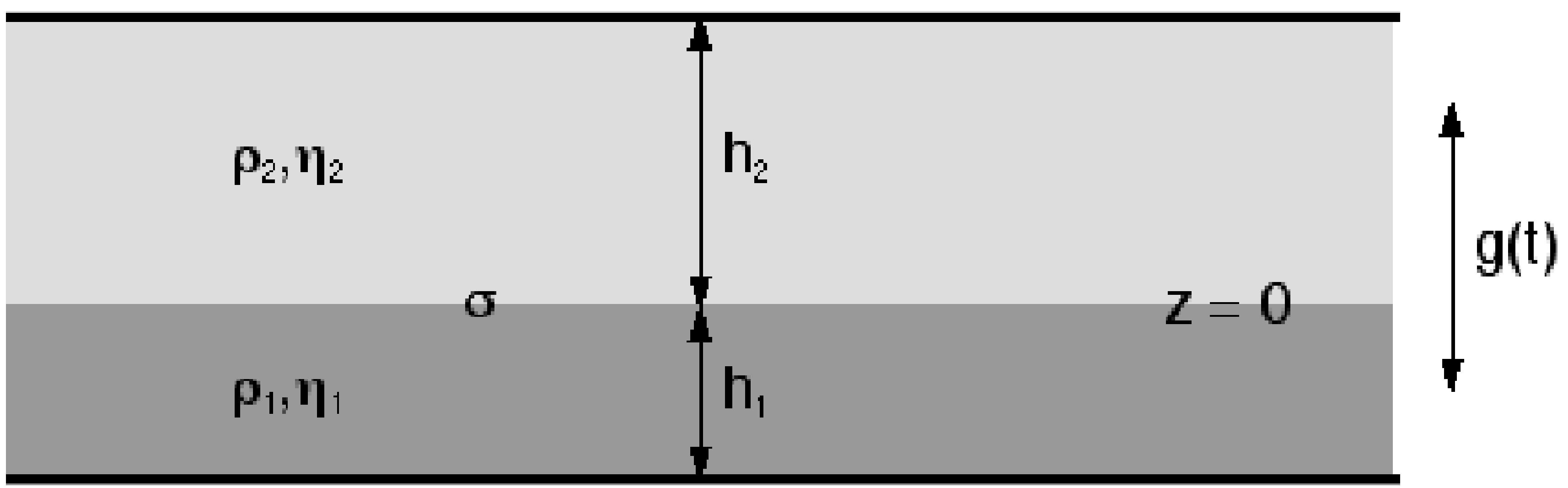


Faraday Instability



Parameters that influence the response

1. Various container geometries (boundary conditions),
2. Fluids (viscosity, surface tension, density),
3. Forcing functions (sine, square, triangle, etc)
4. Frequency and amplitude of vertical vibrations,
5. Surface area and the height of the fluid layer.



2 - Theoretical Background

How to describe the Faraday Instability

The liquid is lying on the horizontal (x, z) plane, and its height at rest is H0. The viscosity and density of the liquid are constant, and the air above it is inviscid. The pressure of the gas phase is the reference pressure of the system and is taken equal to zero.

$$\nabla \cdot \mathbf{v} = 0\,; \qquad \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \frac{1}{Re}\nabla\cdot\left[\nabla\mathbf{v} + (\nabla\mathbf{v})^T\right] + \frac{1}{Fr}\left[F\cos(2\pi t) - 1\right]\mathbf{j}.$$

where $Re=\rho\omega\pi H_0^2 / 2\mu\alpha^2$ is the Reynolds number, $Fr=\omega^2 H_0/4\pi\alpha g$ is the Froude number, and $F=a_0\omega^2/g$ gives the ratio between the external imposed force and gravity force. The characteristic scales adopted are $\pi/k = \pi H_0/\alpha$ for lengths, $2\pi/\omega$ for time, $\omega H_0/2\alpha$ for velocities, and $\rho(\omega H_0/2\alpha)^2$ for pressures and stresses. The initial perturbation imposed to the free surface is

$$h(0,x) = \alpha/\pi\left[1 - \varepsilon\cos\pi x\right],\ 0 \le x \le 1.$$

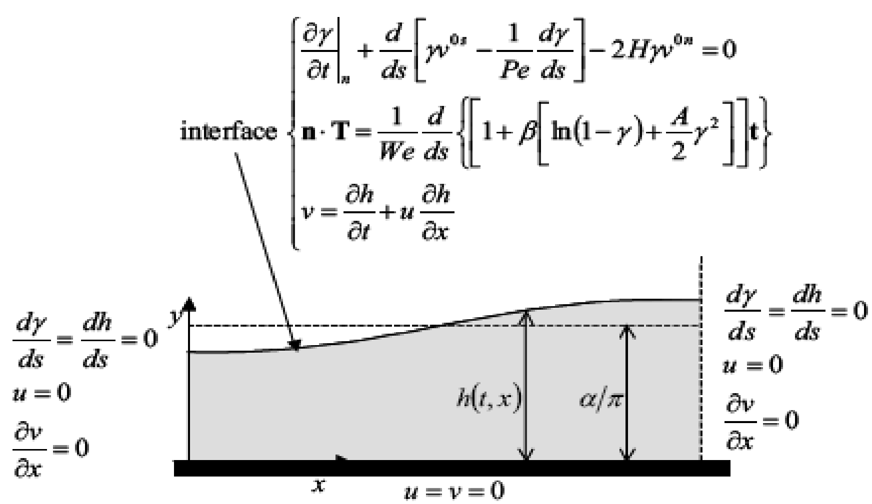


Fig. 1. Schematic representation of the flow domain, boundary conditions, and coordinate system adopted.

Zhang-Viñals Model

Zhang and Viñals model is derived from the Navier-Stokes equations assuming small amplitude surface waves on a deep, nearly inviscid fluid layer. It describes the free surface height ***h(x, t)*** and surface velocity potential ***Φ(x, t)*** of a fluid subjected to a dimensionless periodic vertical acceleration function ***G(t)***

$$(\partial_t - \gamma\nabla^2)h - \hat{D}\Phi = N_1(h,\Phi)$$

$$(\partial_t - \gamma\nabla^2)\Phi - [\Gamma_0\nabla^2 - G_0 + G(t)]h = N_2(h,\Phi)$$

$$N_1(h,\Phi) = -\nabla\cdot(h\nabla\Phi) + \frac{1}{2}\nabla^2(h^2\hat{D}\Phi) - \hat{D}(h\hat{D}\Phi) + \hat{D}\left[h\hat{D}(h\hat{D}\Phi) + \frac{1}{2}h^2\nabla^2\Phi\right]$$

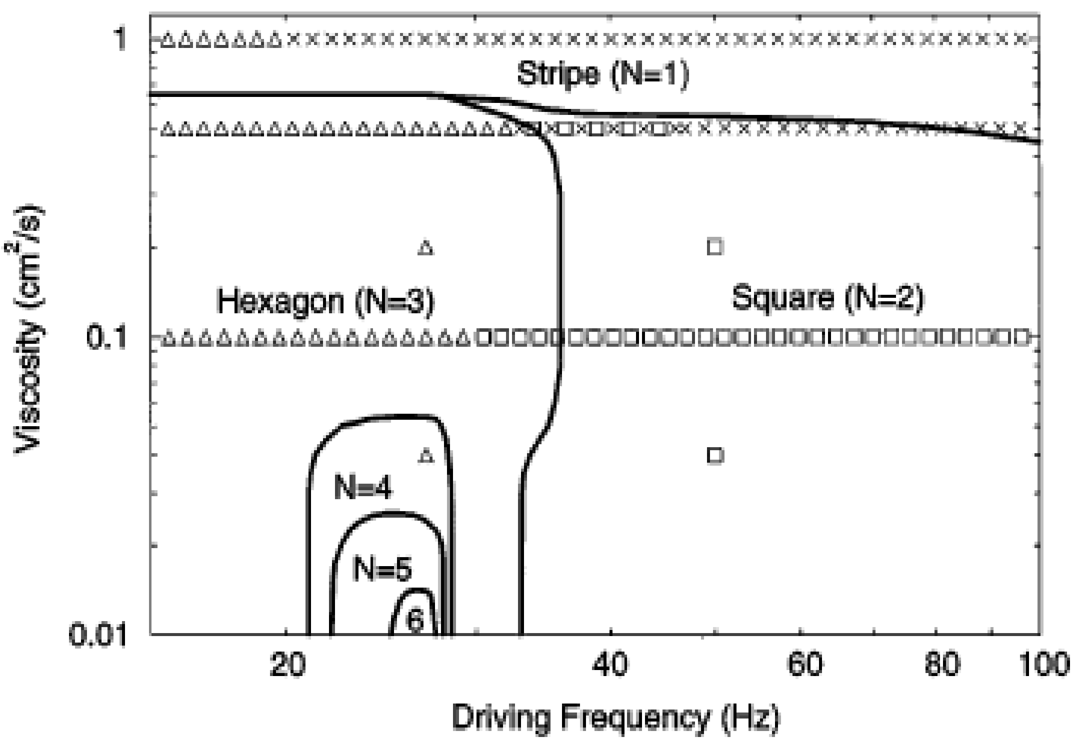
$$N_2(h,\Phi) = \frac{1}{2}(\hat{D}\Phi)^2 - \frac{1}{2}(\nabla\Phi)^2 - \frac{1}{2}\Gamma_0\nabla\cdot[(\nabla h)(\nabla h)^2] - (\hat{D}\Phi)[h\nabla^2\Phi + \hat{D}(h\hat{D}\Phi)]$$

$$\gamma \equiv \frac{2\nu k_0^2}{\omega}, \quad \Gamma_0 \equiv \frac{\Gamma k_0^3}{\rho\omega^2}, \quad G_0 \equiv \frac{gk_0}{\omega^2}, \qquad gk_0 + \frac{\Gamma k_0^3}{\rho} = \left(\frac{\omega}{2}\right)^2$$

$$\frac{dB_1}{dT} = \alpha B_1 - g_0 B_1^3 - \sum_{m\neg=1} g(\theta_{m1}) B_m^2 B_1 \, ,$$

Zhang-Viñals model

- At high viscous dissipation, the observed wave pattern above threshold consists of parallel stripes.
- At lower viscous dissipation, patterns of square symmetry are observed in the capillary regime of large frequencies.
- At low frequencies, higher symmetry patterns have been observed like hexagonal, and eight- and ten-fold quasi-periodic patterns.

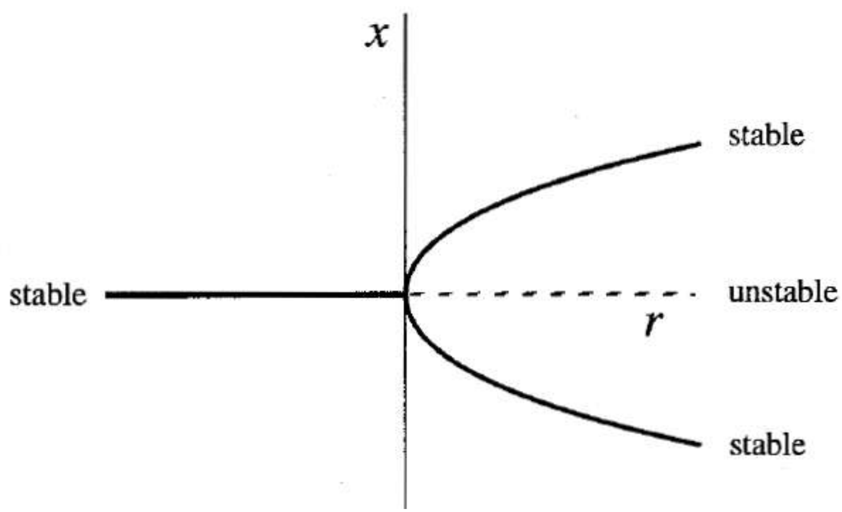


Subcritical Bifurcations in Faraday Waves

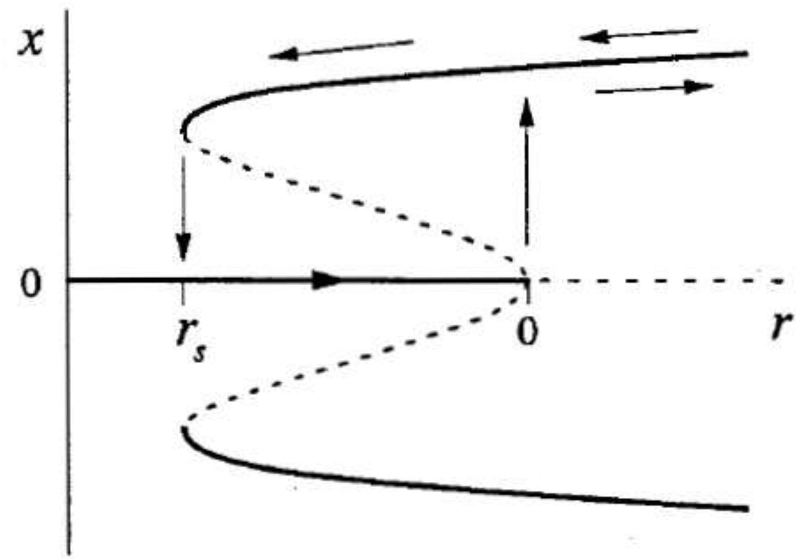
The universal description of small amplitude oscillations is given by the complex Ginzburg-Landau equation (CGLE) for the amplitude of oscillations.

$$\tau_0 \frac{\partial A}{\partial t} = \xi_0^2 \frac{\partial^2 A}{\partial x^2} + \varepsilon A - g|A|^2 A$$

$$\tau \dot{A} = h + \varepsilon A - gA^3 - kA^5$$



$$\dot{x} = rx + x^3$$



$$\dot{x} = rx + x^3 - x^5$$

Why hysteresis in the system?

- Hysteresis of the FW instability was observed when the system is excited beyond the critical acceleration at a given frequency, indicating a sub-critical bifurcation.
- Amplitude equations up to the 5th order were needed to predict the hysteresis boundary.
- These calculations were again based on the Lagrangian method, which neglects the rotational component of the flow.
- The lowest order contributions to the cubic damping coefficient are of the same order for both irrotational and rotational components of the flow. Hence the latter cannot be neglected in a nonlinear theory, even in the limit of small dissipations.

3-Experimental Setup

The equipment

The experiment station includes three main parts:

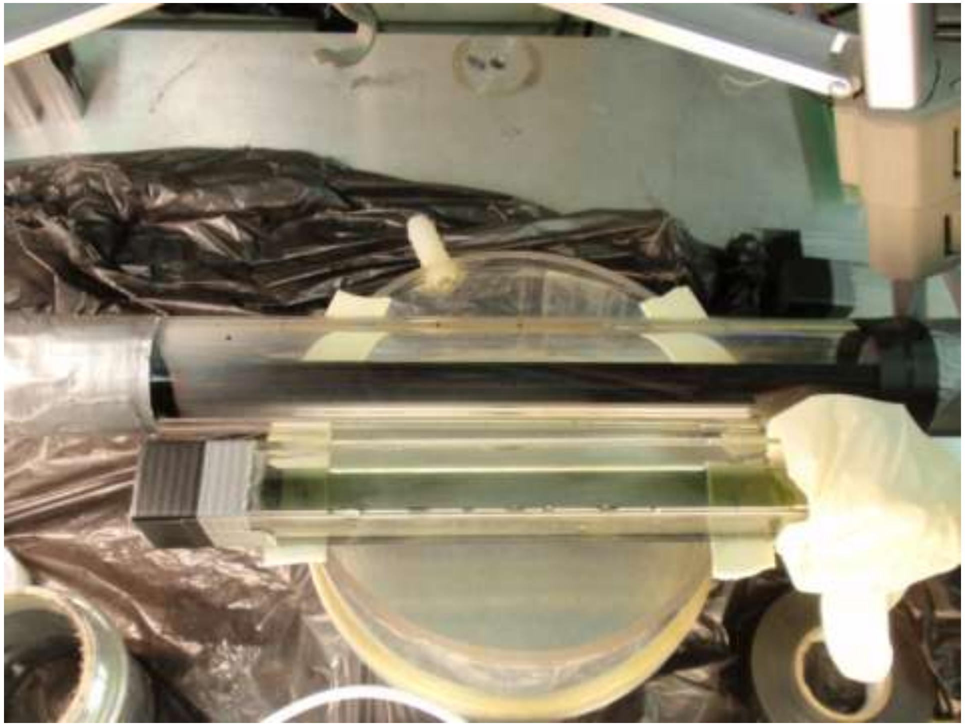
I) a shaker with attached accelerometer; a vessel with fluid; a digital camera, connected to a computer; and a stroboscope.

II) a signal generator; an amplifier; and a digital oscilloscope.

III) a sample preparation station with a fluids; vessels; beakers;

System Set-Up

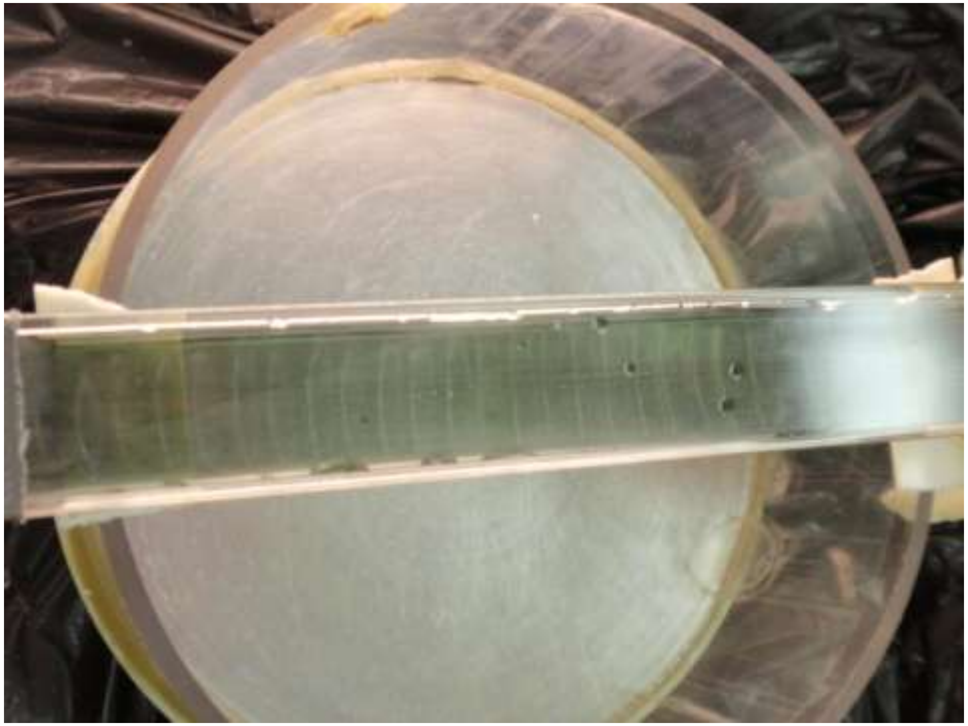
Cylindrical container 2D



Cylindrical and Rectangular container 1D

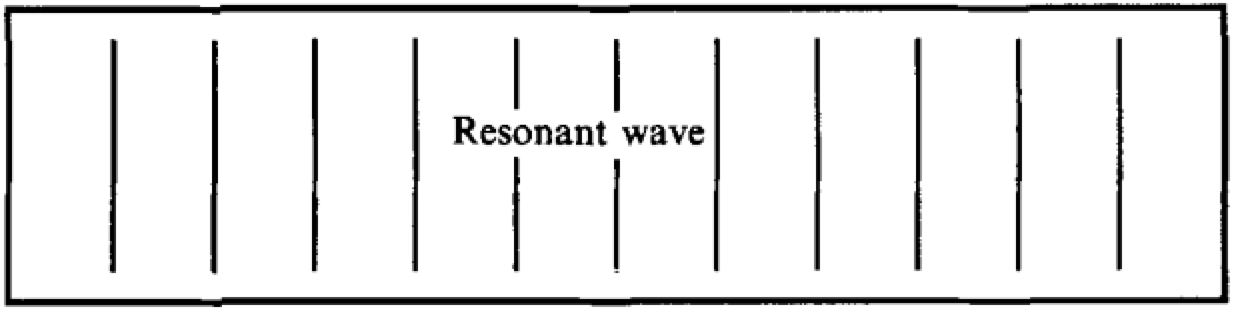
1) We worked with Water, Oil, and a Non-newtonian fluid
2) Different excitation signal, sine, triangular, square
3) The range of frequencies are between 30Hz to 110 Hz

Setting-up our System



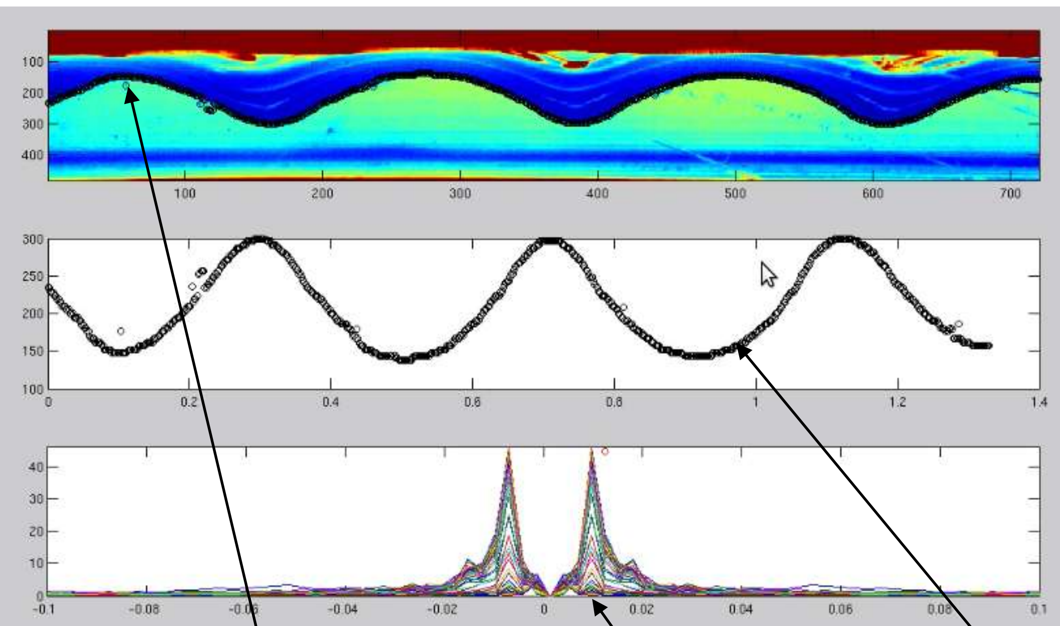
Different 1D containers

The stripe Patterns



Measurements

Video 2 Post Processing



Captured Video

Spectrum of Wave Profile

Capture Wave Profile

4 - Visualization Of Faraday Waves

-Water, Video 3

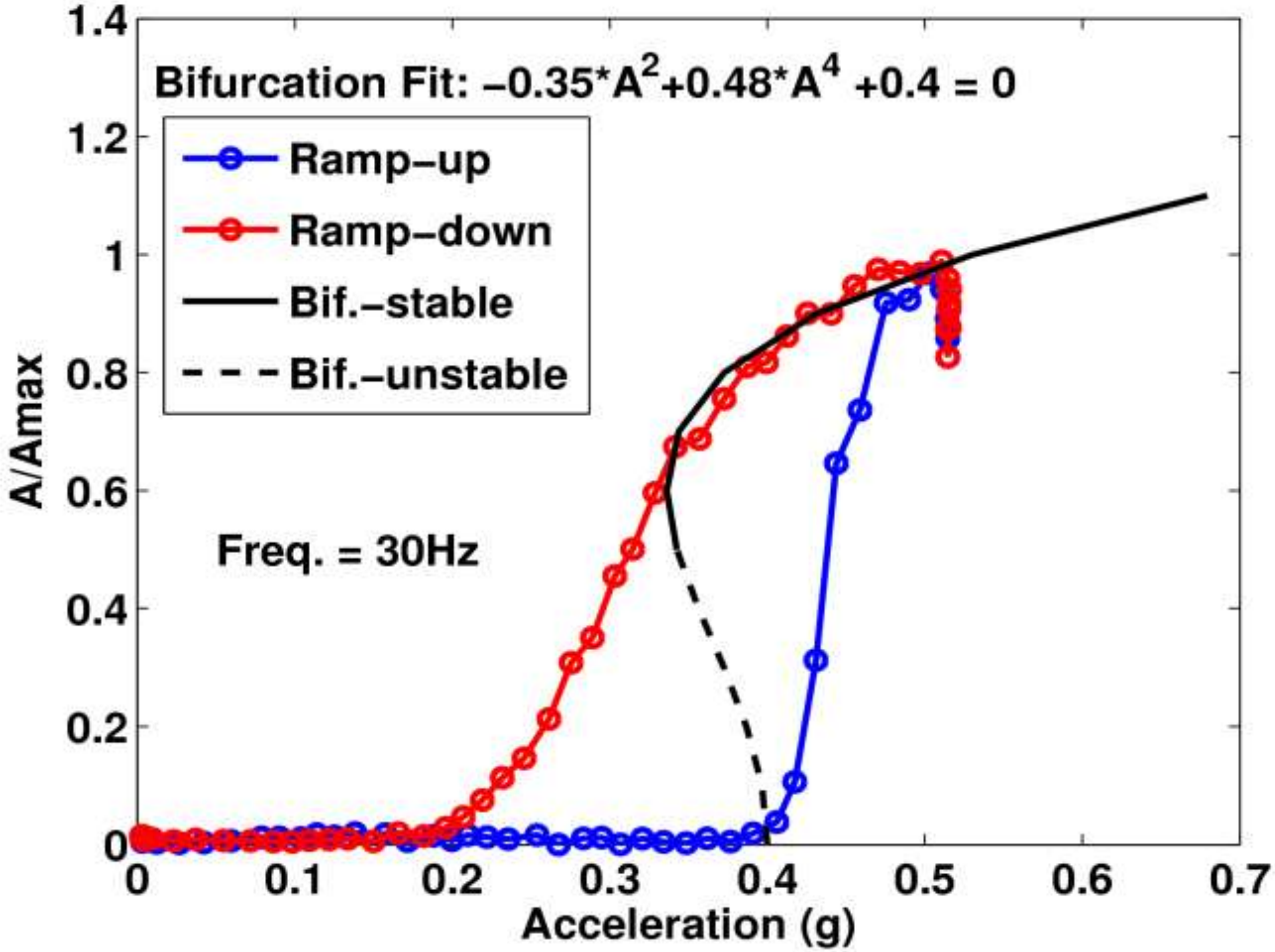
-Oil, Video 4

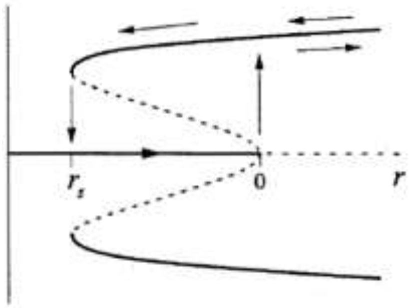
-Non-Newtonian, Video 5

5 – Results & Discussion

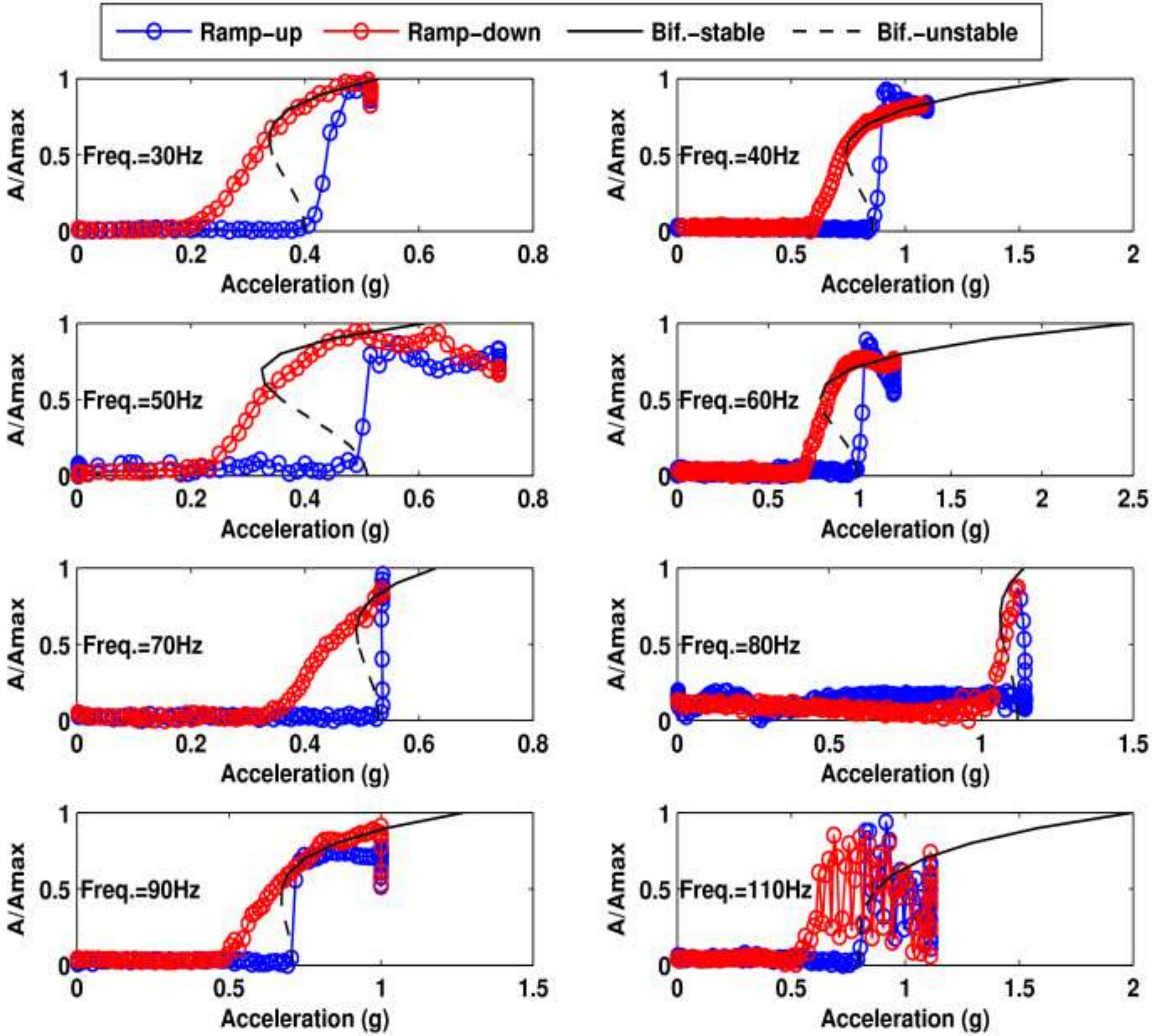
Bifurcation Water 30Hz

$$\tau\dot{A} = h + \varepsilon A - gA^3 - kA^5$$

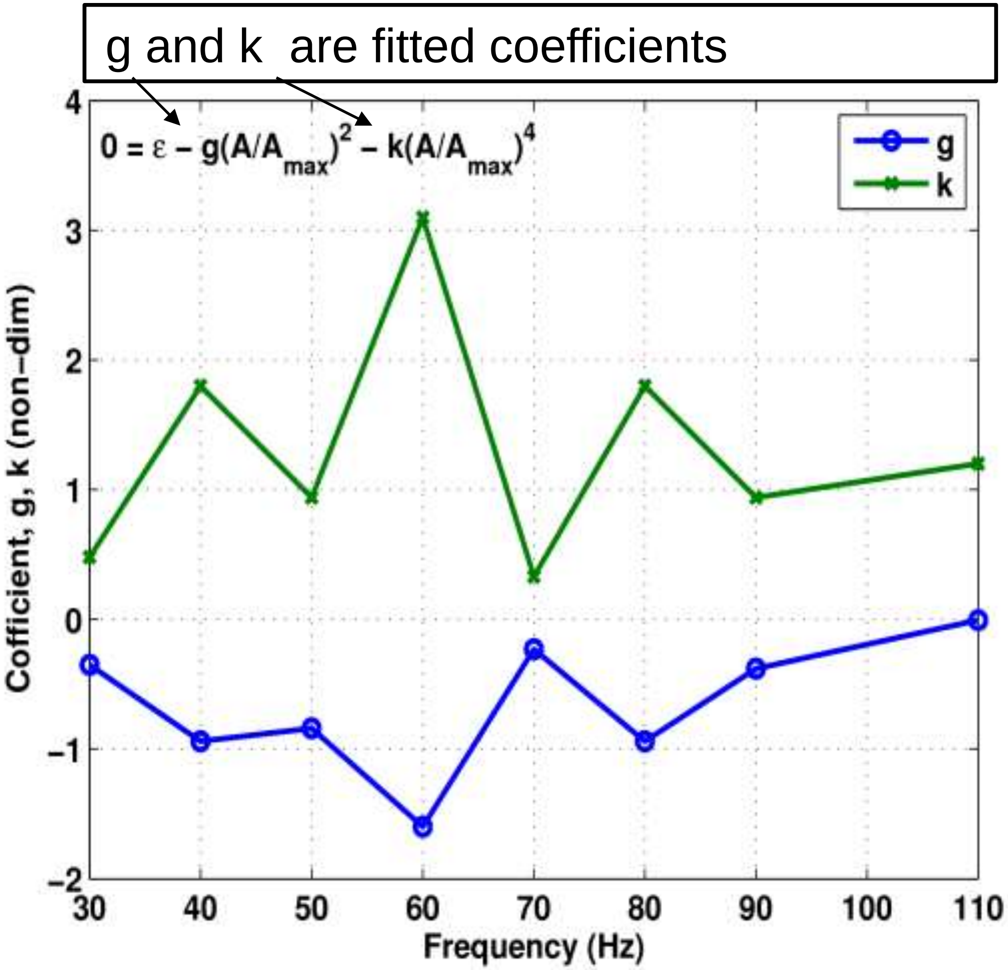




Bifurcations Freq. 30-110Hz Water



Subcritical Bifurcation Coefficients

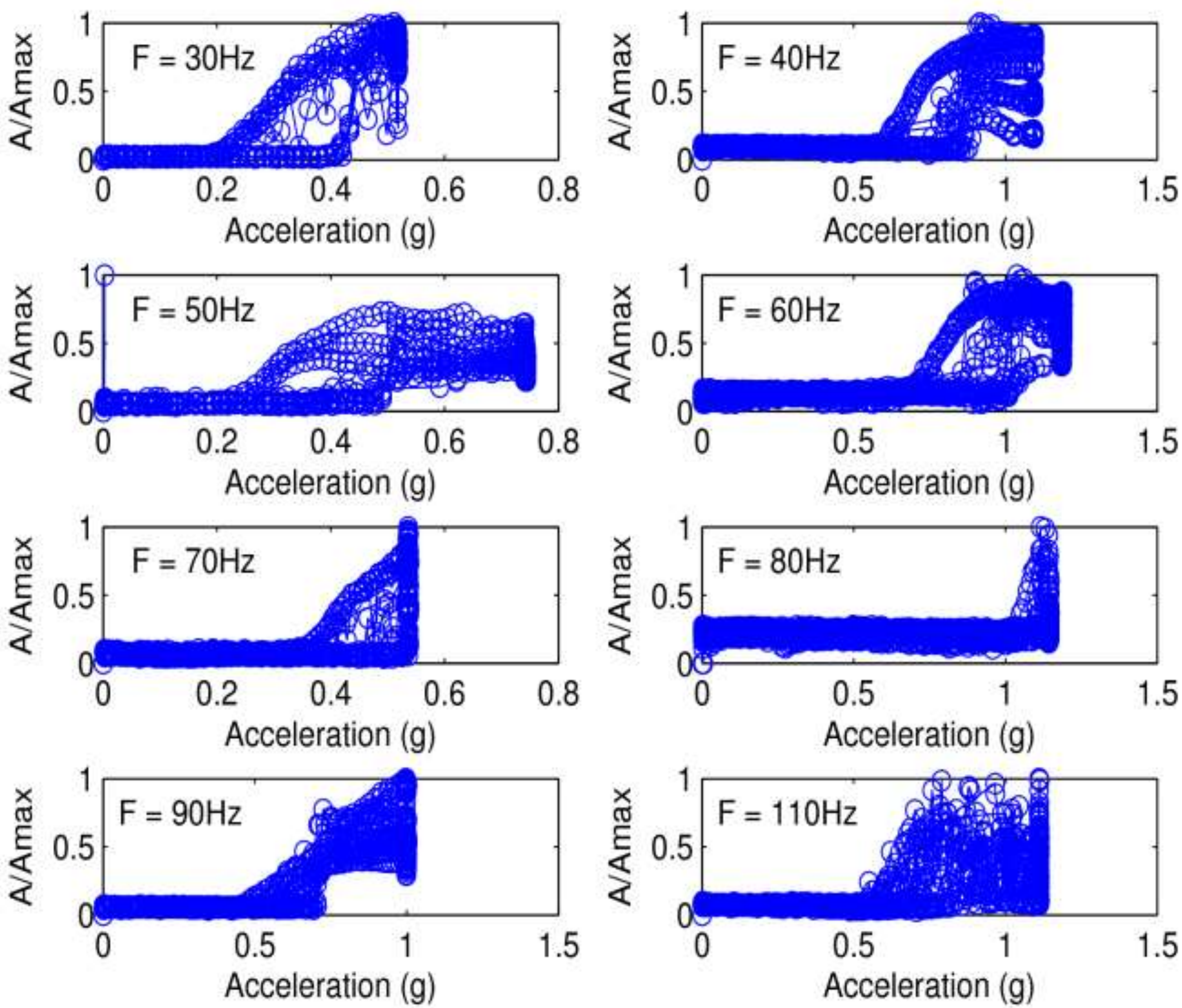


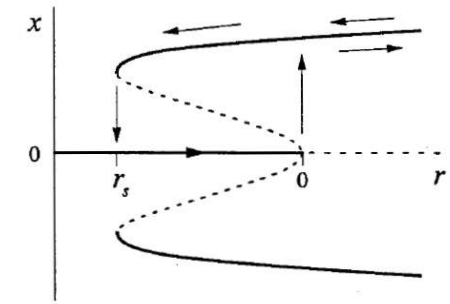
- g < 0 and k > 0 as predicted by the Subcritical Bifurcation Theory

- e is reduced acceleration e = (a/ac -1), and is a known parameter.

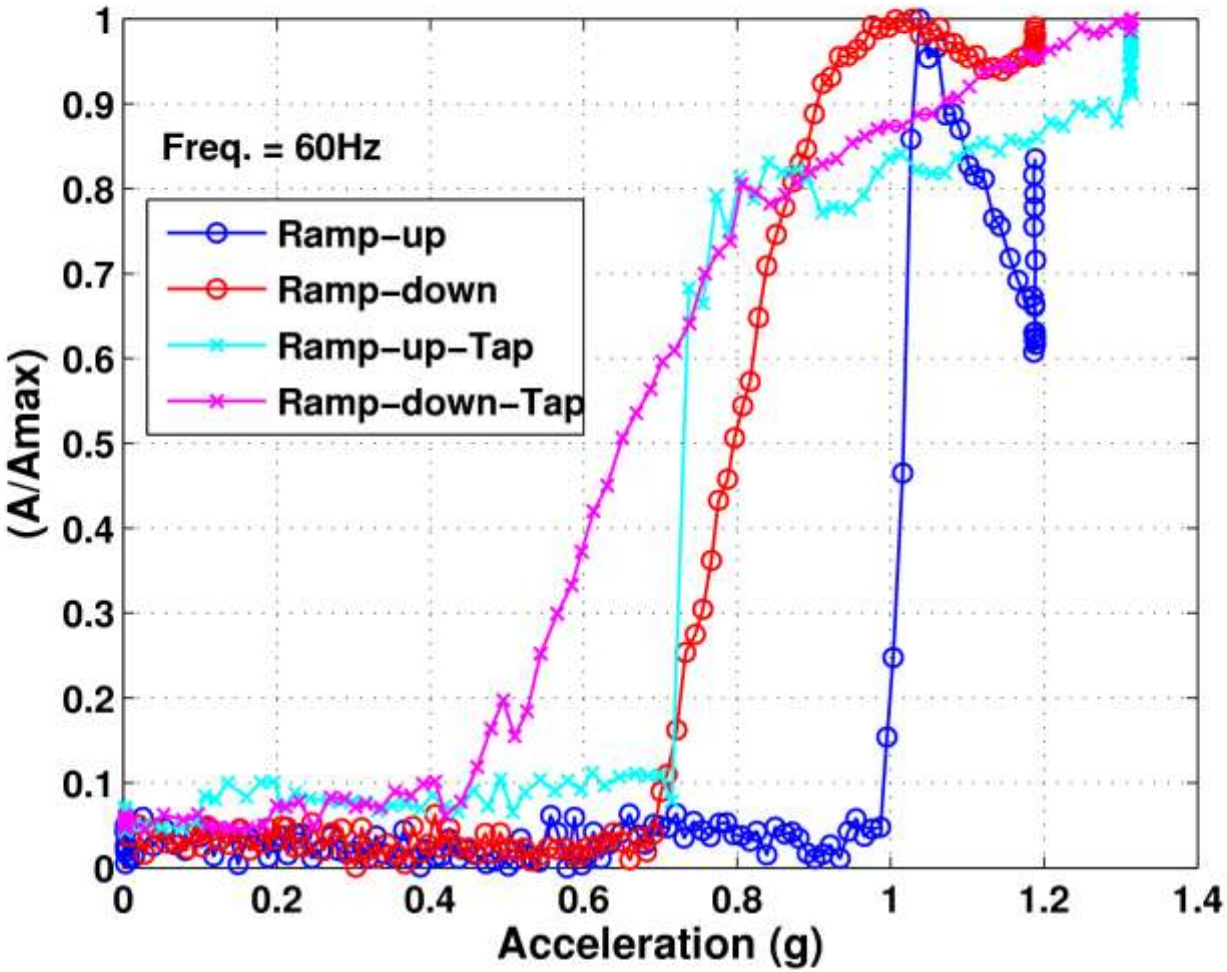
Frequency (Hz)	g	k	e
30	-0.35	0.48	0.4
40	-0.94	1.8	0.86
50	-0.84	0.94	0.51
60	-1.6	3.1	0.99
70	-0.23	0.33	0.53
80	-0.25	0.27	1.12
90	-0.38	0.94	0.71
110	-0.006	1.2	0.8

Bifurcations: Multiple Realizations

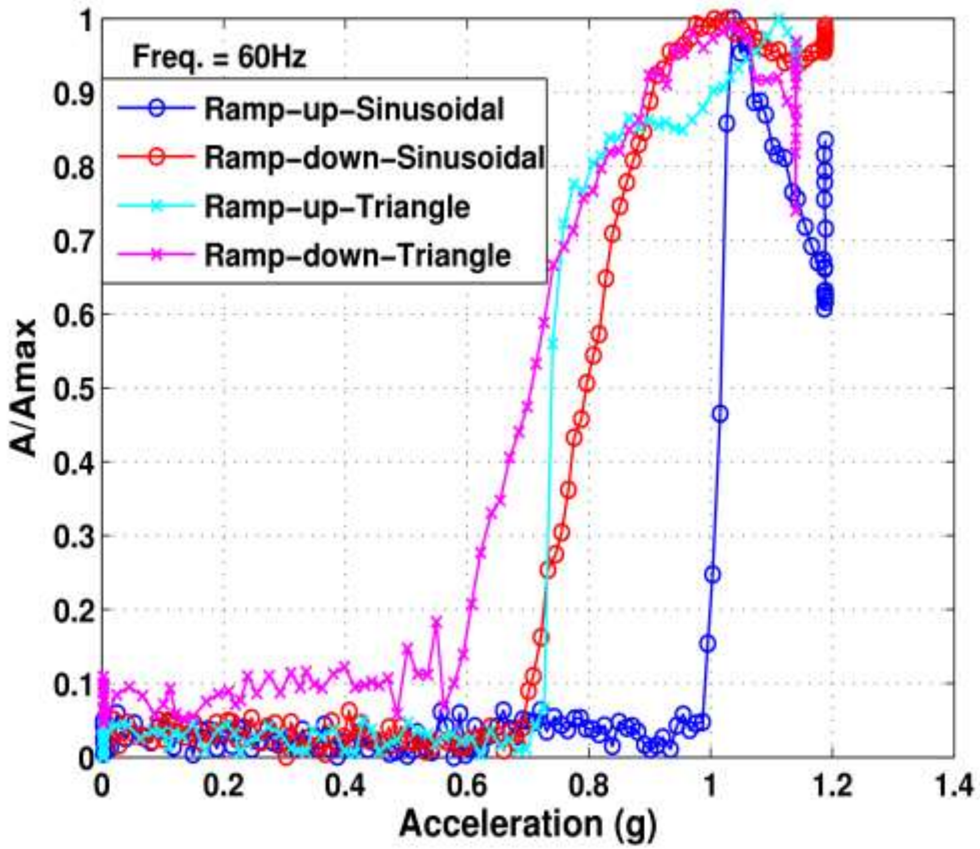




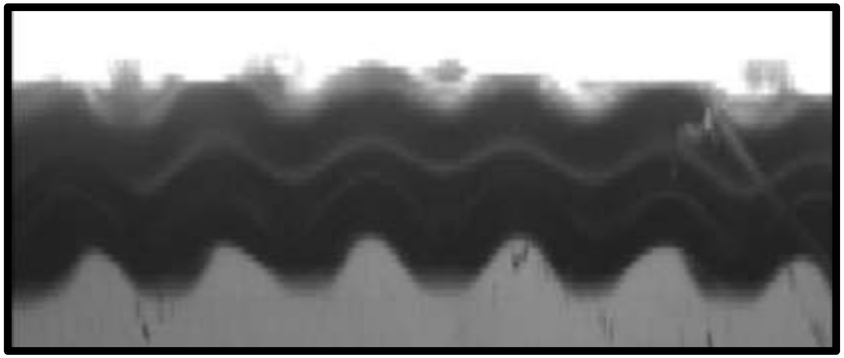
Bifurcation Tap Test 60Hz



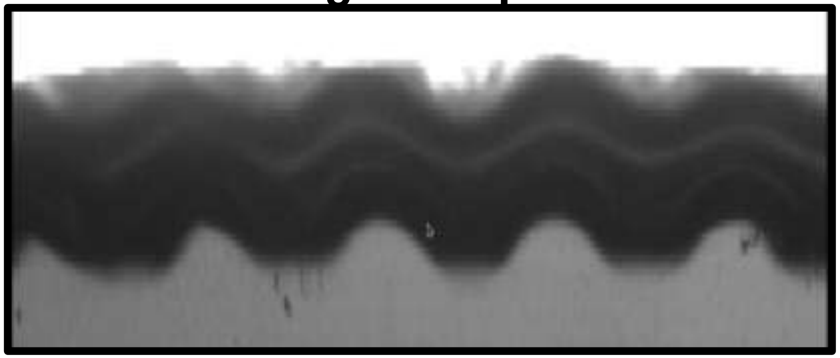
Sinusoidal Input vs. Other Wave Forms



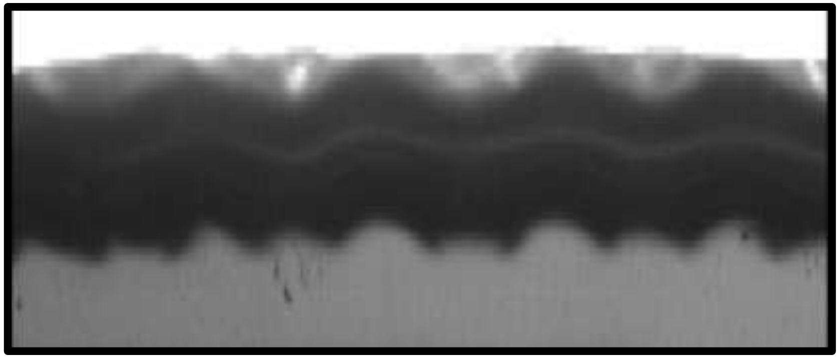
Sinusoidal Input 60Hz



Triangular Input 60Hz

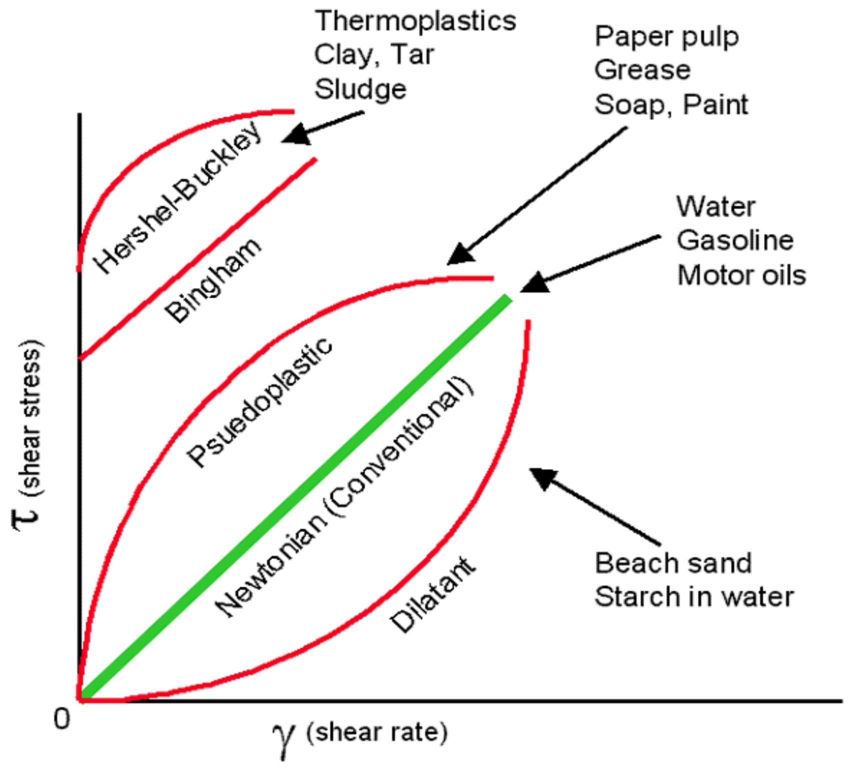


Square Input 60Hz



- **Square waves generate multiple wave numbers as shown in figure -->**
So amplitude measurements for square waves become ambiguous.

Newtonian vs. Non-Newtonian Fluid



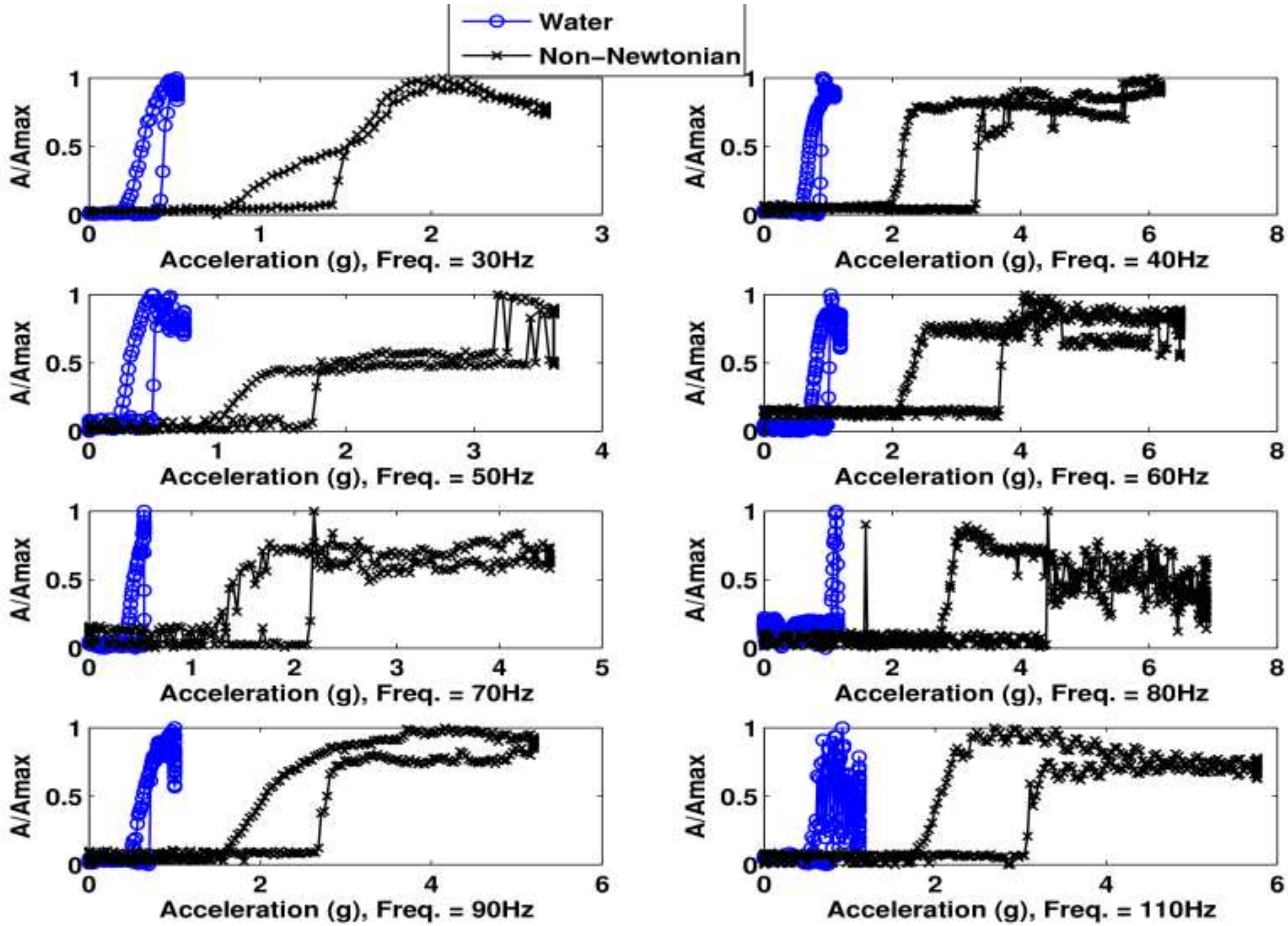
<u>Newtonian Fluid Used:</u>

-Water, Oil

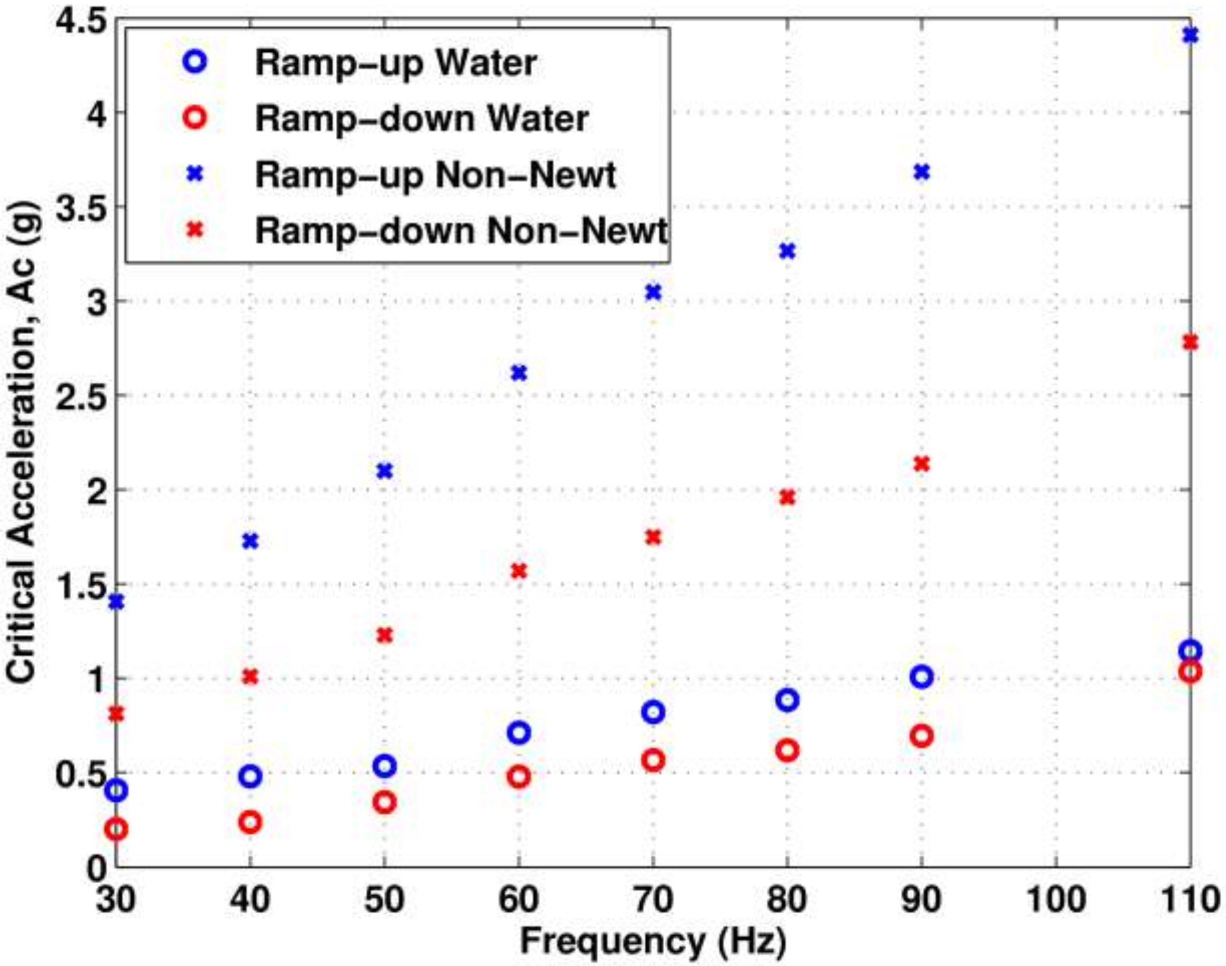
<u>Non-Newtonian Fluid Used:</u>

-Mixture of Cornstarch and water with a volume fraction of ~2.5

Newtonian vs. Non-Newtonian

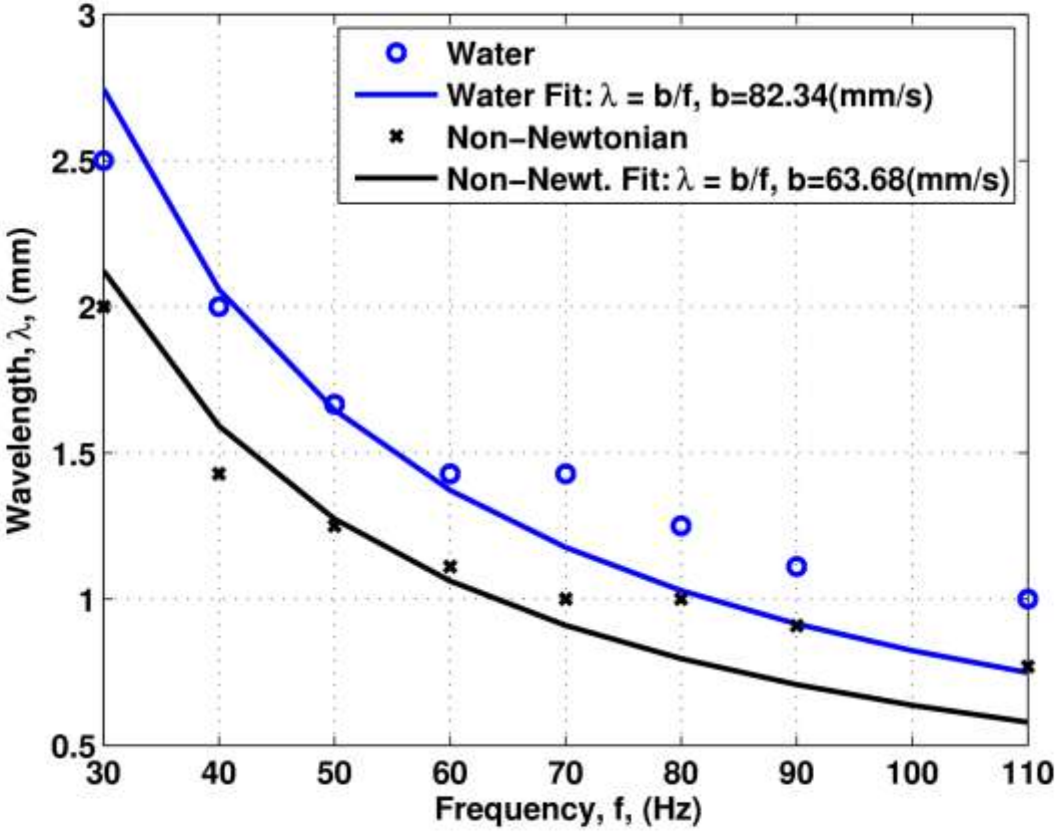


Critical Acceleration vs. Frequency
(Potential New Findings)

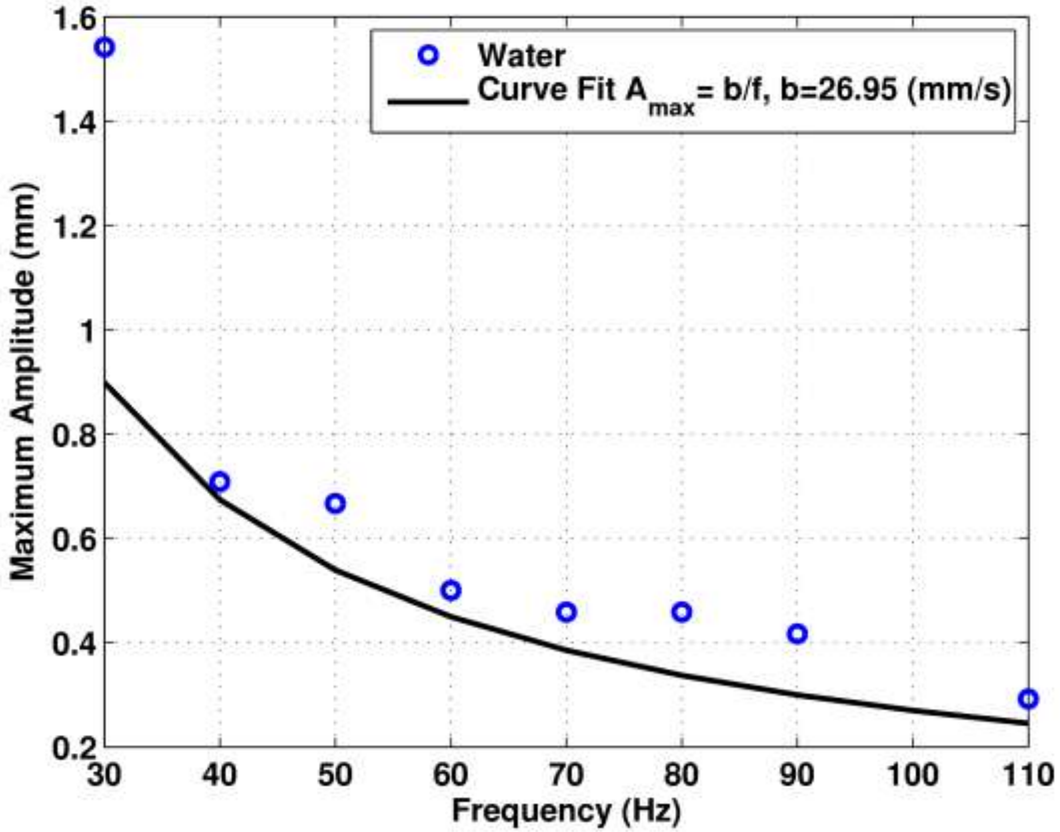


Wavelength vs. Frequency

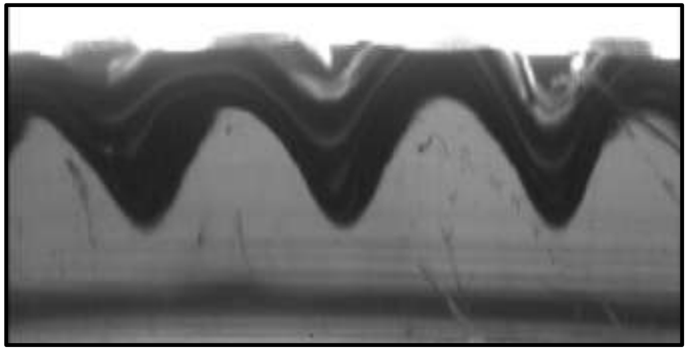
Theory (Dough Binks et al. 1997): Wavelength ~ 1/Frequency



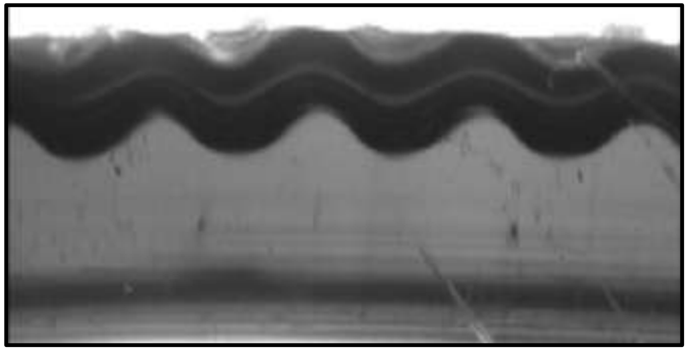
Maximum Amplitude vs. Frequency
(Potential New Finding)



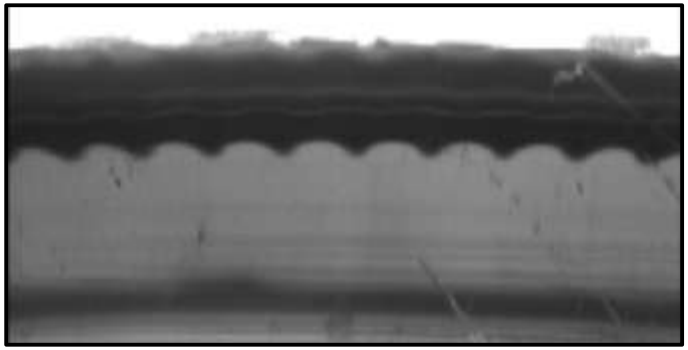
30 Hz



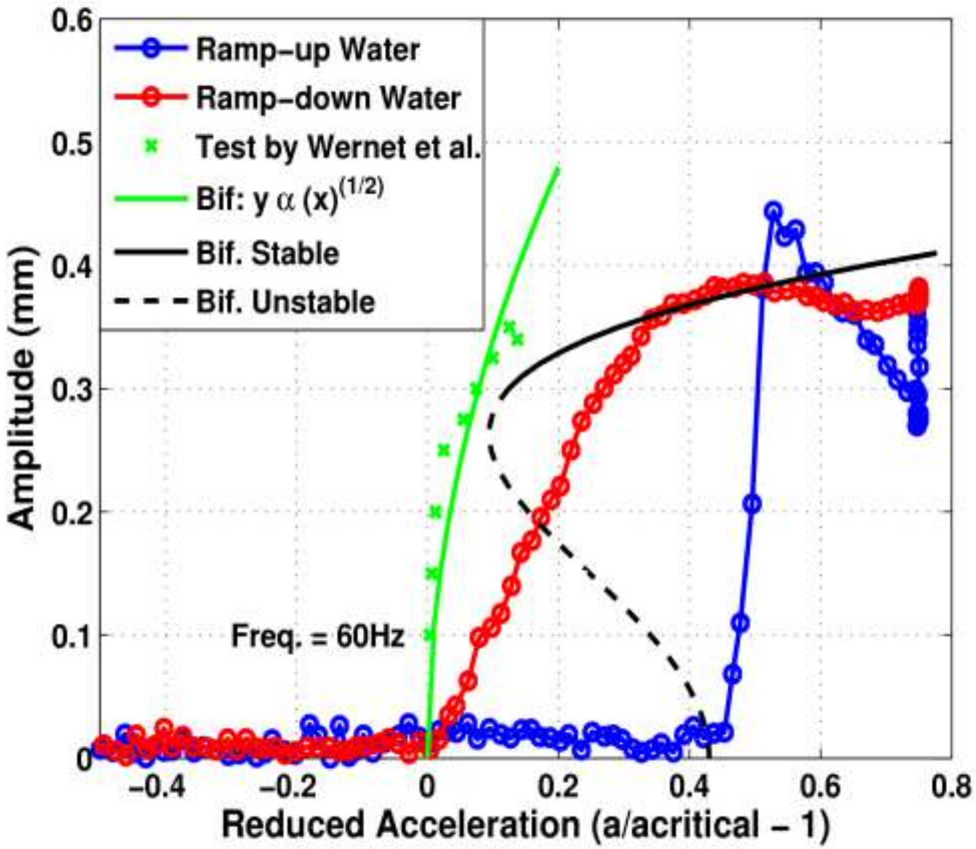
40 Hz



110 Hz



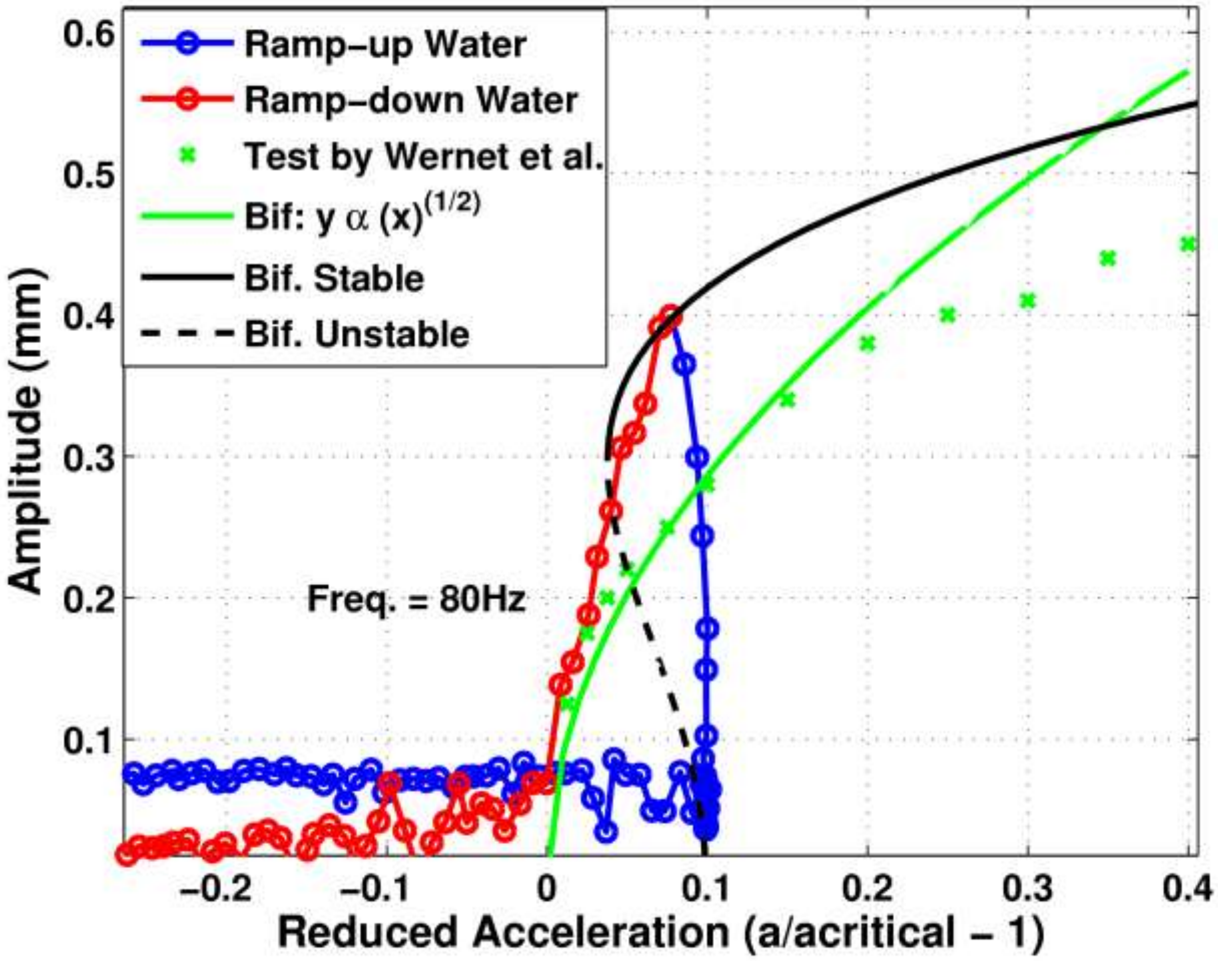
Amplitude vs. Reduced Acceleration: 60Hz



Experimental results from Wernet et al.:

- -Working fluid has ~10 times viscosity of water
- -In general, maximum amplitudes are larger for higher viscosities
- -However, our results show overall agreement with this experimental data

Amplitude vs. Reduced Acceleration: 80Hz



6 - Conclusions

For the chosen working fluids, (water and mixture of water and cornstarch), and this experimental setup:

- Subcritical behavior of Faraday waves is observed
- Different input signals will generate different wave forms, with different critical accelerations, and amplitudes
- A confirmation of wavelength ~ 1/freq. is observed for newtonian and non-newtonian fluids
- Maximum amplitudes are in overall agreement with experimental results of Wernet et al.

<u>Potential New Findings:</u>

- Critical accelerations seem to vary linearly with frequency for both newtonian and non-newtonian fluids
- Maximum amplitudes tend to ~1/freq for increasing frequency

7-Acknowledgements

-Dr. Goldman

-Nick Gravish

-Crew at the CRAB LAB

8 – Other Captured Phenomena

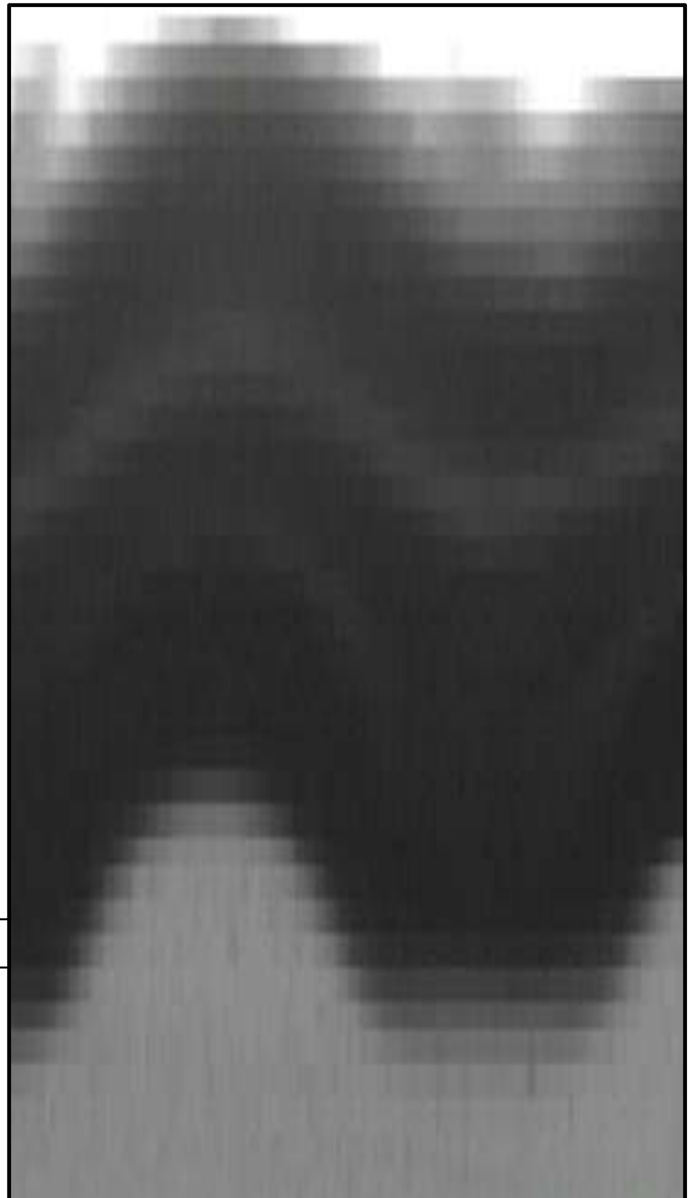
-Particles: Video 7

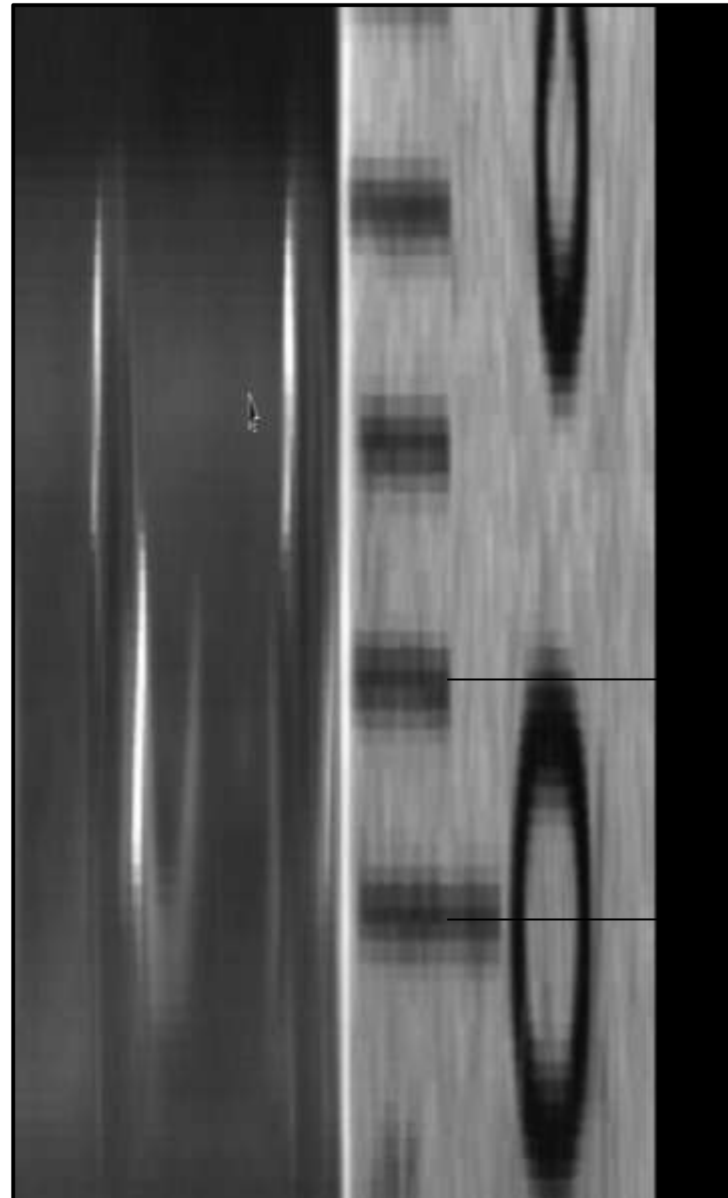
-Modulation with Different Boundary Conditions: Video 8

Back-Up

Amplitude Measurements

~1/13 of mm
~Diameter of
a human hair





1 mm