Analysis of a Hopping Robot

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Abstract

The behavior of a simple hopping robot, with its motion fixed in one dimension, was analyzed. A spring was affixed to the bottom of an actuator, and the motion of the actuator was made to be sinusoidal, with varying frequencies and amplitudes of oscillation. The motion of the robot was captured using 100fps video, and the resulting behavior analyzed using various accepted techniques. The jumping robot exhibited 1-cycle, 2-cycle, 3-cycle and chaotic motion, which was mostly consistent with available models of similar systems, with important differences. Mathematical techniques are used to determine how chaotic the system is, and the bistability of different n-cycles with the same starting parameters is examined in detail.

1 Introduction

One-legged robots are nothing new. Marc Raibert, of the MIT "Leg Lab"[RBC84], first built hopping robots to study stability with the simplest possible gait. Because a unipedal robot (or animal) can only hop, it has only one possible leaping, running, or walking gait, all identical to each other. With this in mind, Raibert set out to create a very simple stable robot, one that maintained stability with little in the way of sensors or complicated calculation. He found that by using feedback from the limb, he could create a mostly stable robot that could not only stay stable hopping in one place, but that could also respond to perturbations, such as someone giving it a good shove.

This project's robot was nowhere near so elaborate, but provided an excellent glimpse into the behavior of such a robot. Through Raibert's example, many have built hopping robots with simple feedback stability systems. It has been established that when forcing parameters are pushed beyond certain limits, chaotic motion is observed in these systems. This research is an attempt to begin quantifying those limits, as well as to explore other fascinating features of the parameter space available. Previous research on a mathematical model similar to both Raibert's hopper and the robot built for this experiment was done by Vakakis et al in 1991[VBC91]. This model predicted some interesting behavior of the hopper robot, but startling differences were also found.

2 Equipment and Methods

The robot consists of a Copley Controls Linear Actuator that bounces on a rigid spring, restricted to one dimension via a guide pole. The actuator can be made to vary in a sinusoidal motion, thereby causing the robot to begin bouncing off the ground. The amplitude and frequency can be varied, giving us a rich range of parameters to explore. Initially, the motion of the robot was intended to be recorded via a high bandwidth accelerometer, but tangled wire and difficulty in maintaining the rig for longer runs led us to fall back onto using a motion capture system. Video was recorded at 100fps, and a small wooden ball was placed on the top of the actuator to aid in tracking its motion. Due to some problems with control software, excellent granularity was able to be obtained in the amplitude of the actuator's oscillation, but only poor granularity was available in the frequency of oscillation. With this in mind, the decision was made to sweep through the interesting, safe amplitudes at 4Hz through 8Hz in integer spaces.

Each run was limited to 30-40 seconds, though the robot was permitted to hop for longer periods on certain runs where interesting behavior was observed. The peaks from each run were then gathered with a MatLab script, and plotted together on an orbital map to look for bifurcations. Some bifurcations were examined in more detail by taking several runs with the same parameters; these yielded interesting results dealt with in more detail below. Unfortunately, it was impossible to correlate certain parameters with SI units; the program available for use allowed for adjustment of oscillation amplitude in "counts", and we were unable to ascertain the value of a count in terms of meters. Throughout this paper and all graphs, this "count" unit (abbreviated ct or cts) is used as a descriptor of oscillation amplitude. We were however able to determine the height of the robots jump in meters from the pixel count given by the image tracking software. Fortunately, because of the nature of the measurements and what is being studied, the units used are not as critical as they might be. In addition, this paper draws mostly qualitative rather than quantitative conclusions from the results.

3 Results and Analysis

3.1 Overview

When looking at an entire run at once (Figure 1), several things are apparent. First, the motion of the robot around its constraining pole shows up here as an offset in the height of the robot. While it looks like there is long term patterned behavior in the height of the jumps, this is actually just the robot moving horizontally. The camera registers this as a varying height, and this introduces noise into the analysis. Second, the initial transient behavior just introduces further noise into the system. For this reason, most of the data analysis focuses on the final quarter of each run, eliminating these transients.

During the very first run, and in most runs after that, the initial transient behavior for the robot was usually 3-cycle motion. From Sharkovsky's Theorem[LY75], it can be inferred from the presence of chaos from this 3-cycle. Though many different stable cycles should have been observed, the only periods present were 1-, 2-, 3-cycles and chaos. This is easily attributable to the presence of many

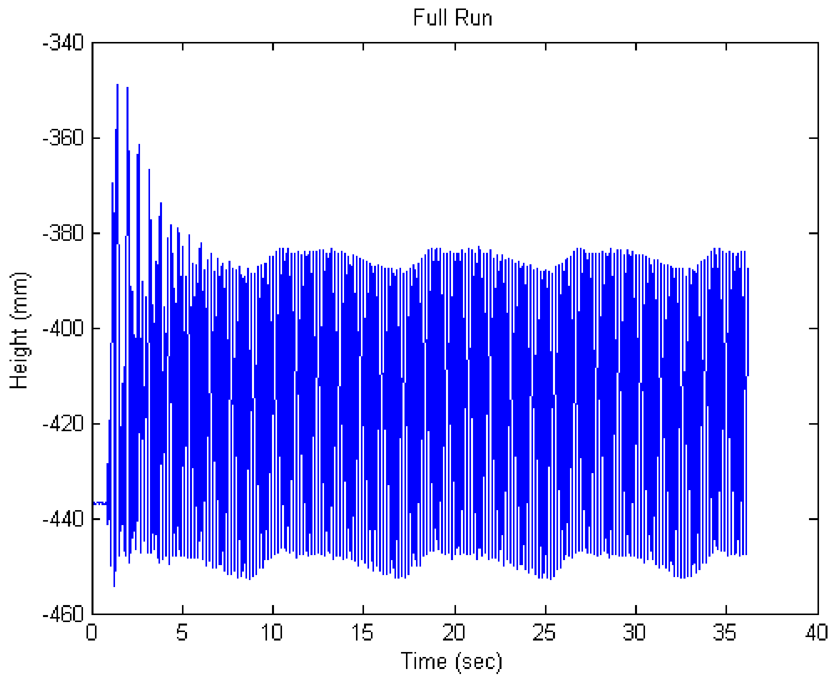


Figure 1: A Full Run

different sources of noise as well as the fragility of the higher order cycles. There was some visual indication of 4-cycles while the experiment was being run, but upon closer examination these were ruled to be merely tricks of the eye rather than real results.

3.1.1 1-cycle

1-cycle behavior was found to be very common at lower frequencies, and less common at higher ones. A typical 1-cycle at low frequencies might be stable, but at higher frequencies and amplitudes the 1-cycle becomes bistable with the 2- or 3-cycle, and at even higher frequencies is unstable. Curiously, as predicted by Vakakis, et al in their mathematical model, the robot's chaotic behavior seems to start to collapse back towards a 1-cycle at the highest amplitudes.

3.1.2 2-cycle

The 2-cycle was much more elusive than the 1-cycle. In this case, it much resembled the "limping" gait described by Raibert in his research, and also resembled the 2-cycle gait found by Vakakis in his mathematical model. It should be noted, however, that the two are distinctly different, with the robot exhibiting a more "symmetrical" step than Vakakis' model. This is probably due to the fact that Raibert's machines used pneumatic pistons, and as Vakakis took this into account with

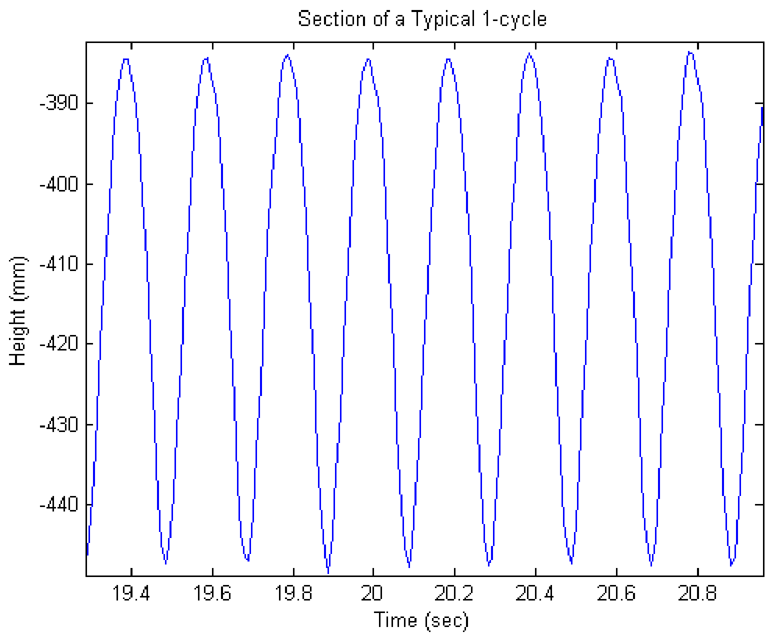


Figure 2: A typical 1-cycle

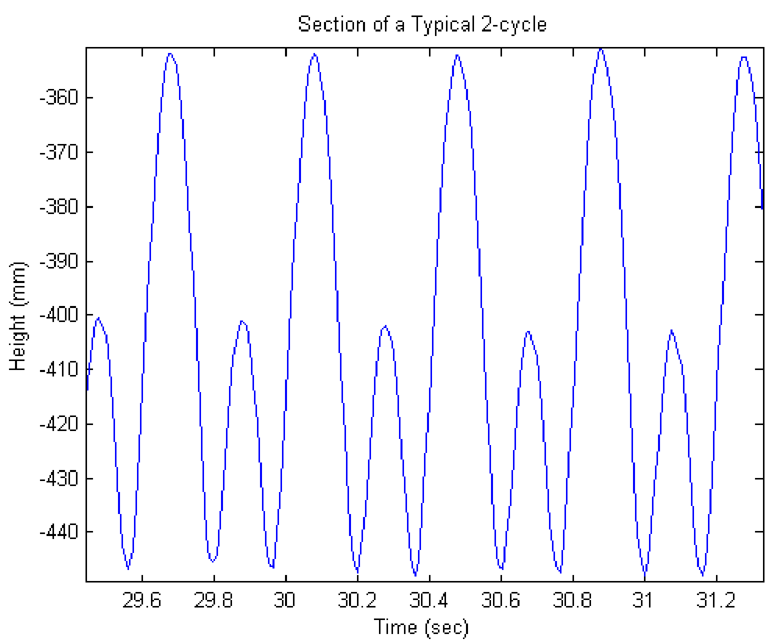


Figure 3: A typical 2-cycle

his model, the difference carried over. The most stable 2-cycle was achieved with a lower frequency, 5Hz. Beyond that frequency, it was always bistable with another gait, and below it, it did not appear.

3.1.3 3-cycle

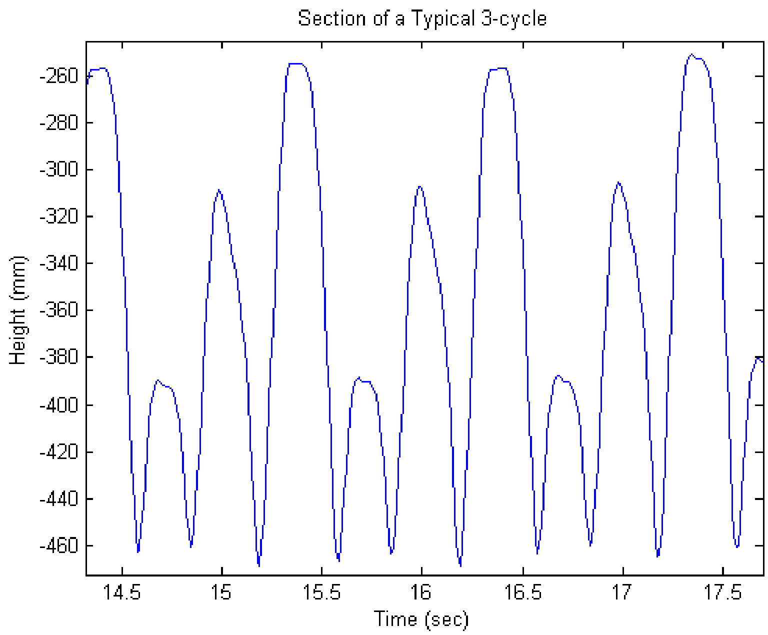
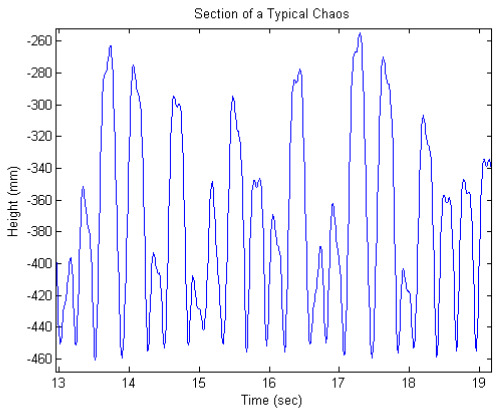
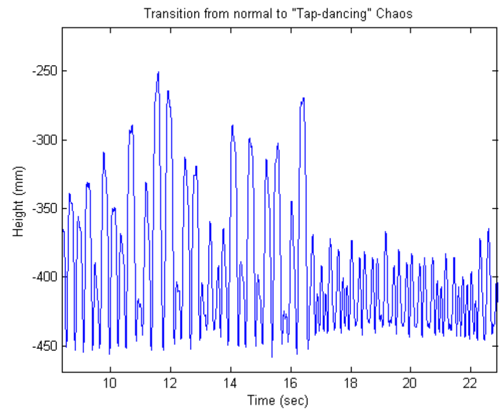


Figure 4: A typical 3-cycle

The 3-cycle dominated for much of the parameter space that was investigated. As mentioned previously, the 3-cycle was often the initial transient seen before stable behavior, and in those cases it was clearly unstable. In addition, a semistable 3-cycle sometimes appeared in the midst of chaotic motion, only to disappear just as quickly. The dominating factor in achieving a 3-cycle, stable or unstable, seemed to be jump height. Once the robot achieved a high enough jump height, a transition into the 3-cycle would take place, which would then either decay into a 1-cycle, 2-cycle, or chaos, or persist as a stable state. However, the 3-cycle was rarely ever stable on its own, instead co-existing with the 1- and 2-cycles, especially at bifurcations. When the 3-cycle appeared in the midst of a bifurcation, bi and even tristable behavior was possible. Video was taken (available at www.hoppingchaos.info) that clearly shows that through perturbing the robot's hopping it is possible to cycle back and forth between 1 and 3-cycle behavior.



(a) Normal Chaos



(b) Coexistence of normal and "Tap-dancing" Chaos

Figure 5: Chaotic Behavior

3.1.4 Chaos

Chaotic behavior in the system was rather interesting. When the system was aperiodic, it seemed to slip into a stable state for a few hops and then back to aperiodic behavior. In addition, there were two distinct kinds of chaos in the system. At lower frequencies, typical aperiodic motion as in Figure 5a was observed. But for higher frequencies, 7 and 8Hz, a sort of "tap-dancing" chaos was present. This chaos had only small amplitude jumps with very small durations, and would continue until the robot jumped so that it launched itself higher, which then propelled it into either the larger scale chaos or a more stable n-cycle. Figure 5b is an example of the "tap-dancing" transitioning into normal chaos is available, taken from a run with amplitude 1075cts and a frequency of 7Hz.

3.2 Orbital Maps

Using this data, orbital maps were constructed to better examine bifurcations. Only a selected number of points were taken from each run to be used in the creation of the orbit maps. This was done to avoid including transient behavior as well as trailing zeroes that were present in the data sets but not relevant to the experiment. Attempts were made to gather as many data points as were necessary to fully explicate the behavior of the system, and in fact resolution was doubled at 6Hz to more fully investigate the strange behavior of that system. Taking of data at higher frequencies was limited by concern for the safety of the apparatus; at some points the robot jumped high enough to collide with other objects around it.

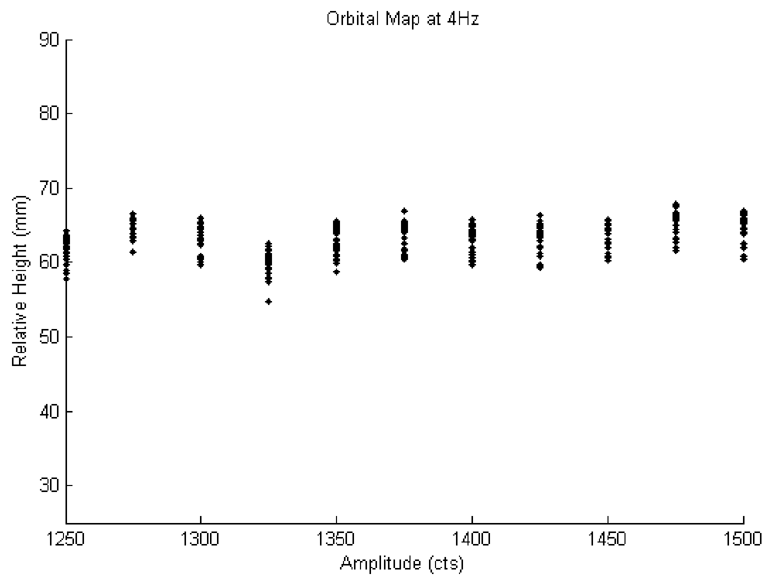


Figure 6: 4 Hz Orbital Map

3.2.1 Orbital Map of 4Hz

The orbital map for 4Hz is rather boring. The original goal was to map from amplitudes of 800 through 1500cts for all frequencies, but below an amplitude of 1250cts, the robot did not leave the ground, and no interesting behavior was observed. With this in mind, data acquisition began at an oscillation amplitude of 1250cts and continued in 25ct increments up to 1500cts. The robot exhibited only a 1-cycle gait at 4Hz, visually confirmed at amplitudes up to 1700cts. This 1-cycle was highly stable and the robot returned to a 1-cycle quickly after being perturbed.

3.2.2 Orbital Map of 5Hz

The sweep through oscillation amplitudes at 5Hz yielded the clearest map. Data was taken at 25ct osciallation amplitude intervals from 800 to 1500cts. Little chaotic behavior was observed, but a clear bifurcation from a 1-cycle to a 2-cycle is quite evident. Also interesting is the appearance of the 3-cycle near the point where the 1-cycle branches into the 2-cycle, with the only appearance of chaos at 5Hz not far behind. Further data was taken (not plotted on this graph) that clarified the amplitude of bifurcation at 1212 counts. Bistability was observed visually and video was taken of the robot being perturbed to switch between 1- and 3-cycle behavior.

3.2.3 Orbital Map of 6Hz

After the initial sweep through oscillation amplitude in intervals of 25cts, an interesting cascade was seen that was thought merited further investigation. It was decided to double the resolution

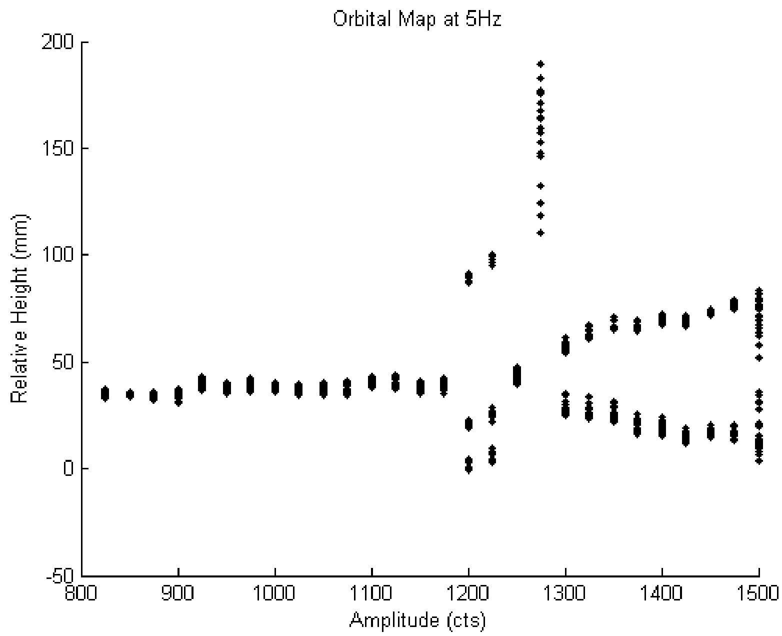


Figure 7: 5 Hz Orbital Map

of this sweep, taking data at a total of 53 different amplitudes. While the initial point is a 1-cycle, the next three points were 2-cycles, and then the next point was a 1-cycle again. This indicates a bistable state in this initial region. At 925cts, sporadic 3-cycles began to be observed, which usually indicated a bifurcation point. Indeed, the points started alternating between 2-cycle and chaos very soon afterwards, finally transitioning to complete chaos for the remainder of the map. Interestingly, the chaotic behavior seemed to reach a peak at around 1375cts, thereafter beginning to collapse. It is thought that this may be the same collapse observed in the model done by Vakakis et al.

3.2.4 Orbital Map of 7Hz

Though it was intended to map through the same parameter space as in the 5 and 6Hz maps, it was found that at an amplitude of 1200cts the rig exhibited violent behavior that could lead to damage to the actuator and other apparatus. Therefore it was decided to cease data acquisition at that point. The 7Hz map begins with a 3-cycle, moves to a clumsy 1-cycle, and then transitions to a 2-cycle for several points until rapidly cascading into chaos. The chaos collapse seen in the 6Hz map did not occur at this oscillation frequency, though this may be because it was impossible to progress an amplitude of 1200cts. A larger, more robust rig may be able to examine this further.

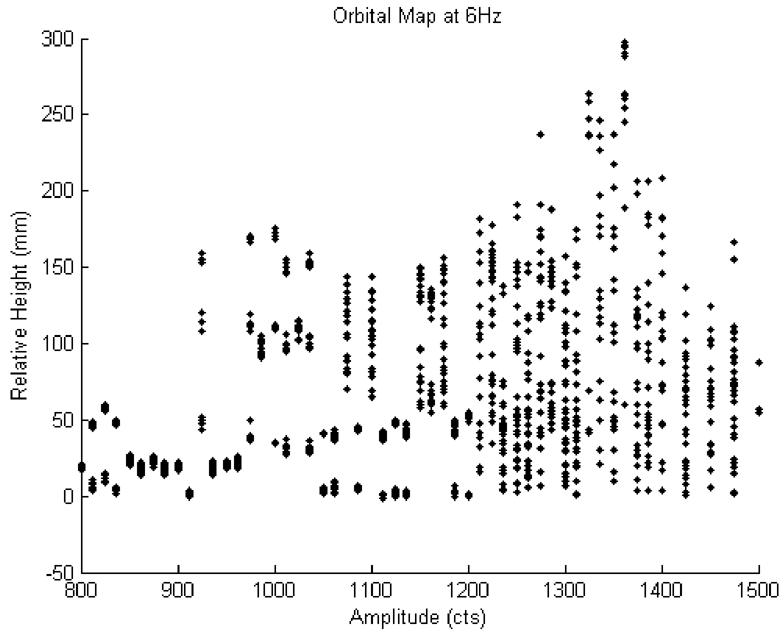


Figure 8: 6 Hz Orbital Map

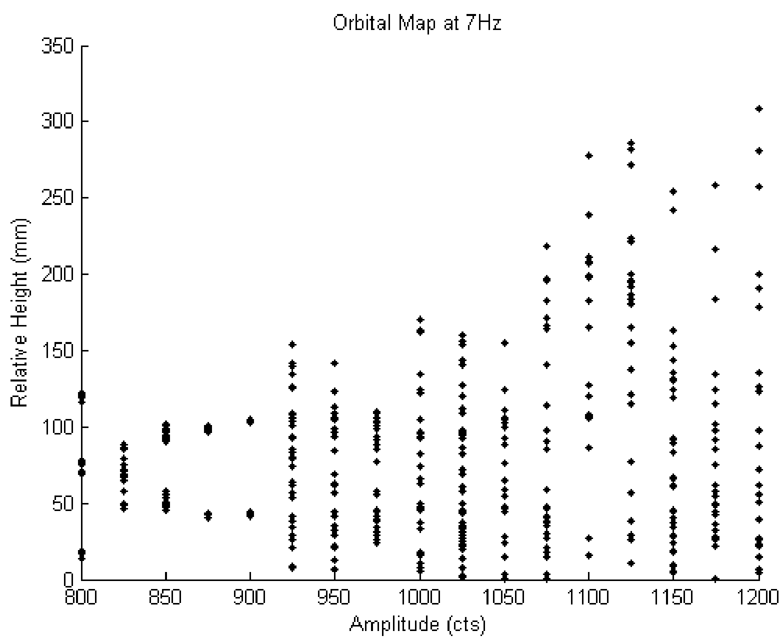


Figure 9: 7 Hz Orbital Map

3.2.5 Orbital Map of 8Hz

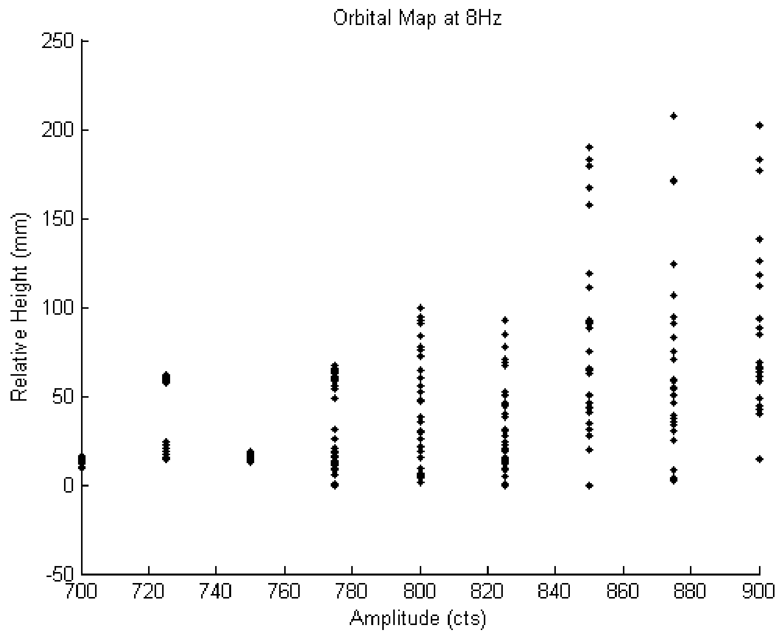


Figure 10: 8 Hz Orbital Map

At an oscillation frequency of 8Hz, even fewer data points could be taken without harming the apparatus. Data was only able to be taken data up to an oscillation amplitude of 900cts. To compensate and attempt to take more usable data, the lower bound was extended to an amplitude of 700cts. The 8Hz map moves more quickly into chaos than any of the other frequencies examined, and no chaos collapse was observed, presumably for the same reasons as the 7Hz map.

4 Discussion

The results found are interesting in their own right, but the presence of bi- and tristability is highly important and novel. In none of the articles describing mathematical models of this system were these mentioned, probably attributable to the fact that small amounts of noise in the system must be present in order for them to occur. Bistability is well known in optical phenomena, and has been described in some other mathematical models, but not in relation to a jumping robot. A 0-1 Test[GM09] for chaos for each of the frequencies with observed chaos was performed (Figure 10). This yielded a measure of how chaotic the system was for each oscillation amplitude at each frequency. As could be predicted, chaos was achieved at lower amplitudes with higher frequencies. More intriguing is the quantifiable evidence that chaos is collapsing at the high end of amplitudes measured for 6Hz. This adds further evidence in agreement with Vakakis' model.

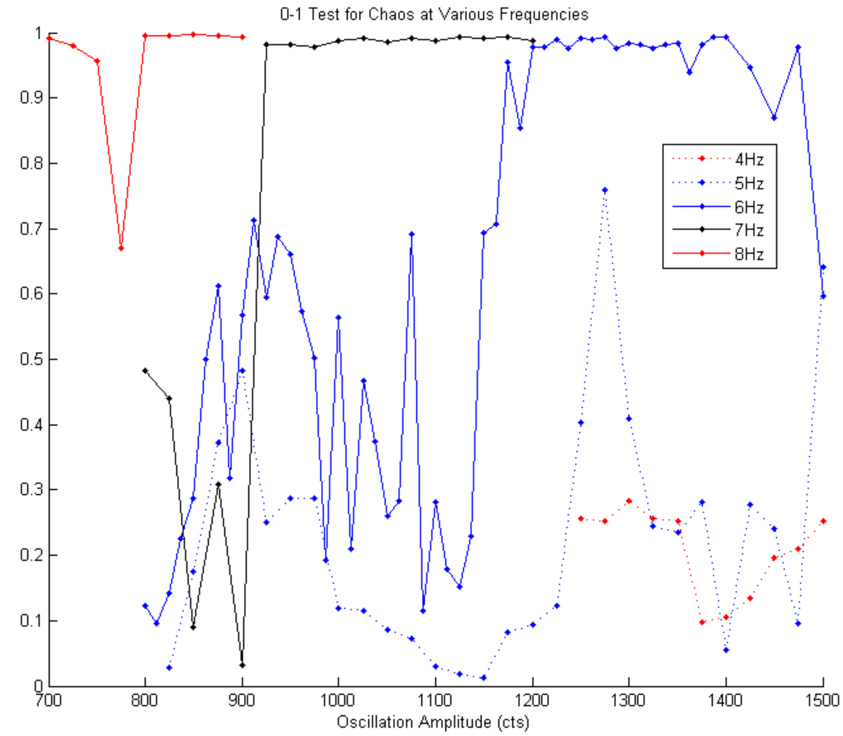


Figure 11: 0-1 Test for Chaos

Also potentially significant is the two distinct chaos types found in this system. The "tap-dancing" chaotic state and normal chaotic state are qualitatively different, and the ability to switch between the two and the conditions under this switching occurs merit further investigation. With more time to gather data as well as a rig with expanded capabilities, the entire problem can be examined in much greater detail.

5 Conclusions

Though the experimental results proved quite fruitful, it is important in any scientific paper to acknowledge the ways that the experiment could have been improved. In this investigation, the main issue encountered was with damping. All mathematical models of similar systems have simple damping models, and though the results show similarities to these systems, it is extremely difficult to produce a model that accurately mirrored this paper's results. Presumably, this was due to the fact that the model not only had spring damping, but also a damping contributed by the floor itself, as well as an aluminum plate placed between the robot's spring and the floor to prevent the robot from gouging out a hole. In addition, friction from the stabilizing pole introduced some force, and the actuator was sometimes unable to provide perfect sinusoidal motion due to being bashed as the rig hit the aluminum plating. All these contributed in some way to the motion of the system as a whole, and most can be corrected or mitigated in some way in order to reduce their influence on the system. Unfortunately, as only one week was available to work on the project, alleviating these problems came after the first priority of obtaining as much data as possible.

Despite these issues, the examination of the parameter space yielded some results that agreed with previous work, as well as others that were new and exciting. The hopping robot system invites much more in depth analysis, and there is much more to be studied about it. For example, what is the effect of forcing waveforms other than sinusoidal? A square or sawtooth wave may provide much more room for analysis, especially as a sawtooth more closely mirrors Raibert's hoppers, which used pneumatic cylinders rather than a linear actuator. Clearly, further investigation into these types of systems is desperately needed.

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