One Dimensional Faraday Waves

Juan, Orphee[1]

School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332, USA

(Dated: 16 December 2011)

Faraday waves for Newtonian and non-Newtonian fluids are studied inside an elongated rectangular container which forces the surface waves into a one dimensional stripe pattern. The surface waves show a subcritical pitchfork bifurcation for a range of forcing frequencies, 30-110Hz. Changes in the hysteresis patterns are observed when perturbations or non sinusoidal forcing functions are applied to the system, which may be due to the "tuning" and "detuning" of the natural frequency of the surface wave and the forcing frequency. The wavelengths and the critical accelerations of the Faraday waves as a function of the forcing frequency are in good agreement with the theory. And the maximum, saturated, amplitudes tend to behave inversely proportional the the forcing frequency, especially for higher frequencies.

I. INTRODUCTION

Faraday waves consist of a standing wave pattern that occurs in a free fluid surface undergoing oscillatory vertical acceleration. When the amplitude of the excitation exceeds a critical value, the standing waves form a pattern on the surface. The standing wave instability occurs at a sub-harmonic frequency i.e. one half that of the driving frequency. The patterns observed are approximately independent of the boundary conditions for large enough containers. The patterns range from squares, hexagons, triangular, and 8-fold quasi-periodic depending on the excitation frequency and fluid properties (viscosity, surface tension, and density). A map of patterns as a function of forcing frequency from Binks et al.[1] is shown in Figure I for illustration. Even though this phenomenon has been studied since its discovery by Faraday in 1831 Ref.[2], a full understanding of the subject has not yet emerged. This study will look at only one dimensional patterns, stripes, and compare results for Newtonian, non-Newtonian working fluids, as well as the effects of forcing frequency and shape of forcing signal, such as triangular and square wave inputs.

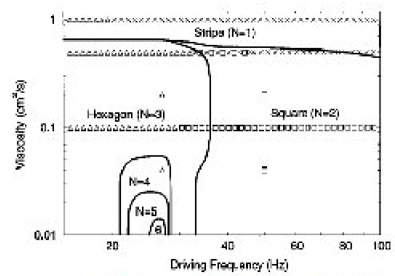

FIG. 2. Selected patterns as a function of the viscosity of the fluid and the angular frequency of the driving acceleration. The symbols represent experimental results: × — stripe pattern, ⊔ — square pattern, and △ — hexagonal pattern. Alternating × and ⊔ indicate mixed-stripe-square patterns.

FIG. 1. Surface wave patterns as s function of viscosity and forcing frequency, from[1].

II. BACKGROUND

Recently, Zhang and Vinals[3] have derived and equation that describe Faraday wave amplitudes in the weakly damped and infinite depth limit from the Navier-Stokes equations, and further extended for non-weakly damped case by Chen and Vinals[4], shown below:

$$\frac{dA}{dt} = \alpha A - g_0 A^3 - \sum_{m=1} g(\theta_{m1}) A_m^2 A, \quad (1)$$

where θ_{m1} is the angle between the wave vectors for multiple waves from different directions. And A is the wave amplitude and α, g_0 and $g(\theta)$ are coefficients to be determined. The above equations is in agreement with a phenomenologically derived amplitude equation, also known as the complex Ginzburg-Landau equation (CGLE):

$$\tau \frac{\partial A(x,t)}{\partial t} = \epsilon A + \xi_o^2 \frac{\partial^2 A}{\partial x^2} - g|A|^2 A \quad (2)$$

Where here, again, A is the amplitude, and τ, ξ and go are coefficients that depend on the physical system studied. However, the second term in the right hand side may be eliminated by a rescaling of the amplitude $A = \overline{A} e^{i\Delta x}$, so this term will not play a role in the dynamics, reducing the amplitude equation to:

$$\tau \frac{d\overline{A(x,t)}}{dt} = \epsilon \overline{A} - g|\overline{A}^2| \overline{A} \quad (3)$$

For the case of the Faraday waves, τ is the characteristic time of linear damping, ϵ is the reduced acceleration, and go is the non-linear dissipation coefficient. The reduced acceleration is given by $\epsilon = (ac - a)/ac$, where a is the acceleration provided by the shaking base(input forcing function) and ac is the critical acceleration at which the surface waves start to emerge. The simplified amplitude equation represents a supercritical pitchfork bifurcation for $g > 0$. For a subcritical bifurcation, the amplitude equation is usually written with a higher order term which moderates the growth rate of the amplitude, as first proposed by Penney and Price[5], and dropping the overbar from now on, $\overline{A} = A$, the amplitude equation becomes:

$$\tau \frac{dA(x,t)}{dt} = \epsilon A - gA^2 A - kA^5 \quad (4)$$

Where the coefficients $g < 0$ and $k > 0$ correspond to the subcritical case. For a subcritical bifurcation, a hysteresis effect is found. This hysteresis and the fact that the amplitude growth rate is not as large as in the supercritical case, can be attributed to a detuning effect as explained by Douady[6]. This detuning results from the nonlinear dispersion relation $\acute{\omega_o} = \omega_o + \beta|A|^2$. where $\acute{\omega_o}$ is the actual frequency of the Faraday wave and β is a negative detuning coefficient. And ω_o is the nominal Faraday wave response frequency, usually $\omega_o \sim 1/2\omega$ since it is a subharmonic response, where $\omega/(2\pi)$ is the forcing frequency. The detuning coefficient, β, is negative because, usually, the frequency of an oscillator decreases with an increase in amplitude. Thus, the response of the Faraday wave in terms of its amplitude depends on whether its natural frequency $\acute{\omega_o}$ is close to being resonant with the forcing frequency. When the amplitude is zero, $\omega_o = \acute{\omega_o}$ and the system is resonant and thus unstable as the forcing acceleration is increased. However, once the amplitude is non zero, the system departs from resonance, and a limiting growth in the amplitude, saturation, occurs, represented by the fifth order term in (3). As the acceleration decreases to zero, the non-zero amplitude still causes a detuning and the system traces a different path than when the acceleration is increased. Therefore, for a subcritical behaviour, the amplitude is expected to behave as shown in Figure II. The critical acceleration at which the Faraday waves begin for form when ramping up the forcing amplitude from rest is here referred as "critical acceleration up," corresponding to $r = 0$ in Figure II. While due to the hysterises, when the forcing amplitude is decreased, the critical acceleration at which the standing wave amplitude decreases to zero is different, and smaller than the ramping up case, and will be referred here as "critical acceleration down," $r = r_s$ in Figure II. The critical acceleration is a parameter of interest, and as shown by Douady[6], the critical acceleration, both up and down, will increase with forcing frequency, caused by an increase in dissipation of energy at the boundaries, (walls and bottom). So, smaller containers and shallower fluid layers should experience a higher critical acceleration. For a one dimensional wave, stripe pattern, there is a maximum amplitude for a given frequency at which if the forcing acceleration is increased the wave transitions into a modulating wave and then the stripe pattern becomes unsteady. Once the Faraday waves are formed, their wavelength λ is, to a good approximation, given by the inviscid deep fluid dispersion relation: $(\Omega/2)^2 = tanh(kh)[gk + (\sigma/\rho)k^3]$, where $\Omega/2\pi = f$ and f is the forcing frequency, while here k is the wave number $k = 2\pi/\lambda$, and h is the height of the fluid. As shown by Binks et al.[1], for shallow fluid height on the order of few millimetres, the wavelength is expected to be, approximately, inversely proportional to the forcing frequency $\lambda \sim 1/f$.

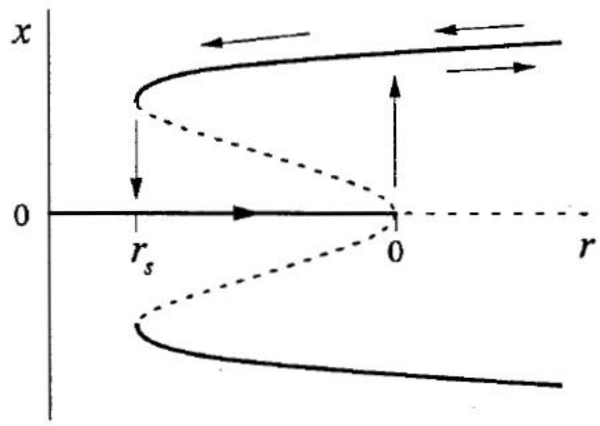

FIG. 2. Subcritical behavior represented by $\dot{x} = \epsilon x - gx^3 - kx^5$.

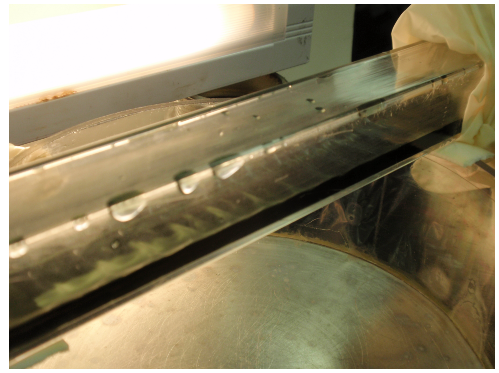

FIG. 3. Photograph of the container used with surface waves.

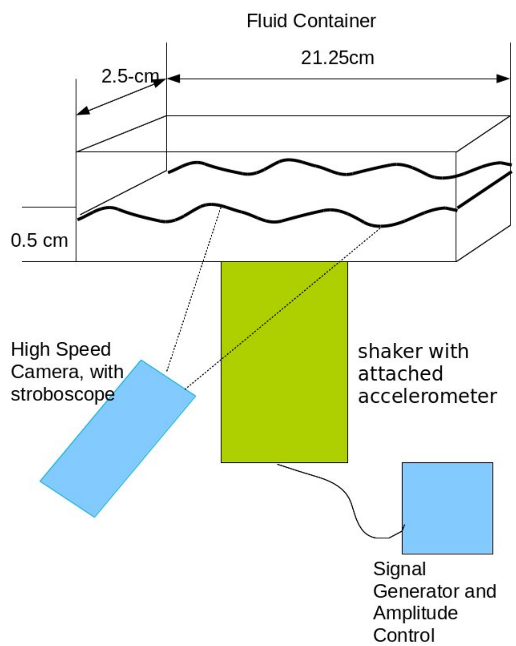

FIG. 4. Schematic of the experimental .

III. METHODOLOGY

A long, rectangular, container is used as shown in Figure III and III to constrain the surface waves to a one dimensional pattern, stripes. A fluid depth of 5mm is used. And a shaking base is connected to a signal generator and an amplifier that provides the forcing accelerations at the base of the container. A high speed camera is placed at an approximately perpendicular angle to the container's longitudinal side. The camera is stroboscopic and can be set to capture the sub harmonic frequency of the standing waves. The image gathered is post processed to extract the surface wave profile. And the spectral analysis of the surface profile provides the wave number, its corresponding wavelength, and amplitude intensity. Water is used with a black colourant to provide more reflection of the incident light to be able to capture the surface waves more distinctively. Also, a non-Newtonian fluid is used composed of a mixture of water and cornstarch of approximately 2.5 volume fraction.

IV. RESULTS AND DISCUSSION

Water is excited at 30Hz, and the forcing acceleration (the base shaking amplitude) is increased continuously from zero up to a maximum value, referred as the "ramp-up" in Figure IV. As the forcing acceleration increases, a critical acceleration is reached, (critical acceleration "up"), at which the Faraday waves start to form. At this critical acceleration "up" the wave amplitude exhibits a rapid increase with increasing forcing acceleration. If the forcing acceleration is increased further, the amplitude of the Faraday wave keeps increasing until the pattern becomes unstable. At this point, the input acceleration is decreased, "ramp-down", and the amplitude decreases back to zero. However, the "ramp-up" path and the "ramp-down" path are different, showing a hysteresis behaviour, as discussed in section II. This hysteresis is consistent with a subcritical bifurcation, as depicted by the curve fit in Figure IV. The hysteresis and amplitude saturation arise due to a detuning effect of the natural frequency of the Faraday wave with respect to the excitation frequency. This detuning is caused by the fact that the natural frequency of the Faraday wave is a nonlinear function of its amplitude. Therefore, the natural frequency will be different for the case in which the amplitude is zero, acceleration increasing "ramp-up" path from rest, than when the amplitude is non zero, corresponding to a decrease in acceleration "ramp-down" path. The different natural frequencies means that the wave will be close or far from being in resonance with the forcing frequency. As the critical acceleration up is reached, the wave frequency response is in close proximity to being resonant; and thus, very unstable, with any perturbation causing the wave to form and increase in amplitude rapidly. However, as the amplitude grows, the natural frequency of the wave shifts away from resonance and the amplitude diminishes its growth rate, it saturates. This saturation is model by including a fifth order term

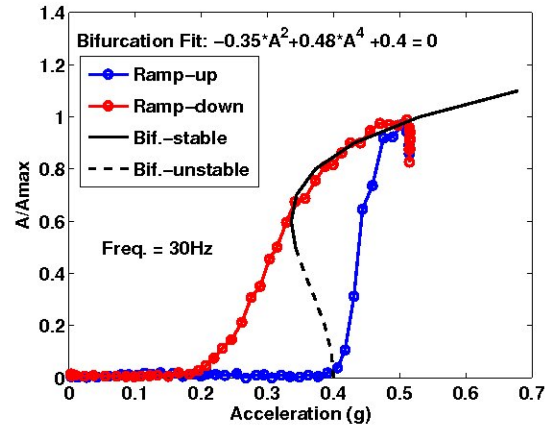

FIG. 5. Normalized amplitude (amplitude/maximum amplitude) for water excited at 30Hz, for "ramp-up" path and "ramp-down" path, and corresponding bifurcation curve fit of the type: $0 = \epsilon - gx^2 - kx^4$.

in the amplitude equation, which balances the growth of the cubic term. Similarly, the subcritical behaviour can be observed for a range of excitation frequencies 30-110Hz, as shown in Figure IV, but the actual bifurcation shape will change for each frequency. For higher frequencies, the amplitudes and wavelengths become smaller. And for the highest frequency tested, 110Hz, the amplitudes are much smaller and the pattern becomes unstable. The bifurcations have been fitted with equation (4), and the fit coefficients are shown in Table IV. As predicted by the theory for a subcritical bifurcation, $g < 0$ and $k > 0$. Where k is the coefficient of the fifth order term, which is stabilizing and responsible for modelling amplitude saturation. And $g < 0$, corresponds to the cubic, de-stabilizing, term. Also, in order to explore the consistency of the particular bifurcation for each frequency, the experiment is repeated 5 times for each frequency, Figure IV. The general bifurcation shape is repeated, and thus there is a unique response of the fluid for a given excitation frequency. Also, there are approximate bounds for the critical accelerations "up" and "down".

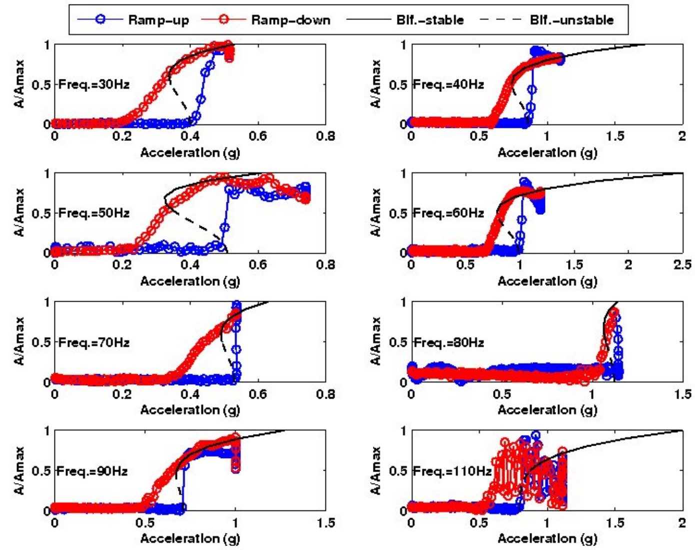

FIG. 6. Normalized wave amplitude (amplitude/maximum amplitude) for water excited a 30-110Hz

Frequency (Hz)	g	k
30	-0.35	0.48
40	-0.94	1.8
50	-0.84	0.94
60	-1.6	3.1
70	-0.23	0.33
80	-0.25	0.27
90	-0.38	0.94
110	-0.006	1.2

Similarly, for the non-Newtonian fluid, the bifurcation behaviour is examined for a range of frequencies and compared with water, Figure IV. For the non-Newtonian fluid, the critical accelerations required are much higher due

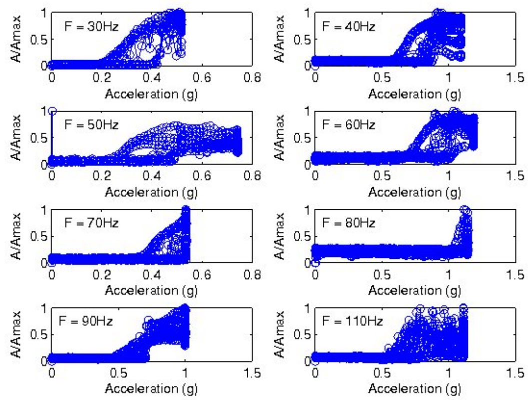

FIG. 7. Normalized wave amplitude (amplitude/maximum amplitude) for water excited a 30-110Hz

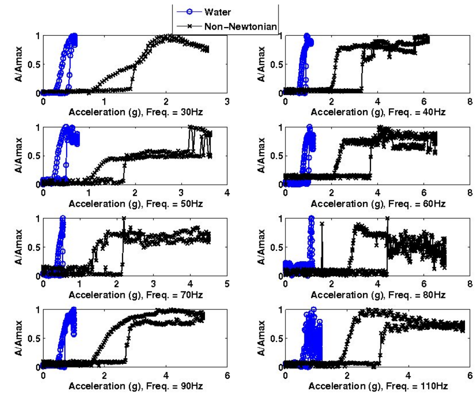

FIG. 8. Normalized wave amplitude (amplitude/maximum amplitude) for Newtonian (water) and Non-Newtonian (mixture of water and cornstarch) excited a 30-110Hz

to a higher viscous dissipation, higher viscosity value as compared with water. Also, the shape of the bifurcation and its hysteresis are quite different as compared to water. For non-Newtonian fluids, the pattern may continue to exist for higher forcing accelerations, but its amplitude response shows a strong saturation, remaining almost flat, or in some cases even decreasing with increasing forcing acceleration. Since for a subcritical bifurcation, the system becomes unstable as the acceleration gets closer to the critical "acceleration up," a perturbation experiment is conducted, Figure 9. In this experiment, the container is physically tapped continuously as the acceleration is increased. As expected, the Faraday waves form before the critical acceleration up of the undisturbed system is reached. Also, as the acceleration is ramped down, the amplitude for the disturbed system reaches a critical "acceleration down" that is lower than the undisturbed system. This latter effect is surprising and may be due to the introduction of noise in the system which may widen the forcing frequency bandwidth, and increases the chances for the natural frequency of the Faraday wave to be closer to resonance with the forcing frequency, thus attenuating the amplitude decay with decreasing acceleration. This supposition is further reinforced by the next test, which compares a triangular wave input, with the base case, a sinusoidal input, Figure 10. The triangular wave in spectral space is composed on infinite number of wave numbers, and thus again this widens the forcing frequency spectrum; thus,

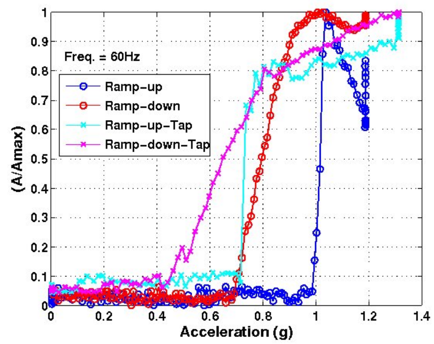

FIG. 9. Normalized wave amplitude (amplitude/maximum amplitude) for water excited a 60Hz for undisturbed system, and disturbed system ("Tap").

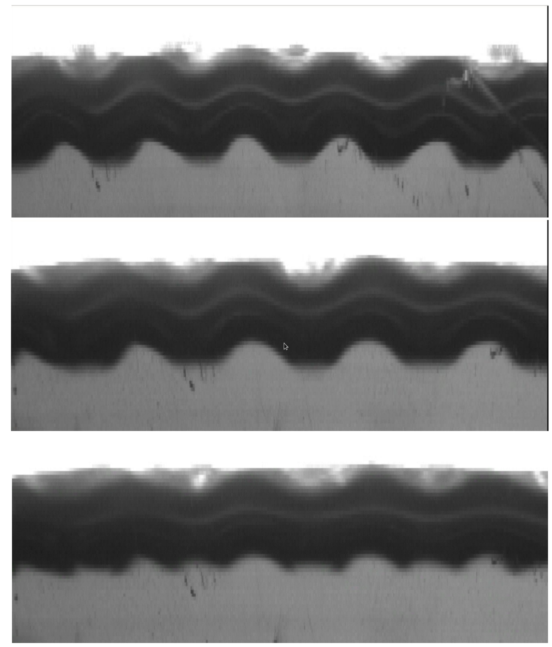

FIG. 10. Faraday wave pattern for water excited a 60Hz for top to bottom: sinusoidal,triangular, and square forcing input wave form

increasing the chances that the natural frequency of the Faraday wave is tuned to some of the forcing wave numbers, and the amplitude decreases at a lower critical acceleration down. This same reasoning applies for the increasing amplitude case, which explains why the critical acceleration up happens before than for the sinusoidal input. Furthermore, square wave inputs have been explored, but resulted in a wave pattern containing multiple wave numbers, Figure 10, and amplitude measurements become ambiguous. The critical acceleration up and down as a function of frequency are shown for water and the cornstarch mixture in Figure 11. As explained in the II section, the critical acceleration increases with increasing frequency and viscosity due to the increase in dissipation at the boundaries of the container. And since in this experiment the viscosity of the non-Newtonian fluid is much higher than water, much higher critical accelerations are required to form a pattern. The wavelength as a function of frequency for water and cornstarch mixture is also explored, Figure 12, and compared with the theoretical approximation of $\lambda \sim 1/f$ for shallow fluid depth. The results are in overall agreement with the theory, curve fits shown, for Newtonian and Non-Newtonian fluid. As mentioned, there is a maximum, saturated, amplitude for each forcing frequency, as shown in Figure 13. While to the author there is no explicit theory that predicts the behaviour of the saturated

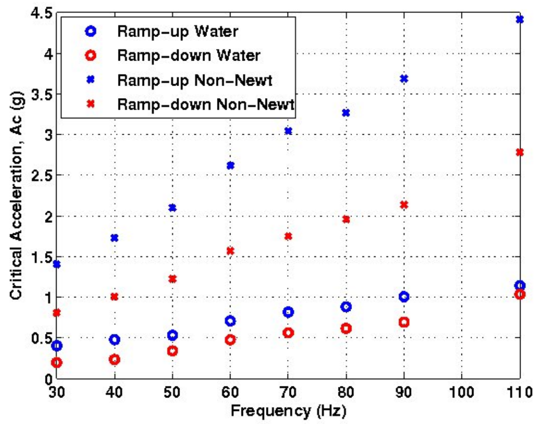

FIG. 11. Critical acceleration for the ramp-up and ramp-down paths, for Newtonian (water) and non-Newtoninan (cornstarch mixture)

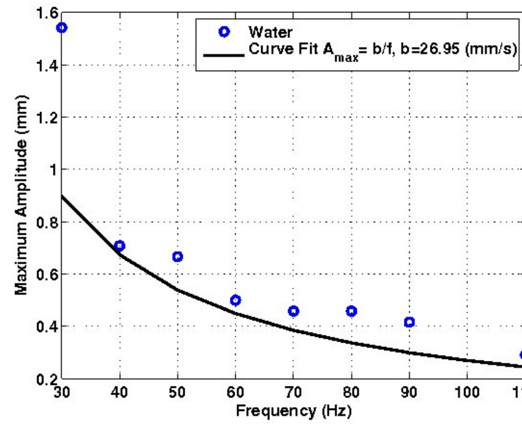

FIG. 13. Maxiumu wave amplitude as a function of forcing frequency for Newtonian (water) fluid

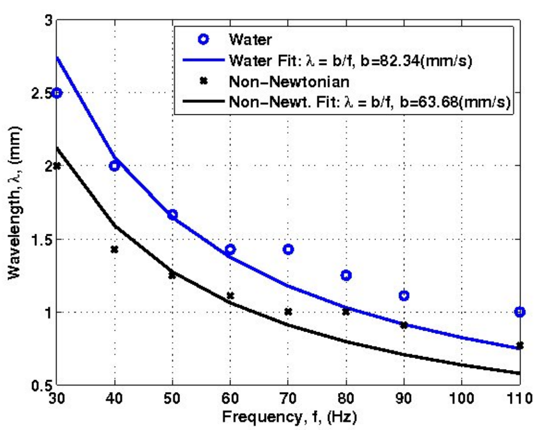

FIG. 12. Wavelength as a function of forcing frequency for Newtonian (water) and non-Newtonian (cornstarch mixture)

amplitude versus forcing frequency, it seems that for larger frequencies $A_{max} \sim 1/f$. However, at 30Hz the maximum amplitude seems not to fit this approximation well, being surprisingly, about twice the amplitude of the 40Hz case, Figures 13 and 14. Finally, the bifurcation departure of our experiment are compared in a qualitative manner with results from Wernet et al.[7], since experimental set-up and working fluids are different. Wernet et al. explored an amplitude departure for a working fluid with 10 times the viscosity of water and the container used was carefully designed to avoid meniscus induced waves, "soft-beach" boundaries. Wernet et al. also tried to model the bifurcation without a fifth order saturating term; however, his work does not indicate whether this set-up induces a hysteresis behaviour. For the 60Hz, Wernet et al. shows a rapid increase of amplitude that resembles the sharp departure at "ramp-up" exhibited by water, Figure 15. And at 80Hz, the results from Weret et al. show a slower departure from zero than

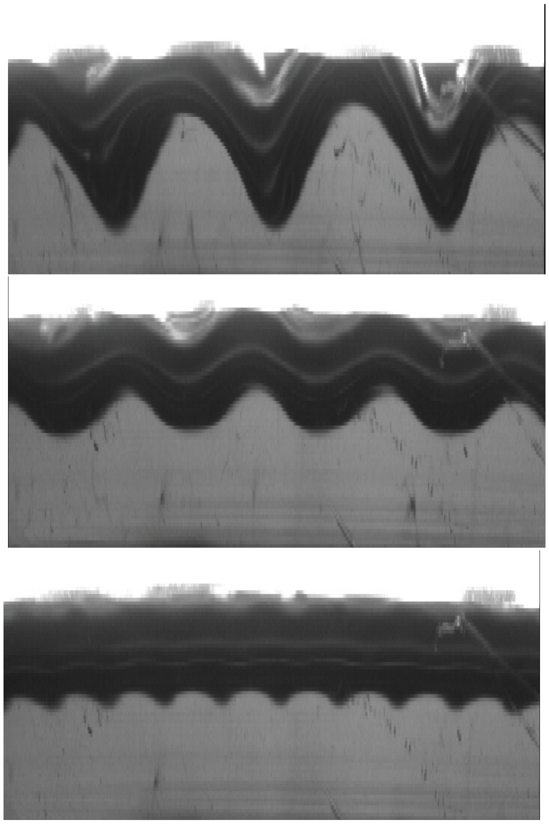

FIG. 14. Faraday waves at maximum amplitude for water excited at (from top to bottom):30Hz, 40Hz, and 110Hz

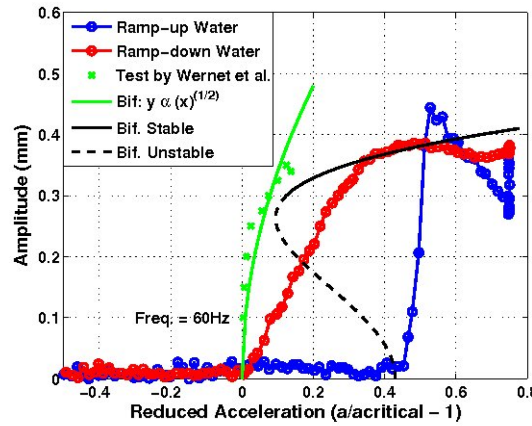

FIG. 15. Amplitude as a function of reduced acceleration $a/ac - 1$ for water, and Wernet et al. working fluid for 60Hz excitation frequency

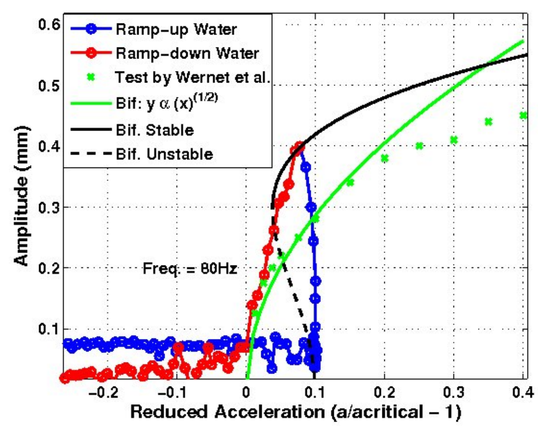

FIG. 16. Amplitude as a function of reduced acceleration $a/ac - 1$ for water, and Wernet et al. working fluid for 80Hz excitation frequency

water, but the cubic term amplitude departure (represented by the square root curve fit) seems not to agree well the amplitude saturation, which appears better captured with a fifth order term, shown by the subcritical bifurcation curve fit, Figure 16.

V. CONCLUSION

The subcritical bifurcation behaviour of the Faraday waves has been explored for a range of forcing frequencies, input wave forms, and Newtonian and non-Newtonian working fluids. It is observed that different excitation frequencies determine a unique shape for the bifurcation behaviour for a given fluid. Critical accelerations for the ramp-up and ramp-down paths increase for increasing frequency, confirming previous observations for Newtonian fluid, which are here also extended for non-Newtonian fluids. Furthermore, the hysteresis process may be altered by introducing perturbations or utilizing other wave input forms which broaden the bandwidth of the excitation frequency, allowing for the Faraday wave to remain close to resonance for a wider range of forcing frequencies. Also, the wavelength of the Faraday waves decreases with frequency in agreement with the inviscid dispersion relation approximation. Finally, the maximum, saturated, amplitudes seem to decrease approximately inversely proportional to the excitation frequency, especially for larger excitation frequencies.

REFERENCES

[1] D. Binks, M. Westra, and W. van de Water, "Effect of depth on the pattern formation of faraday waves," Physical review letters **79**, 5010–5013 (1997).

[2] M. Faraday, Philos. Trans. R. Soc. London , 319 (1831).

[3] W. Zhang and J. Vinals, "Square patterns and quasipatterns in weakly damped faraday waves," Physical Review E **53**, 4283–4286 (1996).

[4] P. Chen and J. Viñals, "Pattern selection in faraday waves," Physical review letters **79**, 2670–2673 (1997).

[5] W. Penney and A. Price, "Part ii. finite periodic stationary gravity waves in a perfect liquid," Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences **244**, 254–284 (1952).

[6] S. Douady, "Experimental study of the faraday instability," J. Fluid Mech **221**, 383–409 (1990).

[7] A. Wernet, C. Wagner, D. Papathanassiou, H. Müller, and K. Knorr, "Amplitude measurements of faraday waves," Physical Review E **63**, 036305 (2001).