Team Metronome

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Synchronization

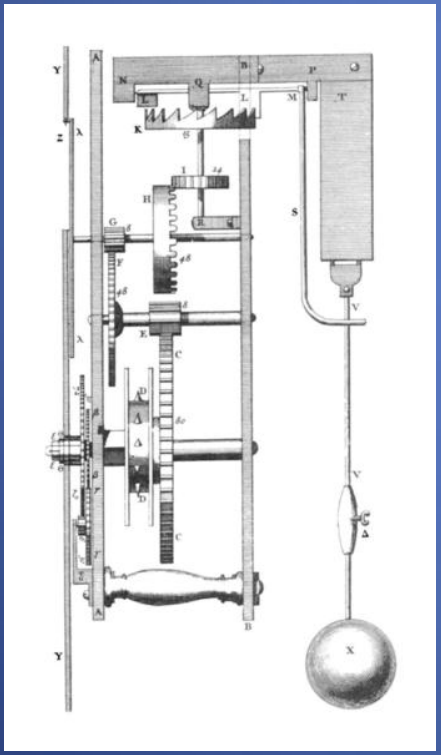
- Fundamental in nonlinear phenomena
- Commonly observed to occur between oscillators
- Synchronization of periodic cicada emergences
- Synchronization of clapping in audiences
- Josephson junctions

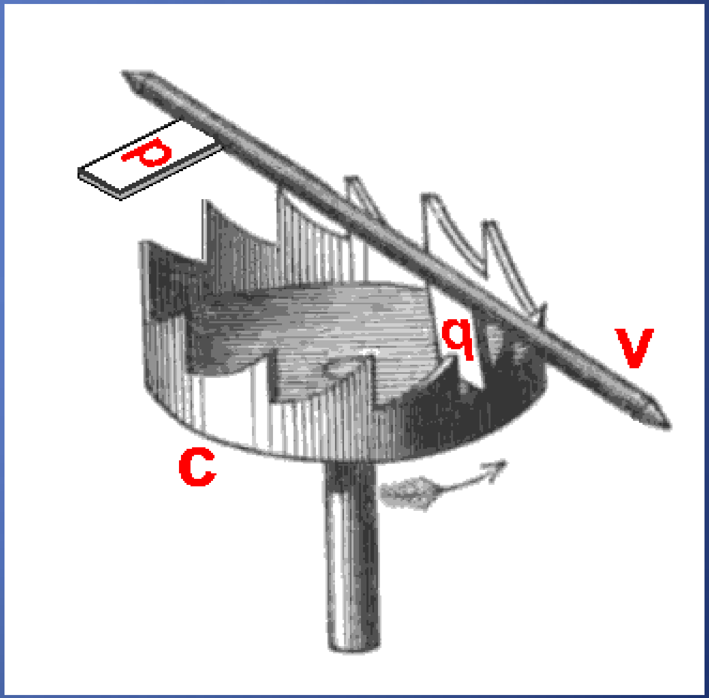
Pendulum Clock

- 1657 Huygens builds first pendulum clock
- Most accurate clock of the day (accurate to within 10 mins/day)
- Built to solve the longitude problem
- “An odd sort of sympathy”
- Observed anti-phase locking

Escapement

- Verge escapements vs. modern escapements





Escapement

- verge escapement required a large amplitude and light bob (hence inaccurate)
- provides for the nonlinearity of the system

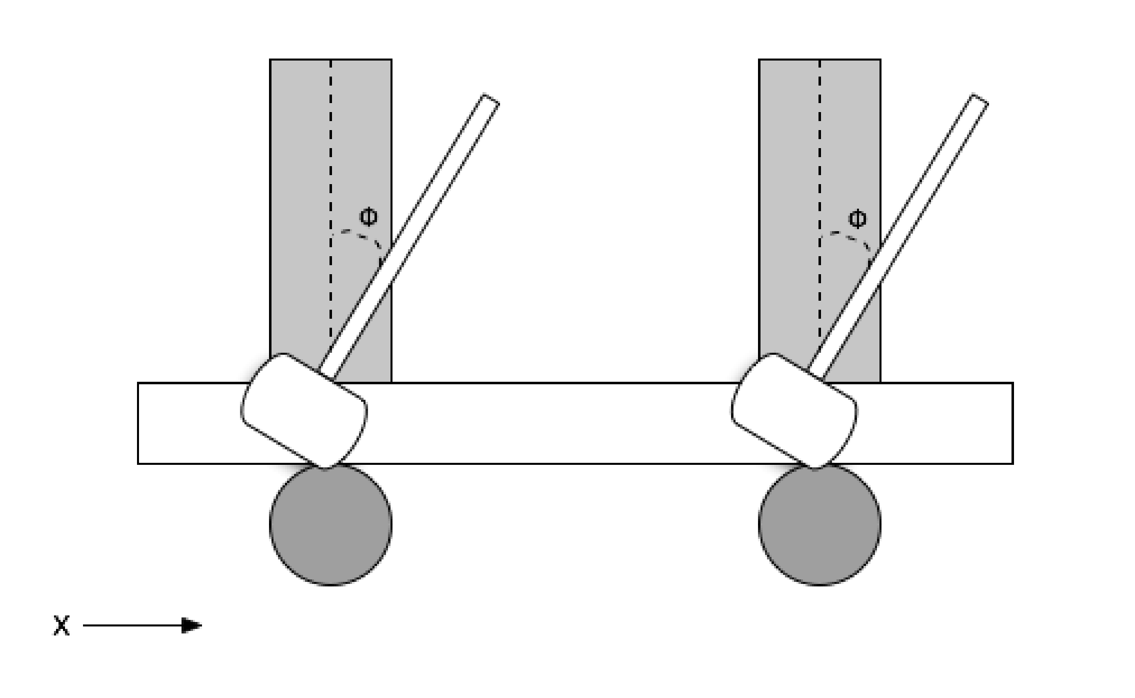
Explanation

- Huygens first thought the synchronization was caused by air currents between the pendulum bobs
- After many experiments he concluded the synchronization was cause by imperceptible vibrations in the wall
- Ultimately Huygens’ clock was unsuited for solving the longitude problem due to its sensitivity

Modeling The System

- Kortweg - Linear mode analysis
- Blekhman - Van der Pol Oscillators
- Pantaleone - Method of averaging
- this model shows that large oscillation of the pendulum destabilizes the anti-phase synchronization
- anti-phase synchronization can be produced through adding significant damping

System



l = pendulum length
g = gravity
M = total mass - pendulum masses
m = pendulum mass

Equations of Motion

$$\mathcal{L} = \tfrac{1}{2}(M+2m)\dot{X}^2 + m\dot{X}\ell(\cos\phi_1\dot{\phi}_1 + \cos\phi_2\dot{\phi}_2) + \tfrac{1}{2}m\ell^2(\dot{\phi}_1^2 + \dot{\phi}_2^2)$$
$$+ mg\ell(\cos\phi_1 + \cos\phi_2) - \tfrac{1}{2}KX^2,$$

$$\ddot{\phi}_k + b\dot{\phi}_k + \frac{g}{\ell}\sin\phi_k = -\frac{1}{\ell}\ddot{X}\cos\phi_k + \tilde{f}_k,$$

$$(M+2m)\ddot{X} + B\dot{X} + KX = -\sum_j m\ell(\sin\phi_k)^{\cdot\cdot},$$

Phenomena

- phase locking
  - favored by the escapement
- anti-phase locking
  - favored by the damping force
- beating
- beating death
  - caused by the limit of the escapement

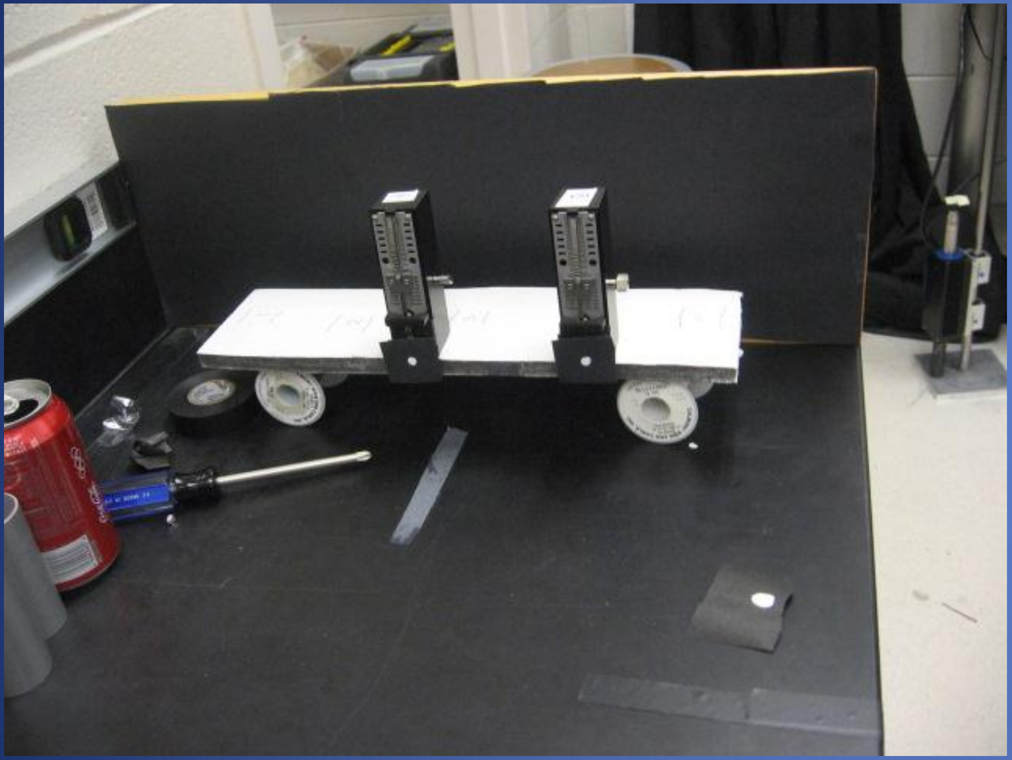
Parameters

- Most important parameter is the coupling strength
- Coupling strength depends on the mass ratio of the pendulum masses to the total mass
- Coupling strength can be varied by simply adding mass to the platform

Synchronization of Metronomes

Experimental Procedures and Observations

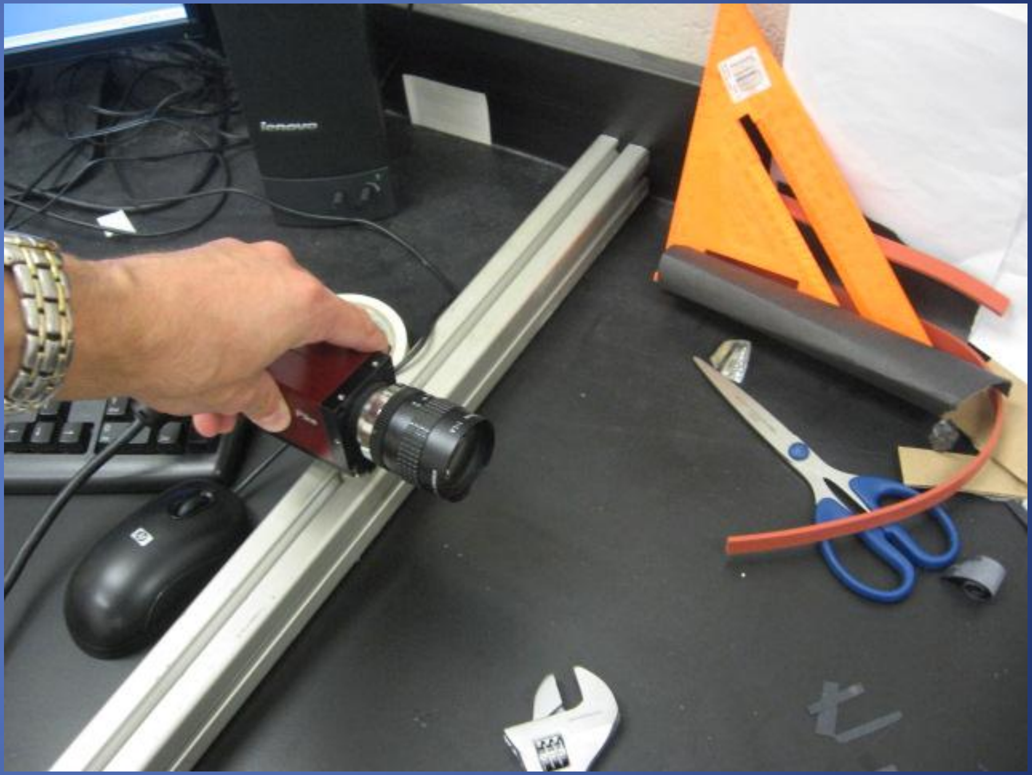
Board and metronomes



- Metronomes were mounted on a foam board. This board rested on two rollers. Initially we used cans, but they proved unreliable so we switched to spools.

Camera and tracking

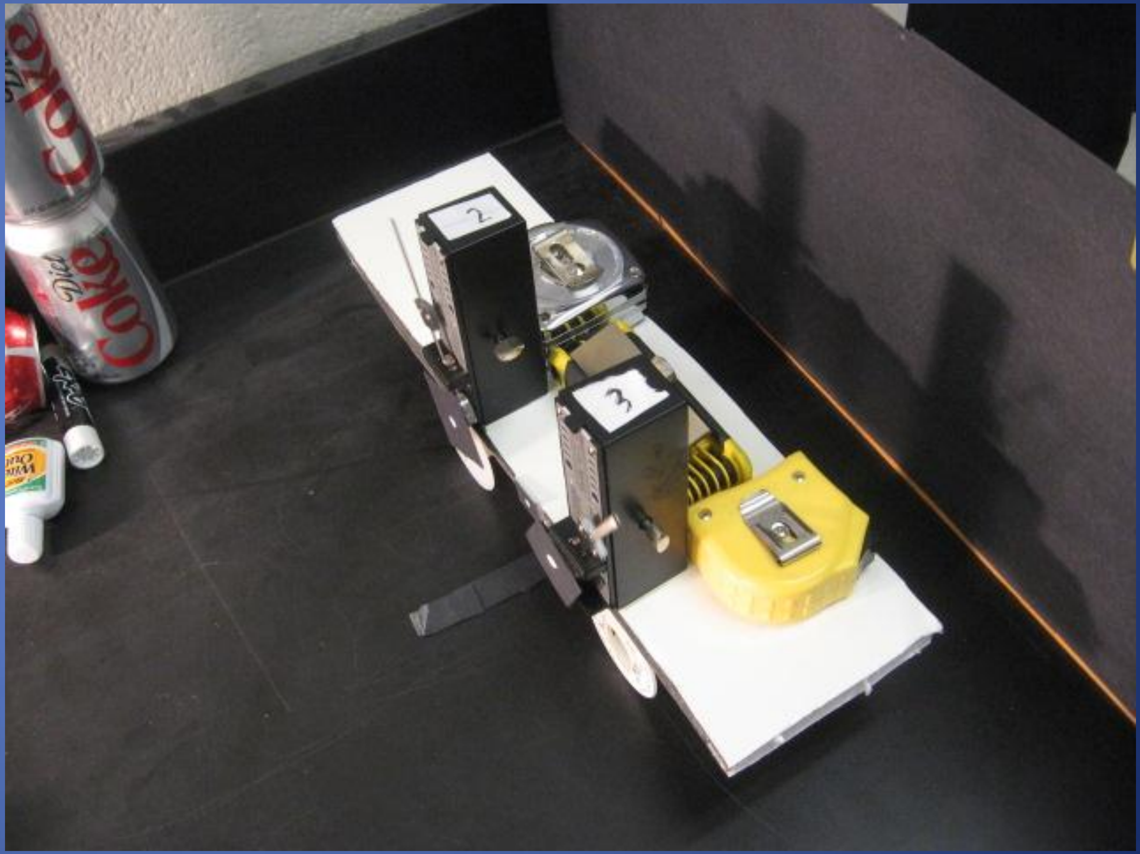
- We used a camera to track white dots on each metronome and the board.
- Lab view took the video from the camera and created tables of x and y position over time.



2 metronomes (demo)

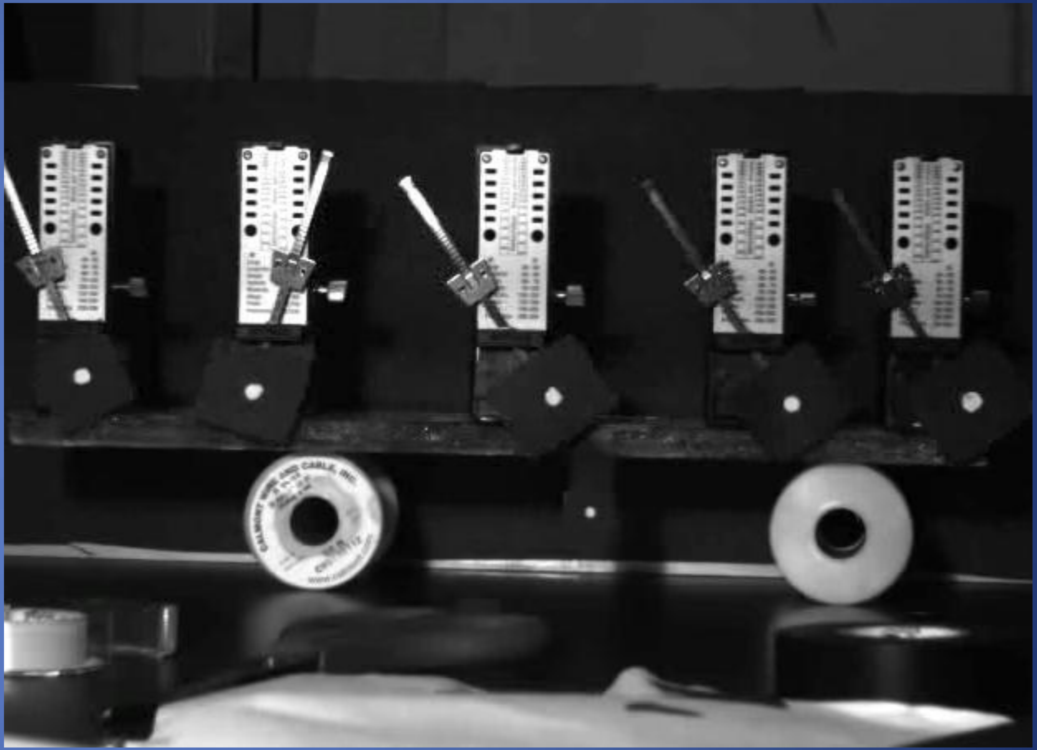
Results 2 metronomes

- For high coupling we tended to see the metronomes locking in phase
- For low coupling they would lock anti-phase or not at all.



Results 5 metronomes

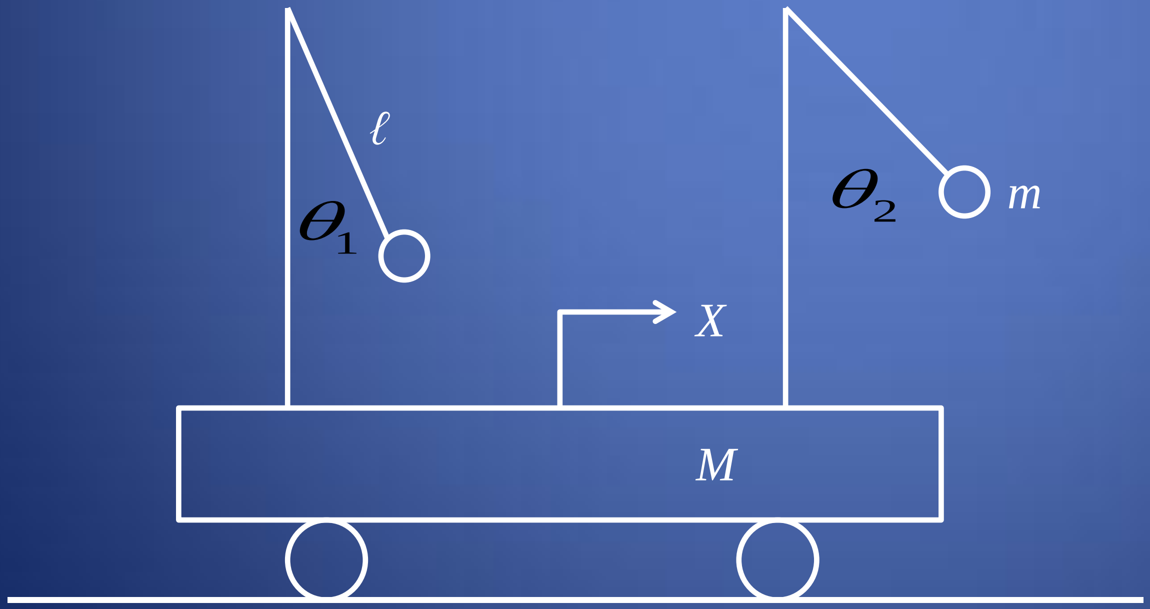
- We saw all 5 in phase.
- Or, 4 in phase and 1 anti-phase.
- Beating death was also observed.



Metronome Analysis

- Coupled oscillator model
- M = 180.06 g, m = 17.7 g
- Mass ratio μ, Here = 0.08

$$\mu = \frac{m}{M + 2m}$$



Coupled Oscillator Equations

- Metronome oscillation (1)

$$\frac{d^2\theta_i}{d\tau^2}+(1\pm\Delta)\sin\theta_i+\alpha\left(\left(\frac{\theta_i}{\theta_0}\right)^2-1\right)\frac{d\theta_i}{d\tau}-\beta\cos\theta_i\frac{d^2}{d\tau^2}\left(\sum\sin\theta_i\right)=0$$

- Base motion (2)

$$x=-\mu r_{\mathrm{CM}}\left(\sin\theta_1+\sin\theta_2\right)$$

- Other parameters

Coupling strength

Uncoupled frequency

Pivot to metronome CM

$$\beta=0.0089$$

$$\omega=10.9\ \mathrm{rad/s}$$

$$r_{\mathrm{CM}}=8.223\ \mathrm{mm}$$

Averaged Equations of Motion

- Phase relationship (3)

$$\frac{d\psi}{d\tau}=\frac{1}{8}\left[-3\gamma\left(A^2-B^2\right)+8\Delta+4\beta\left(\frac{B}{A}-\frac{A}{B}\right)\cos\psi\right]$$

- Amplitude relationship (4)

$$\frac{dB}{d\tau}=\frac{1}{8}\left[\alpha B\left(4-B^2\right)+4\beta A\sin\psi\right]$$

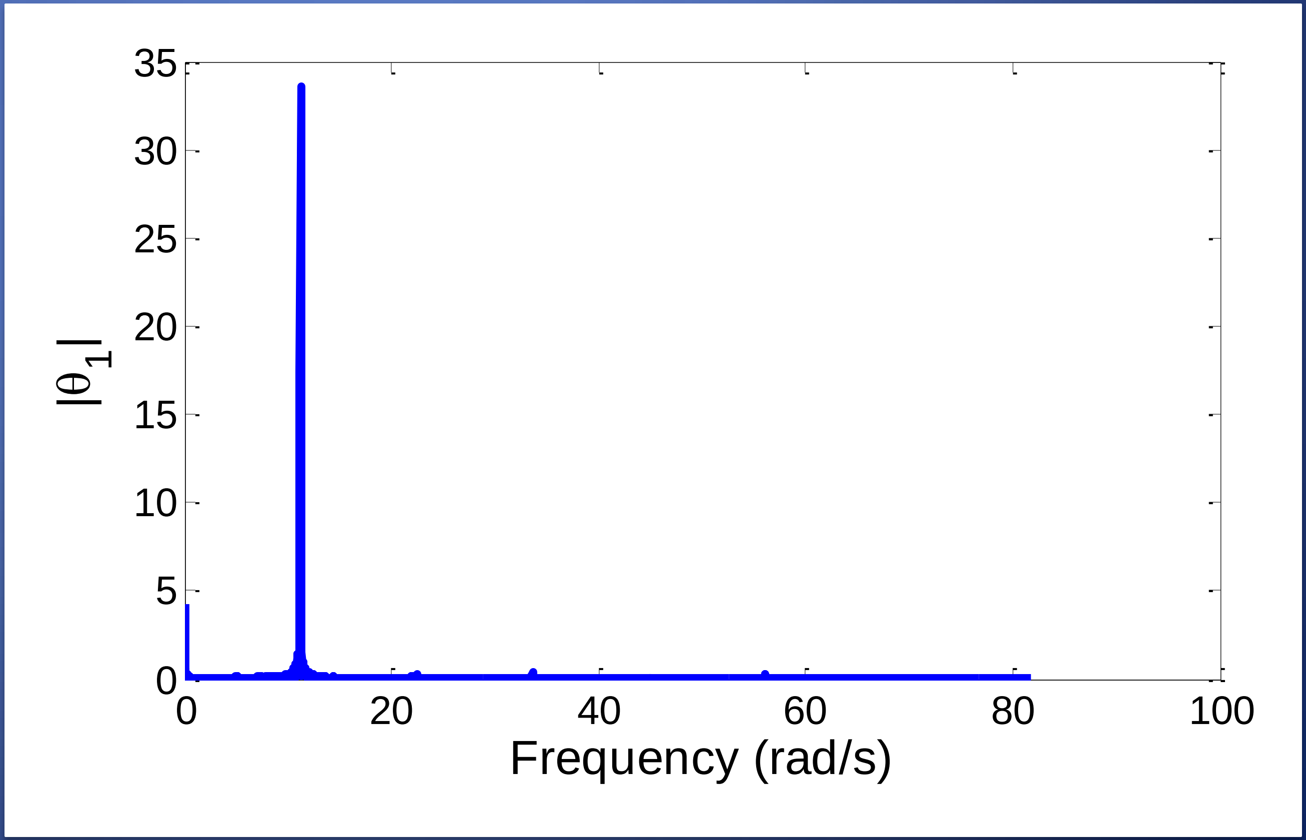
Fourier Analysis

- Spectral analysis showed two different coupled frequencies
- In phase

ω = 11.217 rad/s

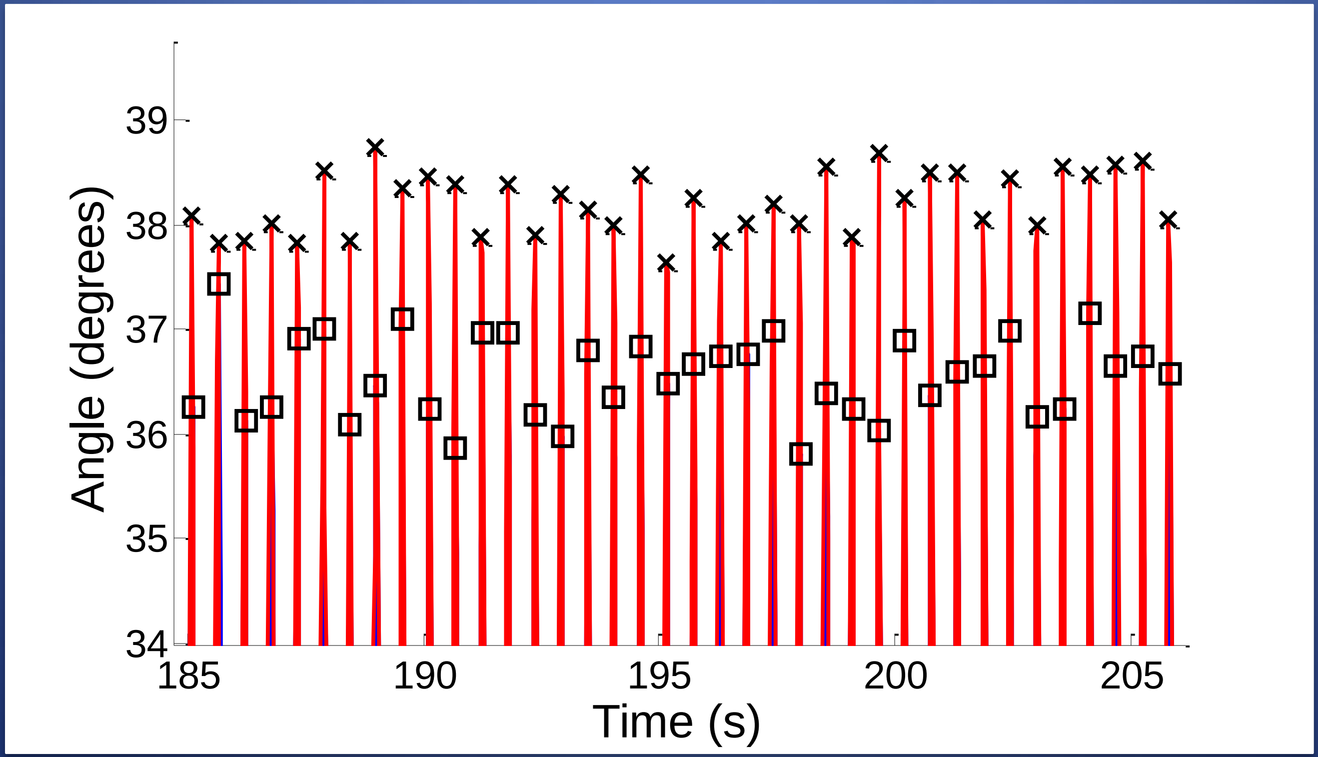
- Anti-phase

ω = 11.019 rad/s

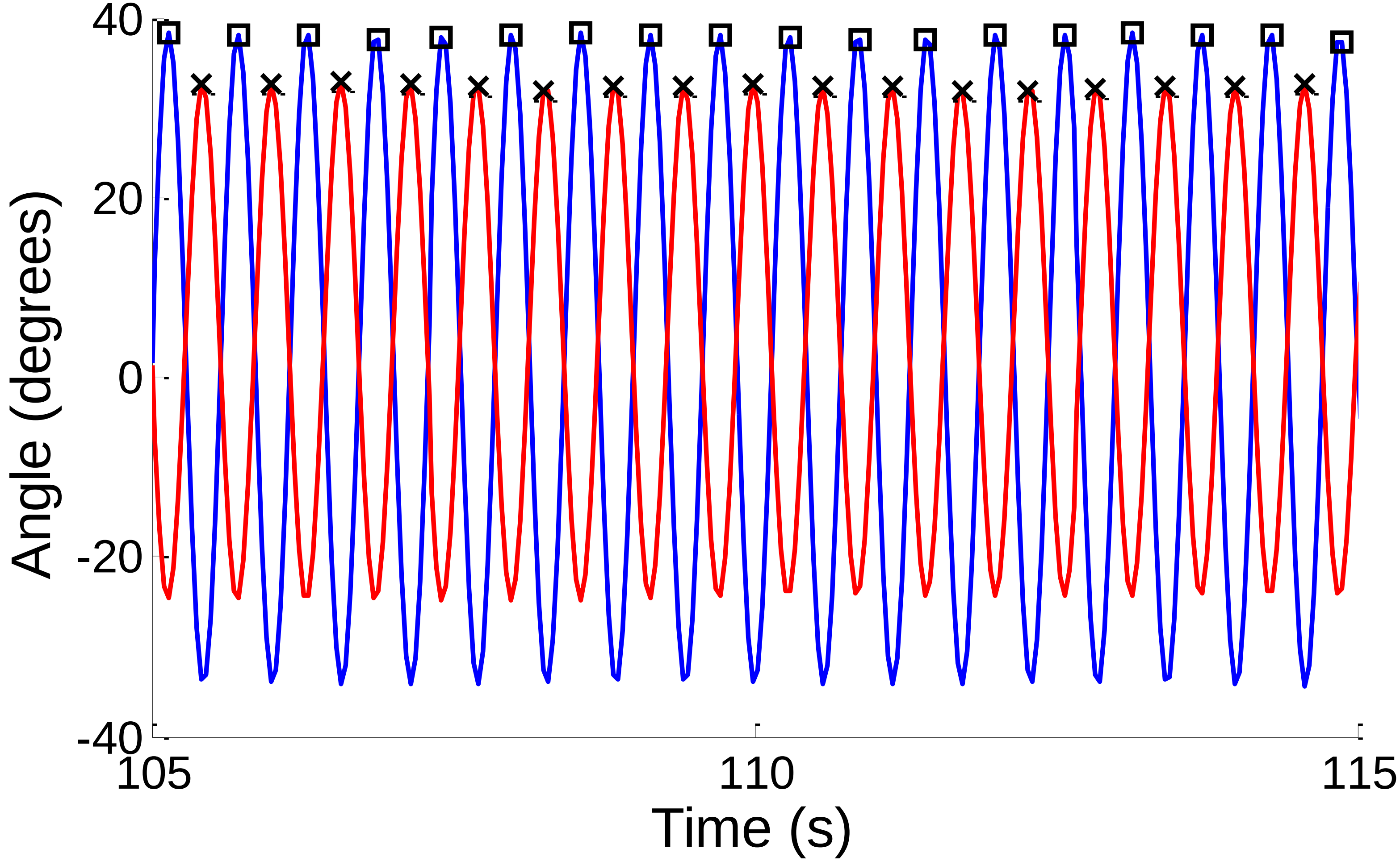


Examine Data Points

- Look at time of successive maximums to determine phase difference
- Example – in phase response $\Delta\theta = \omega\Delta t$

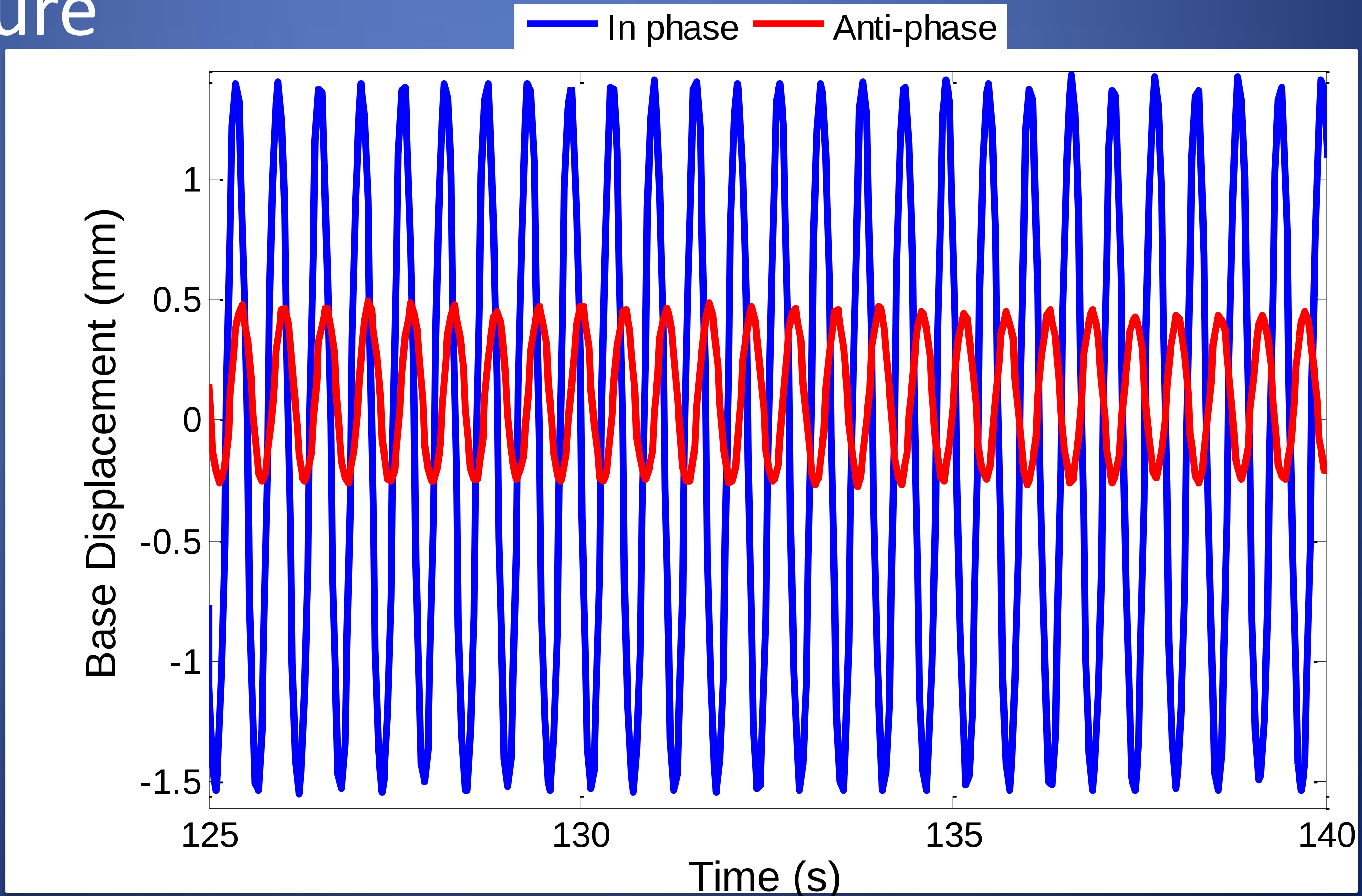


Example Anti-Phase



Base Motion

- Base motion has not been examined in literature

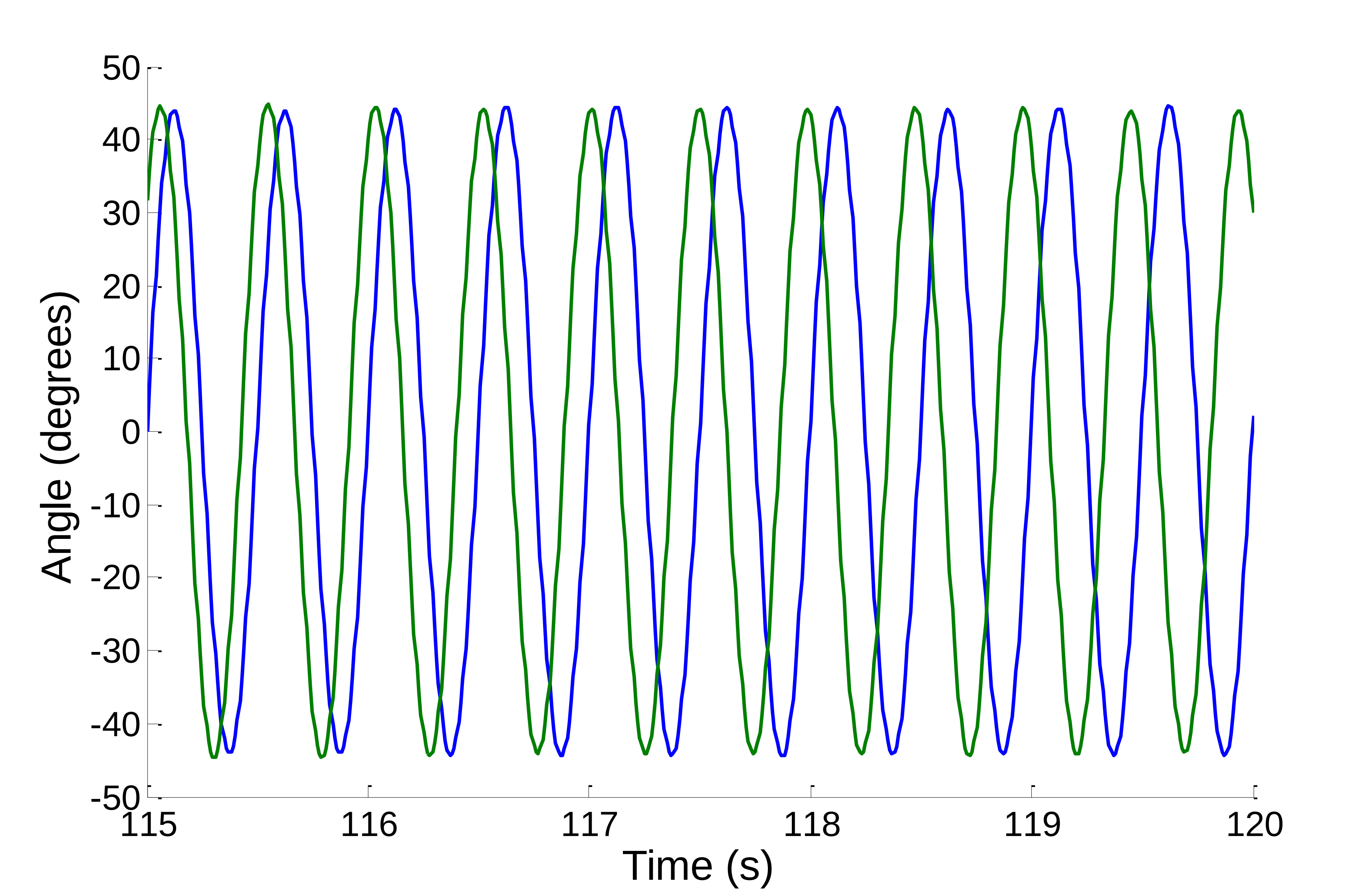


Results

- Averaged equations qualitatively agree with experimental results
- Metronomes exhibited phase drift
- Here mass ratio $\mu = 0.02$

- Other options included iterated map analysis which models the escapements

Simulated Displacement of Metronomes – Phase Drift



Conclusion

- Some results agree with theory
- Both in and anti phase states exist
- System is sensitive to mass
- Additional analysis needs to be performed

Thank you!

References

- 1. Bennett, M., et al., *Huygens's clocks.* Proceedings of the Royal Society of London Series a-Mathematical Physical and Engineering Sciences, 2002. **458**(2019): p. 563-579.
- 2. Pantaleone, J., *Synchronization of metronomes.* American Journal of Physics, 2002. **70**(10): p. 992-1000.
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- 5. Lepschy, A.M., G.A. Mian, and U. Viaro, *Feedback Control in Ancient Water and Mechanical Clocks.* Ieee Transactions on Education, 1992. **35**(1): p. 3-10.