Analysis of a Hopping Robot

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Abstract

The dynamics of a one-legged jumping robot was studied. The robot was restricted so that its motion was only one dimensional. The robot consisted of an actuator driving a rod with a spring attached to the end. The actuator provided a sinusoidal driving force, and the amplitude and frequency of this driving force was varied to analyze its effect on the system. The robot experienced 1-cycles, 2-cycles, 3-cycles, and chaotic motion while under the driving force. The robot had many similarities with other one-legged hopping robots, but demonstrated some unique characteristics not seen in similar setups.

1 Introduction

Considerable progress has been made towards legged transportation. Marc Raibert of the MIT “leg lab” built a one-legged hopping robot that was able to maintain stability when moving around using fairly simple methods. The robot used feedback from the limb to determine how much force it should apply to maintain itself [1]. Most of the research on one-legged transportation has been done using this original robot Raibert developed. One example is the model done by Vakakis et al where they demonstrated that the system would have a period doubling cascade into chaos followed a reverse cascade back to a 1-cycle and that the strange attractor for the robot could actually vanish under certain conditions [2].

Legged transportation provides several advantages over the usual wheeled or tracked transportation being used currently. Wheels are useful for travel on prepared surfaces, such as roads or rails, but have difficulty traveling over terrain that is soft or uneven. This severely limits the areas where wheeled vehicles can travel; they can currently only travel on about half of the Earth’s landmass, while animals are capable of traveling over a much larger portion. The goal is to develop legged transportation so that vehicles can be capable of traveling everywhere that animals can [3]. Legged travel works better over tough terrain because it allows for separate footholds that can be chosen based on the support and traction offered, while a wheel has to travel over all the difficult terrain. Legs also inherently provide an active suspension that allows the body to travel smoothly, regardless of the terrain. It also provides the vehicle with the option to step over obstacles [3]. This ability to travel over more types of terrain than wheeled transportation has led to legged transportation being suggested for Martian and lunar exploration [4].

2 Experimental Setup and Methods

The jumping robot is comprised of a Copley Controls Linear Actuator applying a driving force to a “pogo stick”. The “pogo stick” consists of a short rod with a rigid spring attached to the end of it, and the robot is restricted to vertical motion by a guide pole. The actuator produced a sinusoidal motion on the pogo stick, giving the robot a sinusoidal forcing and causing it to hop. The amplitude and frequency of the sinusoidal motion were varied and the resulting dynamics of the system studied.

Originally the motion of the robot was to be recorded using a high bandwidth accelerometer, but difficulties with tangled wires and maintaining the apparatus for longer runs caused us to instead turn to a motion capture system. The video was taken at 100fps and tracked the motion of a small wooden ball placed on top of the pogo stick.

While the setup could produce excellent granularity for the amplitude, difficulty with the software caused the actuator to only be able to produce waves with integer frequencies. This led us to fix the frequency and then run a sweep through a range of amplitudes. We were limited on the range of amplitudes because if it was too low the robot would not make it off of the ground, and if it was too high then we ran the risk of the robot damaging itself and the surrounding equipment. We took these restraints into account and chose an appropriate range of amplitudes to run through. This sweep was done for each of the integer frequencies from 4Hz to 8Hz. The robot was allowed to run for 30-40 seconds in each run, unless it demonstrated interesting behavior that warranted more time. A MatLab script was then used to capture the peaks on each run which were then plotted on an orbit diagram for each frequency.

Unfortunately, the setup used some nonstandard units that we were unable to relate to any corresponding SI units. The software for the actuator used a measurement called “counts” for its position on the pogo stick, and the motion capture measured the movement of the ball in the number of pixels it traversed in the pictures the camera took. However, the nature of our analysis is such many of the results are qualitative and units were not very important for what numerical analysis was done.

There were other difficulties with the apparatus. The actuator attempted to produce perfect sine waves, but it was unable to when the robot would impact the ground. Instead it would produce sine waves with tapered peaks. The robot also had a tendency to rotate around the guide pole as it hopped, which the motion capture program registered as slight changes in height. These changes were not significant compared to the actual heights the robot reached, so it only introduced a small amount of noise into the measurements. The robot also was strong enough to gouge the floor, so we were forced to put an aluminum plate on the floor for the robot to bounce on. This likely provided some complex dampening into the system that we were unable to find a model for.

A picture of the group with the apparatus and videos of the robot running are available at the website http://www.hoppingrobotchaos.info/index.html.

3 Results and Analysis

3.1 Overview

The robot demonstrated chaos along with 1-, 2-, and 3-cycles (Fig 1). While there were only stable 1-, 2-, and 3-cycles for the data we took, higher order cycles probably exist for the system. We were most likely unable to obtain them because of the many sources of noise in the system and the frailty of these cycles. The system usually experienced a transient behavior of 3-cycles before settling into its long term behavior. Therefore, by Sharkovsky's Theorem [5], the existence of these 3-cycles implies that our system must experience chaos.

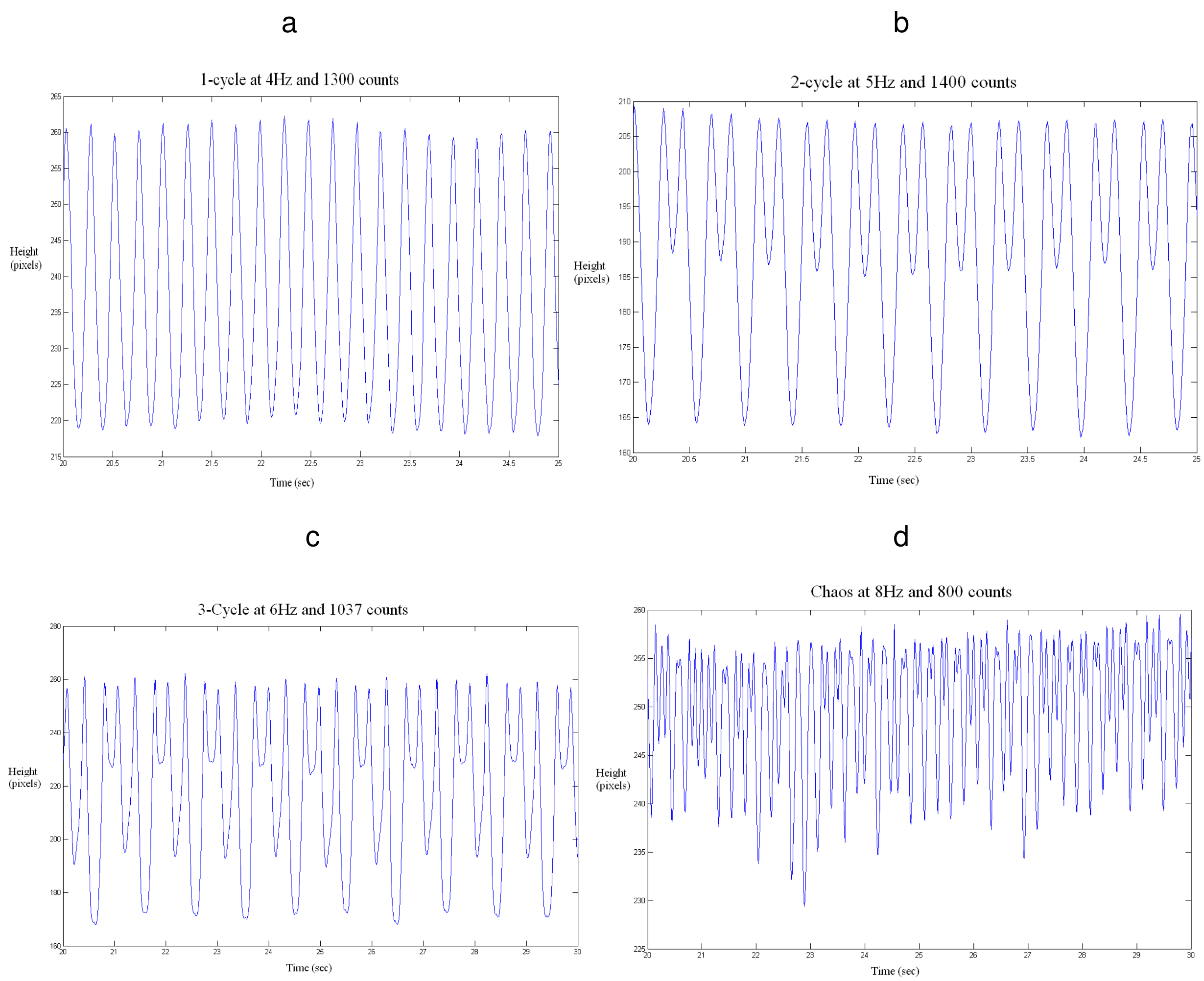

Figure 1

Fig 1a, b, c, and d are examples of a 1-cycle, 2-cycle, 3-cycle, and chaos respectively. Due to the setup of the camera for the motion capture, the graphs are “inverted” from the actual motion of the robot. The top of the graphs are where the robot hits the ground and the bottom peaks are where the robot is actually airborne. The ground level seems to shift during these runs. This is caused by the robot rotating around the pole and the camera registering this horizontal motion as a slight change in height.

3.2 Orbit Diagrams

An orbit diagram was made for the data at each frequency. When making the diagrams, only the peaks of the last fourth of the run were plotted so that the transient behavior would not be included. The runs are done at increments of 25 counts for the amplitude, with the exception of 6Hz, where twice the resolution was done once the system demonstrated an interesting cascade.

3.2.1 4Hz Orbit Diagram

Figure 2: 4Hz orbital diagram

Again because of the way the camera was setup, the y-axis is “inverted”. Lower values for the height correspond to the robot jumping higher. This is true for all of the orbit diagrams. These are 1-cycles; the spread of the points is caused by the robot rotating around the guide pole which the camera registered as a slight change in height.

The measurements start at a higher amplitude of 1250 counts for 4Hz (before this the robot would not make it off of the ground), and are done up to an amplitude of 1500 counts. Only 1-cycles were seen at 4Hz. An additional run was done at 1700 counts to insure the system still only experienced 1-cycles at higher amplitudes. We also investigated the stability of these 1-cycles and discovered they are stable at least up to small perturbations. The robot was perturbed multiple times during a run at 1700 counts and each time the robot returned the original 1-cycle. The system still experienced transient behavior of 3-cycles before finally settling into the stable 1-cycles.

3.2.2 5Hz Orbit Diagram

For 5Hz, the runs began at 800 counts and ran up to 1500 counts. The 5Hz orbit diagrams shows a standard period doubling bifurcation from a 1-cycle to a 2-cycle. This bifurcation is different than in the usual cascade though because there are 3-cycles at 1200 counts and 1225 counts, near the end of the 1-cycle region, and a chaotic trajectory at 1275 counts, near the beginning of the 2-cycle region. The system never reached a region of purely chaos in the range of amplitudes taken. The system also experienced an interesting bi-stability where there were 3-cycles. The chaotic motion at 1275 counts in the 2-cycle region still demonstrated a local behavior similar to that of a 2-cycle.

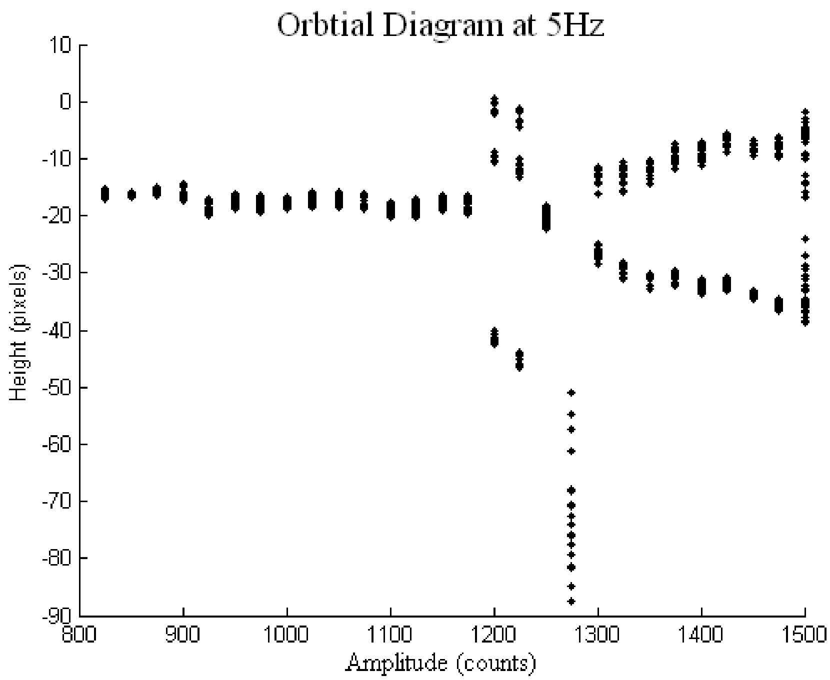

Figure 3: 5Hz orbital diagram

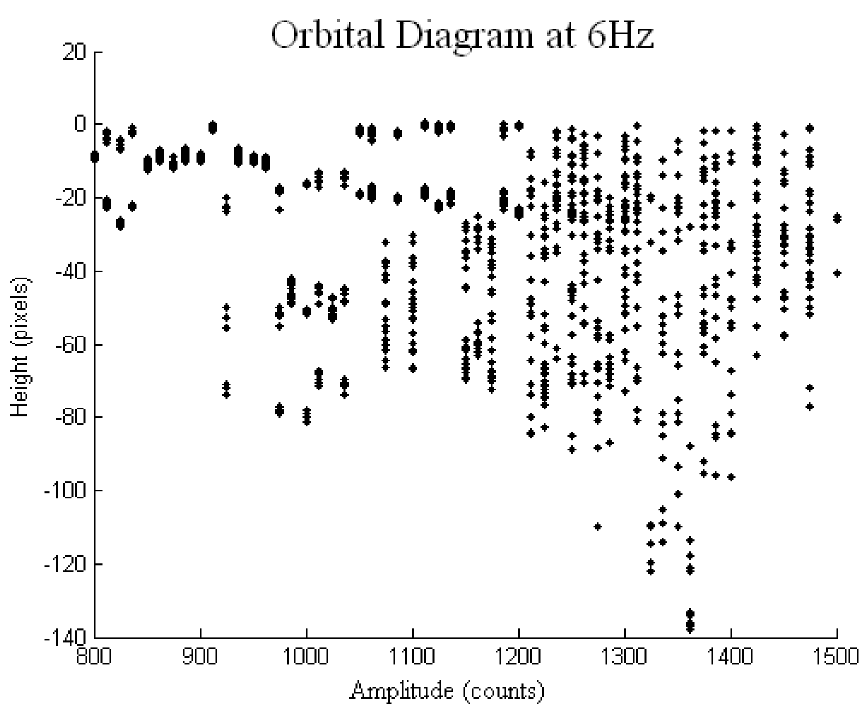

Figure 4: 6Hz orbital diagram

3.2.3 6Hz Orbit Diagram

The sweep for 6Hz was done at twice the resolution of the other frequencies. This was done because the initial run through using increments of 25 counts demonstrated an interesting cascade into chaos. For 6Hz, there were initially 2-cycles at 800 counts. This was followed by a region of 1-cycles before it transitioned back into 2-cycles and eventually chaos at around 1212 counts. Similar to 5Hz, inside the region of 1-cycles there were 3-cycles and inside the second region of 2-cycles there appear to be of chaotic trajectories. Both the chaos and the 3-cycles have higher maxima than the normal 1-cycles and 2-cycles and would occur if the robot's hops achieved some minimum height. The chaotic behavior qualitatively appears to peak at around 1375 counts, after which the chaotic trajectories begin to have shorter hops. This could be an indication of the reverse cascade in the model done by Vakakis et al.

3.2.4 7Hz Orbit Diagram

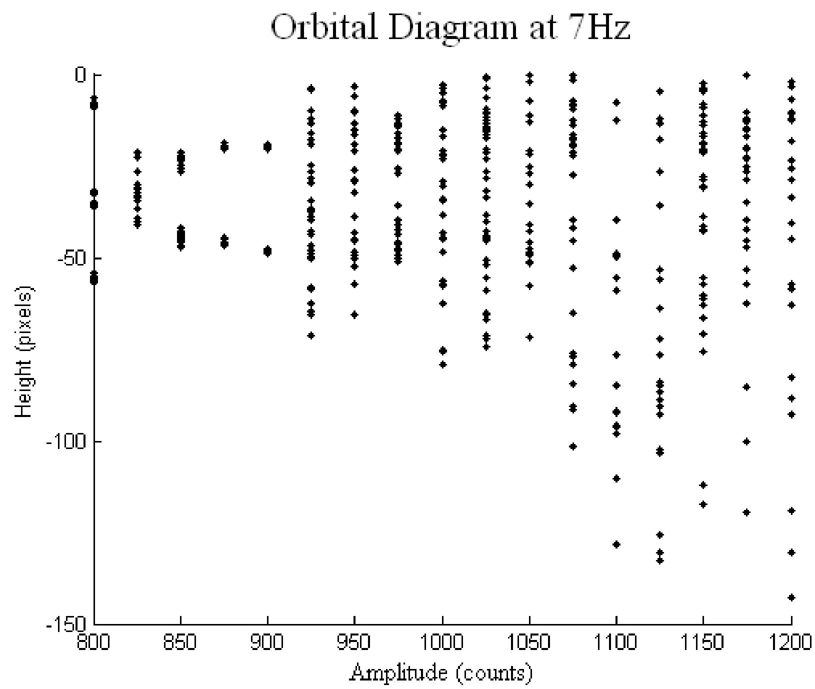

Figure 5: 7Hz orbital diagram

For 7Hz, there initially was a 3-cycle at 800 counts, followed by 2-cycles before it becomes chaotic at an amplitude of 950 counts. There is also a trajectory which appears chaotic in the transition between the 3-cycle and the 2-cycles. This trajectory is similar to the chaotic trajectories in the 2-cycle region for 5Hz and 6Hz in that it experiences local 2-cycle behavior. The data is taken only up to 1200 counts for 7Hz because the robot began to jump high enough that it was at risk of damaging the actuator and other equipment. The collapse of the chaos that was seen at 6Hz did not occur in the range taken for 7Hz, though this is likely due to the shortened range of measurements.

3.2.5 8Hz Orbit Diagram

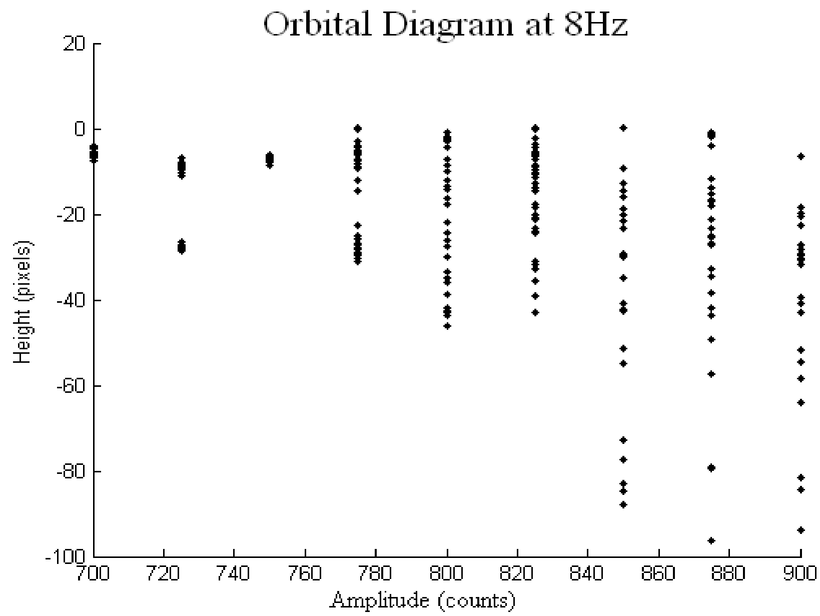

Figure 6: 8Hz orbital diagram

For 8Hz, we started at a lower amplitude of only 700 counts because the apparatus quickly began to jump too high. Looking at the orbit diagram for 8Hz, you will see that the system initially had two 1-cycles with a 2-cycle in between before the system became chaotic at an amplitude of 775 counts, much sooner than for 6Hz and 7Hz. The cascade into chaos occurs much quicker for 8Hz than it does for 6Hz and 7Hz and at it occurs at lower amplitudes.

3.3 Additional Observations

The robot demonstrated an interesting “bi-stability” in the regions where there were both 1-cycles and 3-cycles. Both the 1-cycles and 3-cycles are stable for the system, and the system would switch from the 1-cycle to the 3-cycle if the robot ever made it to some critical height dependent on the amplitude and frequency of the driving force. Further investigation was done into this for 5Hz at 1212 counts amplitude. This amplitude and frequency were chosen because the robot could settle into either the 1-cycle or the 3-cycle for this driving force, and appeared to have no preference. We were able to get the robot to transition back and forth between the two cycles by perturbing it during its run (video of this is available at the website).

The systems behavior in the chaotic regions was also interesting. The system appeared to experience two types of chaotic behavior; “normal” and “tap dancing” (Fig 7). During the “tap dancing”, the system would make many quick and short hops, while in the “normal” chaotic state it would jump to heights on the same order as it did during cycles. The “tap dancing” only occurred for the higher frequencies, 7Hz and 8Hz, with it being most prominent for 8Hz where the system would switch between the normal chaos and the “tap dancing” chaos at random intervals.

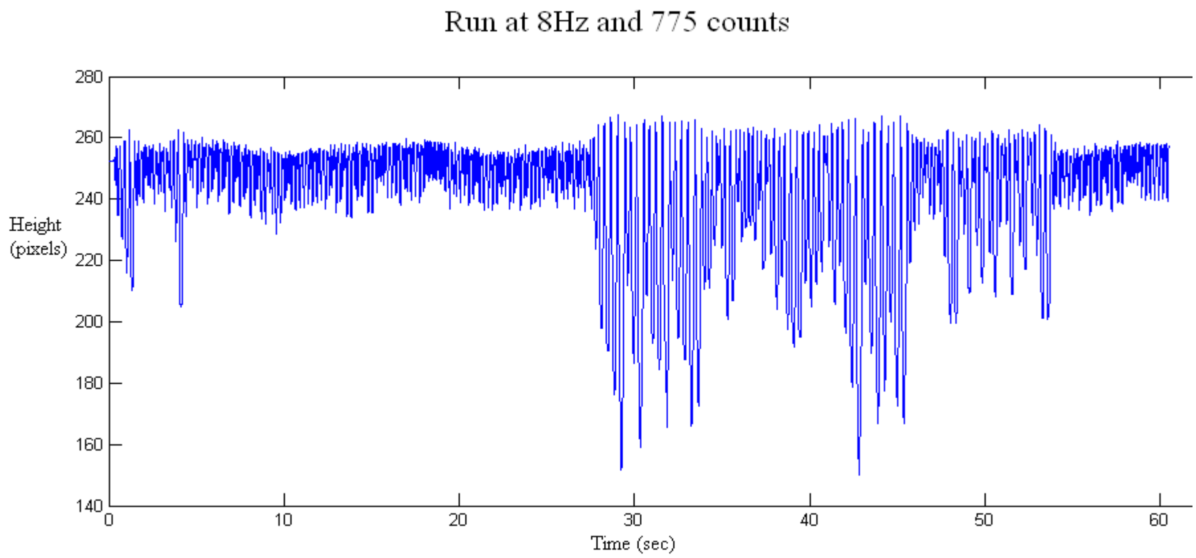

Figure 7

Fig 7 is an example of the robot switching between the "tap dance" and "normal" chaos. The run was done at 8Hz and 775 counts. The robot is initially in the tap dance state, changes to the normal chaos at around 28 sec, changes back to the tap dance state for about a second at 47 sec, and is in the tap dance state from 54 sec till the end.

The chaotic motion that was seen inside the 2-cycle region for 5Hz, 6Hz, and 7Hz was also different than the normal chaotic behavior (Fig 8). The system would still demonstrate some short term 2-cycle behavior, it would always have a large hop followed by a short hop, but the height of the successive large and small hops were both aperiodic.

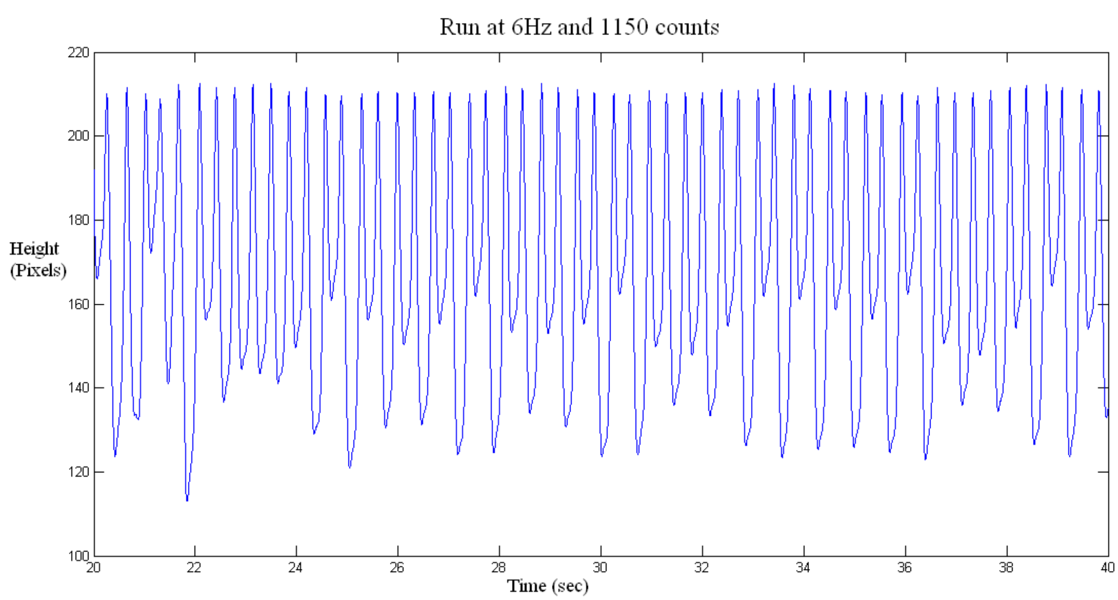

Figure 8

Fig 8 is a segment of the run at 6Hz and 1150 counts. It is an example of the chaotic trajectories in the 2-cycle region where the overall motion of the system is aperiodic, but locally there is a 2-cycle pattern.

A 0-1 test for chaos [6] was done an each run. This test is used to check whether a time series is experiencing chaotic motion. It yields a value of 0 for when the motion of the system is regular and a value of 1 when the motion of the system is deterministically chaotic. This provides us with a measure of how "chaotic" the system was for each run. As expected, the chaos begins at lower amplitudes the higher the frequency is, and the value begins to decrease at the end for 6Hz. This provides quantitative evidence that the chaos begins to contract for 6Hz as in a reverse cascade for high enough amplitudes.

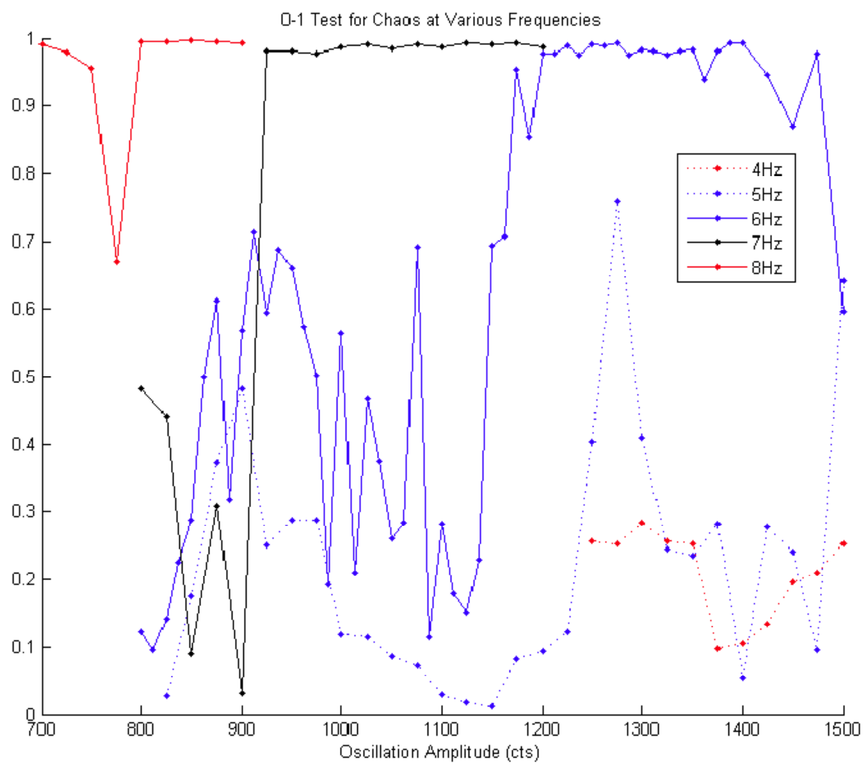

Figure 9: The 0-1 test for chaos

As the frequency is decreased from 8Hz to 4Hz, the amount of chaotic trajectories the system experienced decreases until there is no chaotic motion at 4Hz. This resembles the loss of the strange attractor seen in Vakais' model. The 0-1 test also provides quantitative evidence for this since the values for the test are smaller for 5Hz and 4Hz than the others.

4 Discussion

The bi-stability is something that has not been seen for Raibert's hopping robot and appears to be unique to this system. The system even appeared to have the possibility of tri-stability occurring, though we were unable to fully study this. The robot did have results similar to Vakakis' model for Raibert's robot. In Vakakis' model, the robot would undergo the standard period doubling cascade into chaos, but would then do a reverse cascade from chaos back into a 1-cycle. The size of this cascade would also change depending on the parameters, and would actually not occur below a certain threshold (when the strange attractor disappeared). The system appeared to show the beginning of a reverse cascade at 6Hz. The size of the cascade also appeared to depend on the frequency and did not even occur when the frequency was low enough at 4Hz. Our system does not fit Vakakis' model in a critical area though, Vakakis' analysis used the fact that each minimum of the robot's motion depended only on the minimum before it. Our robot has a sinusoidal driving force that is dependent on time which causes successive extrema of the robot to have a time dependence. Despite this fundamental difference, our robot still exhibited a lot of the interesting behaviors found in their model.

5 Conclusion

The jumping robot demonstrated a lot of interesting behavior, i.e. the period doubling cascade and its disappearance as the frequency decreased and the bi-stability. However, there are many rooms for improvement on this experiment. We were unable to match any simple damping model to the system. This was likely caused by the fact that our robot not only experienced damping from the spring, but also damping from the floor and the aluminum plate it was hopping on. The attempts to model Raibert's robot all used simple models for the dampening of the system. The actuator was also unable to produce perfect sinusoidal motion when the robot impacted the ground, and there was friction between the pole and the actuator. All of these affected the motion of the robot and their impact could be reduced with a more sophisticated device.

There are opportunities for further study of this simple hopping system. We only had a week to work with the robot, so we were unable to solve all the problems we encountered with the robot and take all the measurements we originally intended. We were unable to investigate what happens when the amplitude is fixed and the frequency is allowed to vary because the software used only supported integer frequencies. Being able to vary the frequency would provide a better insight into when the system is no longer able to produce chaos at low frequencies and when interesting states such as the tap dancing begin. Further investigation into whether the chaos actually collapses at higher amplitudes for frequencies with a cascade would also be beneficial. One could also investigate how the system behaves when the driving force is something other than sinusoidal, such as a square wave or triangle wave.

References

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