# Vortex Cannon Dynamics

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## 1 Introduction

Vortex rings are a poloidal pattern of pulsed fluid that forms as a result of viscous friction between dense, high velocity fluid and a mass of stationary fluid which creates a poloidal flow, or by a mass of fluid being pushed through a small opening by some impulse. These rings are known to form at many length scales and serve as benchmark examples of coherent structures in fluid mechanics. (Roberts) Mathematical analysis of vortex rings was first discussed by the famous German physicist Herman von Helmholtz in his 1858 paper On Integrals of the Hydro-dynamical Equations which Express Vortex-motion in which he notes a “peculiar analogy between the vortex motions of water and the electromagnetic effects of electric currents.” Since Helmholtz, the formation, motion and interaction of vortex rings has been extensively studied. The vorticity which drives the propagation of the rings is derivable from the Navier-Stokes equation given the conservation of angular momentum.

There are numerous examples of phenomenon that exhibit a vortex ring pattern including smoke-rings, micro-bursts, mushroom clouds, artillery fire, turbulent flow of water droplets and in blood in the human heart specifically the flow of blood into the left ventricle from the mitral valve. Furthermore, it has been demonstrated in computer simulation; the airborne flow dynamics of Covid-19 resembles vortex ring dynamical behavior originated by tussis i.e. cough which served as our initial motivation to further investigate the necessary and sufficient conditions that cause the formation of poloidal vorticies.

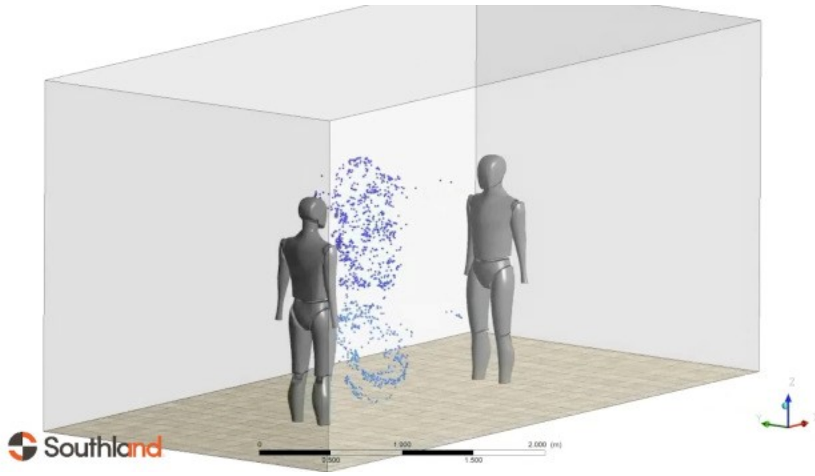


Figure 1: simulated model of Covid-19 transmission via tussis

## 2 Experimental Setup

The experimental apparatus consisted of the vortex cannon constructed from PVC pipes of outer length of 24 in (60.96 cm), and diameter 4 in (10.16 cm) and inner length of 48 in (121.92 cm) and diameter of 3 in (7.62 cm). Cardboard was used to cover the inner diameter of the pipe forming an effective piston. Screws acted as a break for the piston which allowed us to safely bring the piston to rest without damaging the plastic apertures that were used to vary the opening of the cannon and were added at 0.25 in. 1 in, 3 in displacements from the barrel. Two bands (bungee cords) were attached from the outer shell to piston by small incisions allowing the bungee cords to be securely hooked. The apparatus was thoroughly tested to ensure the bands were taut when the piston was pulled back with minimal friction when the piston was released. There were 2 variable points for the bands to be affixed to the main outer barrel which notably changed the stiffness of the bands and thus the velocities of the piston but had no effect on the conditions of ring formation.

Duct tape affixed the plastic apertures to the cannon, as well as the cardboard to the inner pipe forming our piston forming a piston diameter of 3.6 in (9.144 cm). Duct tape also secured the cannon to the bench while trials were conducted. A total of four aperture diameters were tested ($\frac{3}{4}$ in (1.905 cm, 1 in (2.858 cm) in, $1\frac{1}{2}$ in (3.81 cm), 2 in (5.08 cm). A fog machine was used specifically an "AGPTEK 500W Portable Led Smoke Machine with

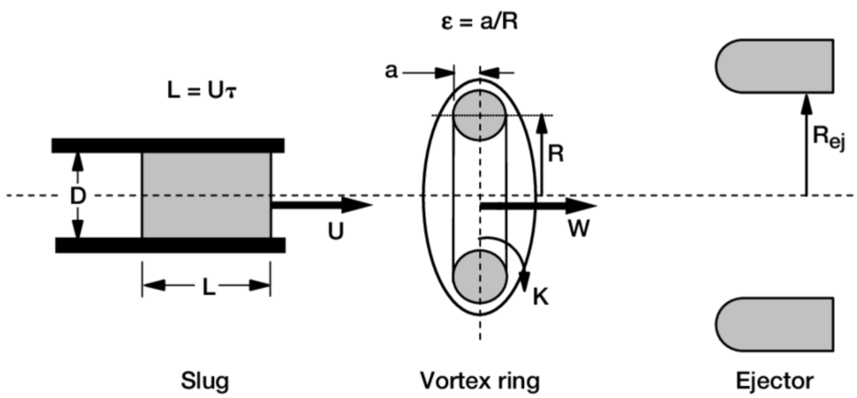


Figure 2: slug-model of a vortex cannon [13]

Lights (Red, Blue, Green) Wireless Remote Control". Finally, wooden beams were aligned on the floor in front of the barrel and marked by 1 ft (30.48 cm) intervals in order to take measurements of translational velocity of the rings.

A "Home Canon" camera was used to record the top view to best capture the displacement and slug velocities of the piston and iPhone cameras were used to record the translational motion of the rings and a construction leveling laser was used to capture cross sectional areas. The fluid used in the experiment was vaporized "fog juice" from a bottle labeled "Fog Liquid" consisting of a mixture of water, non-toxic glycols e.g. propylene glycol, and glycerol.

In general decisions made regarding the construction of our experimental apparatus and data collection procedures were done so with minimizing confounding variables while working within our low budget constraints. This premise served as our primary modus operandi regarding the construction of our experimental apparatus and data collection methodology. The slug model was a well documented model that not only fit our budget, and within our ability to construct but also was the best possible option available to us to simulate a cough which was our initial motivation for exploring vortex ring formation.

| displacement | D =1 1/8 | tenseband | D=2 (tense) | D = 2 (new band) | D = 2 (tense) ( | D = 1 1/8 (tense) | D = 3/4 in (ter | D1 *1/2 | D = 3/4 | D = 2 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | ring | ring | x | ring | x | x | ring | ring | ring | x |
| 1 | ring | ring | x | ring | x | ring | no ring | ring | no ring | x |
| 1.5 | ring | ring | x | ring | x | ring | x | ring | x | x |
| 2 | ring | ring | ring | ring | ring | ring | x | ring | x | ring |
| 2.5 | no ring | no ring (ghost) | x | x | x | no ring | no ring | ring | no ring | x |
| 3 | no ring | no ring | x | ring | x | no ring, splitting patte | split ring | ring | split ring | x |
| 3.5 | no ring | x | x | x | x | x | ring | ring | ring | x |
| 4 | no ring | no ring | ring | ring | ring | x | no ring | ring (messy) | no ring | ring |
| 5 | x | x | x | x | x | x | no ring | split two | no ring | x |
| 5.5 | x | x | x | x | x | x | x | split two | x | x |
| 6 | no ring | no ring | ring | ring | ring | x | no ring | no ring | no ring | ring |
| 6.5 | x | x | x | ring | x | x | no ring | no ring | no ring | x |
| 7 | x | x | x | ring | ring | x | x | no ring | x | ring |
| 7.25 | x | x | x | x | ring | x | x | x | x | ring |
| 7.5 | x | x | DAMAGED | ring | x | x | x | no ring | x | ring |
| 8 | x | x | x | no ring | x | x | x | x | x | no ring |
| 8.25 | x | x | x | no ring | x | x | x | x | x | x |
| 8.5 | x | x | x | no ring | x | x | x | x | x | x |
| 9 | x | x | x | no ring | x | x | x | no ring | x | x |
| 10 | x | x | x | no ring | x | x | x | x | x | x |
| 12 | x | x | x | no ring | x | x | x | x | x | x |
| l = 1/2 in | l = 1/2 in | l = 1/2 in | l=1/2 | l=1 in | l=1 in | l =1 in | l=1inch | l = 1inch | l=3in | l=3in |

Figure 3: Tabulated observed vortex ring formation data

# 3 Results and Discussion

The qualitative data tabulated in the figure shown notes the observations made by our group. The "l" at the bottom is the displacement from the end of the barrel where the piston was stopped by the screws forcing the gas through the aperture. "D" was the diameter of the aperture. We found some interesting trends and one surprising result from our experiment.

While exploring the least upper bounds of vortex ring formation. One observation was that rings stopped forming only after increasing the volume i.e. the displacement of the piston from equilibrium beyond a certain threshold governed by the ratio of the displacement and the diameter of the aperture. Exchanging the bands merely affected the velocity of the rings that formed but did not have any noticeable impact. I suspect that the explanation for this lies in the fact that internal energy which is only a function of the pressure inside the chamber which was held constant by allowing the gas to come to equilibrium inside the chamber and the change in volume remained constant set only by the displacement of the piston. This would explain why the parameters governing ring formation did not change despite changing the bands. We also observed azimuthal instability as the ring began to decay; however, this was difficult for us to capture.

One surprising result was the observation of a ring at a particularly large displacement (3.5 in, D = $\frac{3}{4}$ in that formed due well beyond the regime of displacements where formation of rings was expected. There was an increased value for the circulation at this displacement at a maximum value of 1365 $in^2/s$.

The hypothesis our group was able to come up with drew inspiration from Kolmogorov's theory regarding inertial turbulence. We suspect the increased circulation was due to eddy currents that formed, broke apart transferring energy to smaller eddies which resulted in the increased circulation given by the turbulent flow present at the 3.5 in displacement. We actually managed an approximation of Reynold's number for this trial converting first to SI units, and assuming the gas contained primarily of water vapor which the dynamic properties are such as viscosity is well known 0.00013 Pa*s and the density of water vapor also well-known $0.6kg/m^3$ we were able to find an estimation of the Reynold's number as 4058.26 by dividing the maximum circulation calculated and divided by the kinematic viscosity of water vapor found by simply dividing the dynamic viscosity by the density. which is well into the turbulent regime being greater than 2900. It should be noted that the true Reynold's number should expected to be smaller than 4058.26 given that our fog was also composed of propylene glycol and glycerol both with higher boiling points than water. Further investigation regarding the exact chemical composition of the "Fog Liquid" used would be required in order to obtain a closer approximation.

We couldn't have obtained any Reynolds' number approximation whatsoever if we did not also obtain quantitative calculations of the circulation which was obtained by finding translational and slug velocities obtained by the camera images. The greatest source of error in our approach was the low fps of the camera meant that only small number of time intervals were usable for the slug velocity. This meant in our comparison of the gas circulation and the slug circulation we could only use estimates based on the slug time intervals. A future improvement would be to obtain more precise measurements of the the slug velocity so that we could have made more precise estimation of the circulation ratios for our data.

The circulations were obtained by considering approximate solutions given by the square of the velocities. After confirming the validity this approximation was applicable to both the gas [20] and the slug [19] we were able to obtain estimates of both the slug circulation

$$\frac{\Gamma_{slug}}{dt} = \int_0^{t_{piston\ stop}} \omega u_x d\sigma \approx \int_0^{t_{piston\ stop}} \frac{\partial u}{\partial \sigma} u_x d\sigma \approx \frac{1}{2} U_p^2(t)$$

Figure 4: approximation of slug circulation [19]

as well as the circulation of the gas.

After obtaining the results for the circulation we could then use these results to test if our values fit with current theory regarding vortex ring formation given by the Pullin curve and the Didden curve.

$\Gamma(gas)/\Gamma(slug) = 1.41(L/D)^{(-2/3)}$ (Pullen)

$\Gamma(gas)/\Gamma(slug) = 1.14 + 0.32(L/D)^{(-1), L/D>0.6}$ (Didden)

where L is the displacement of the piston and D is the diameter of the aperture (see Figure 2)

Didden's model tends to work for fluids but not as well for vapor compared with Pullin's model which is noted to typically be better fit for systems with higher Reynolds numbers. The respective curves are shown in figure 5. Our group constructed a SVM Decision model to predict vortex ring formation based on our data and found that our results are in agreement with current literature in that our data was better fit to Pullin's simulation theory based model except we seem to be overestimating of the ratio of the gas circulation compared with the slug circulation. The source of this error is likely due to the low 18 fps of the camera which severely limited the precision of our analysis.

## 4 Conclusion

In summary, we were able to successfully investigate the key parameters that govern the formation of vortex ring systems from the slug-model and obtained results that are in agreement with current literature in many regards. Furthermore, we were able to contribute to a better understanding of dynamical systems by effectively exploring a surprising which seems to corroborate Korogomov's theory of inertial turbulence. Lastly, we were able to

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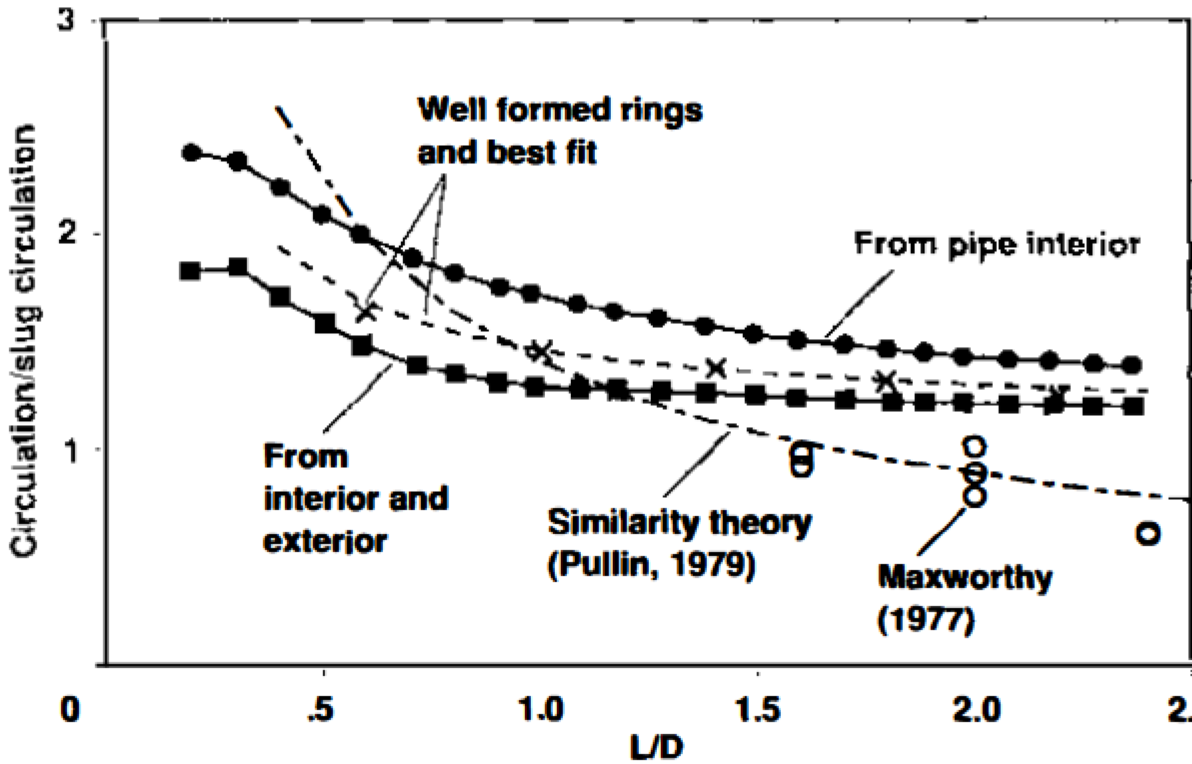


*Figure 3* Circulation of vortex rings formed at a pipe referred to the circulation predicted by the slug-flow model, Equation (2.5). Unless indicated, the data are from Didden (1979). The multiple data points of Maxworthy (1977) at the three $L/D$ values are for different Reynolds numbers, increasing downward. Well-formed rings seem to have a larger circulation (*crosses*) than the total flux at the pipe (*solid squares*) due to slight inadequacy in measuring the flux at the pipe.

Figure 5: Didden's model compared with Pullin [15]

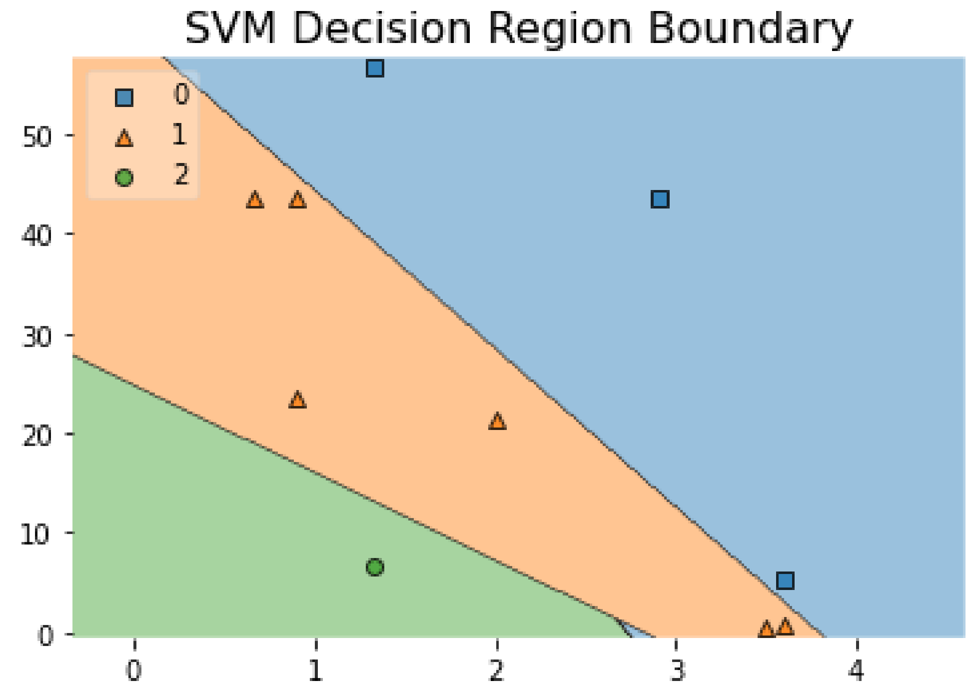


Figure 6: SVM decision model regarding formation of vortex rings as predicted by our data. This is given by the plot of the ratio of circulations of fog vapor to the slug compared with the ratio of the displacement of the piston and diameter of the aperture. Red = ring , Green or Blue = no ring.

construct a SVM Decision model which predicts vortex-ring formation based on our data. Future improvements to our project would be to utilize higher budget cameras with higher fps to capture the slug velocity of the piston as our camera only had 18 fps. This caused our calculations for the circulation ratio to be overestimated compared to the expected outcome since we only had small time windows to compare the circulation of gas with the circulation of the slug. Future work would benefit from revisiting this study with higher FPS cameras so a more precise analysis could have been done.

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