The Effect of Platform Mass on Metronome Synchronization

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Abstract

In this project, we investigate the way that the mass of a platform affects the coupling between an array of metronomes in a synchronization scenario. Specifically, we initially develop video processing code that allows us to track the trajectories of the metronomes utilizing color-coded pointers. Subsequently, we gradually increase the mass of the platform supporting the metronomes to explore how the time to synchronize changes and compare our results with the theoretical predictions in a quantitative way.

I. Introduction

Synchronization phenomena have been extensively studied over the previous centuries due to their emergence in a huge variety of different fields [1]. Initially observed by the physicist Christiaan Huygens in 1657, synchronization was investigated in the mechanical domain. Specifically, Huygens noticed synchronization taking place between two pendulum clocks hanging by an overhead beam in his laboratory. Although his initial conjecture on the nature of the coupling mechanism was wrong – believing it to be due to air fluctuations caused by the movement of the pendulums – Huygens did push forward with his experiments, determining the stability properties and time properties of his pendulum system, albeit without the mathematical tools that enable him to rigorously describe them [2].

The need to study synchronization has become apparent by the variety of different disciplines where relevant models have been employed. The resurgence of interest in synchronization was in part due to their use in modeling biological phenomena [3]. One striking example is the synchronization of firefly light emission [4], while different cell types have been shown to operate with oscillatory behaviors that synchronize as needed. As is often the case, some of the most interesting results regard the operation of the brain. Norbert Wiener, during his studies of human brain waves, observed an increase of activity in electroencephalograms in a narrow frequency band around 10Hz. As such, he hypothesized that oscillating elements in the brain exhibited collective behavior leading to this frequency matching [5]. Although Wiener did not create a theory regarding those findings, subsequent researchers have discovered the way that neurons synchronize their spiking behavior in order to create macroscopic properties such as memory storing [6].

Beyond biological systems, synchronization has been both observed and leveraged in a number of different areas [7]. It is of great interest to mention synchronization phenomena in the social sciences. Due to the influence that personal opinions of individual agents have on their environment, the emergent collective behavior of rational agents has been investigated from a synchronization-theoretic point of view. In [8], the authors propose a Kuramoto-based model of opinion formation that goes beyond the typical consensus-oriented ones that have been studied.

Various engineered systems have been designed based on synchronization principles. Data-mining algorithms have been proposed, which encode data vectors – taken by a database of raw data – into vectors of natural frequencies for an oscillators' dynamical model, expecting that an algorithm simulating synchronization dynamics will group similar data in clusters, thus indicating patterns in the data sets [9]. Similarly, the power production, dissipation, transmission, and consumption of a power grid represents a dynamical problem and the power grid can be seen as an example of a system of oscillators, akin to a second order Kuramoto model [10].

Despite the fact that the synchronization of mechanical oscillators was the first such phenomenon to have been studied, it remains an active research topic even to this day. Delving deeper into the underlying mechanisms and expanding our understanding of the emergent behaviors, efforts have been focused on the experimental study of metronome synchronization. Metronomes have been utilized as a tool for emulating the pendulum clocks of Huygens in a more compact manner. Pantaleone [11] presented a theoretical framework for the synchronization mechanism of metronomes coupled via a platform restricted to move in a single dimension. He further demonstrated how the derived equations could lead to a Kuramoto – a popular theoretical system that captures the phase dynamics of coupled oscillators [12] – model of the process.

The dynamical system proposed by Pantaleone has been investigated with respect to various properties. The nature of the steady state frequency locking has been examined in various publications [13], where it has been observed that in large numbers of metronomes, there are different emergent equilibria. During phase locking, the metronomes converge to the same phase, in anti-phase locking, the system converges a state where two clusters with phase difference of π is found, while in chimera states a part of the metronome population synchronizes while another oscillates incoherently [14] Furthermore, coupled mechanical oscillators have been investigated on a number of their properties. The authors of [15] very recently published a stability analysis of the synchronization of a system of two pendulums, which is indicative of the theoretical interest – and difficulty – of the metronome system analysis in the scientific community. The authors of [16] investigate the behavior of coupled metronomes of different lengths, while in [2] the authors study bifurcation phenomena in a system of two weakly coupled metronomes. Similarly, in [17] the emergent behavior of two metronomes was changed via varying the rolling friction of a PVC rolling basis and bifurcations diagrams were presented and analyzed.

Along those research directions, in this project we will focus on the robustness, stability and convergence properties of a system of three coupled oscillators exhibiting phase locking, and the effect that changes in the mass of the platform has on the aforementioned properties. Towards this, we employed a widely-used structure of metronomes operating on a common platform, itself resting on two soda cans. The mass of the platform was shifted by the addition of weights in equal discrete steps, while visual measurements allowed us to derive the trajectories of the metronomes.

II. Experimental Setup

In this section, we will describe the experimental setup that was constructed in this project. Our main components are the metronomes themselves. During the measurement process, we utilized three metronomes. Although there have been research endeavors where larger numbers of metronomes have been considered, we chose to only use three of them for two reasons; one being the fact that the measurement process itself becomes harder as more metronomes are used (and the need for higher precision equipment becomes more apparent), while from a theoretical standpoint, we have seen that the more quantitative experiments have kept the numbers of devices at most three. The metronomes used were of the brand Wittner, whose frequency was set at 184 beats (half-oscillations) per minute. This corresponds to oscillations with a period equal to $T = 0.652s$. The energy of the metronomes is supplied via a hand-wound spring. The way this energy is utilized will be described explicitly in the sequel.

The coupling was achieved via a board, comprising a simple piece of rigid material. This rigidity is important in the lossless transmission of inertial forces between the metronomes. Furthermore, we tried to keep the weight of the board relatively low, using either foam or a light oven pan. The board and the metronomes, in turn, rest upon two soda cans, which are also rigid as to minimize the deformation and, by extension, the friction of the platform. This mechanism will also become more apparent in the sequel. The soda cans allow the system to move in only one direction. As will be further explained, in order to search the parameter space of the system, we increased the mass of the platform by adding weights. The weights used were small tomato sauce cans, with masses equal to either 170g or 227g. The structure is shown in Fig.1.

The most challenging part of the project was the gathering and processing of the measurements. Towards that, we chose to obtain visual measurements of the system oscillations. The video acquisition system comprises a phone camera, a number of paper wafers and a computer. The camera resolution

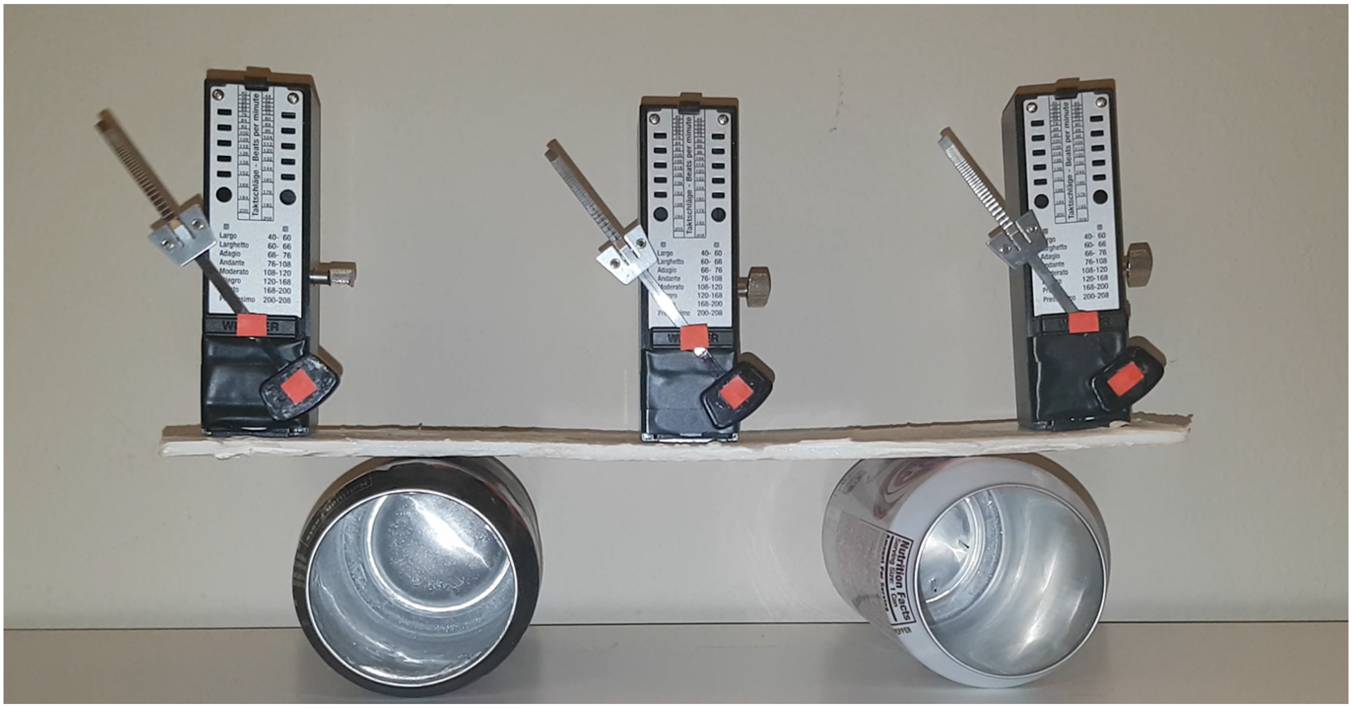


Fig.1: The experimental setup used for the project.

is 3840×2160, recording at 30 Hz. The paper wafers used are made of brightly colored paper, and are leveraged to allow for the tracking of the metronome bobs. Specifically, we attached the pointers in the bottom end of the bobs. This way the movement of the metronome is captured more easily due to the smaller frame needed for the camera to keep all the metronomes in view while being close enough for the processing software to track the positions of the wafers. Furthermore, special care was taken to choose a background that was sufficiently white to allow the software to distinguish the colored pointers.

To capture the trajectory of the oscillating metronomes, we utilized Matlab code, leveraging the video processing and image recognition software integrated therein. After recording the video of the metronome response, the file was read by Matlab and the image was separated into 3 parts, each corresponding to a single metronome. Afterwards, a code was developed that searched each frame of the video for the colored wafer on the metronome bob. Furthermore, to increase the performance of the algorithm, we partitioned the frame into different areas and search for the colored wafers only in those that contained the metronome. Since in the preprocessing phase of the trial we had saved the RGB color code corresponding to each wafer, the code was tasked with finding pixels in the image with the appropriate RGB code. Once the wafers were located, that point was marked as the required point. By locating both the end of the bob and the pivot point of metronome the same way, the angle of the metronome was calculated as $\phi_i = \arctan \delta y_p / \delta x_p$.

III. Theoretical model

Throughout the literature, we have found two different ways of theoretically analyzing coupled metronome systems. The first is a low-level second order model derived by the Euler-Lagrange equations

[18], [19], given, for all $i \in \{1, \ldots, N\}$ metronomes, as

$$\ddot{\phi}_i + b\dot{\phi}_i + \frac{g}{l}\sin\phi_i + \frac{1}{l}\ddot{x}\cos\phi_i + \bar{F}_i = 0, \qquad (1)$$

$$(M + Nm)\ddot{x} + B\dot{x} + Kx + ml\sum_{j=1}^{N}\sin\phi_i = 0,$$

where ϕ_i denotes the metronome bob's angle with the horizontal axis, x the platform position, l the length of the bob, and m, M the mass of the metronomes (assumed to be identical) and the platform, respectively. The mass M will be the parameter changed throughout our trials and its effect of the coupling strength will be investigated experimentally. I can be seen that the system consists of $N + 1$ second order equations of motion, corresponding to the N metronomes and moving platform. This model is quite useful in showcasing the coupling mechanism in the more physical level. Each metronome has an internal dissipative force due to friction, $b\dot{\phi}_i$, which would stop the oscillations in a free metronome. In order to maintain the oscillation amplitude, metronomes are fitted with a mechanism that attenuates the dissipation, called an "escapement" mechanism. The force of the escapement mechanism is denoted by $\bar{F}_i$, and is typically modeled as a van der Pol term. It can also be seen that each metronome is driven by the inertial force of the moving platform due to the term $1/l\ddot{x}\cos\phi_i$. The platform equation demonstrates the coupling mechanism between metronomes. It can be seen that the oscillation of the platform is affected by a friction force $B\dot{x}$ which is kept at a minimum by the use of rigid soda cans. The platform is also driven by the movement of the metronomes. This "feedback" coupling between metronomes and platform is the mechanism by which the metronomes regulate their phase difference.

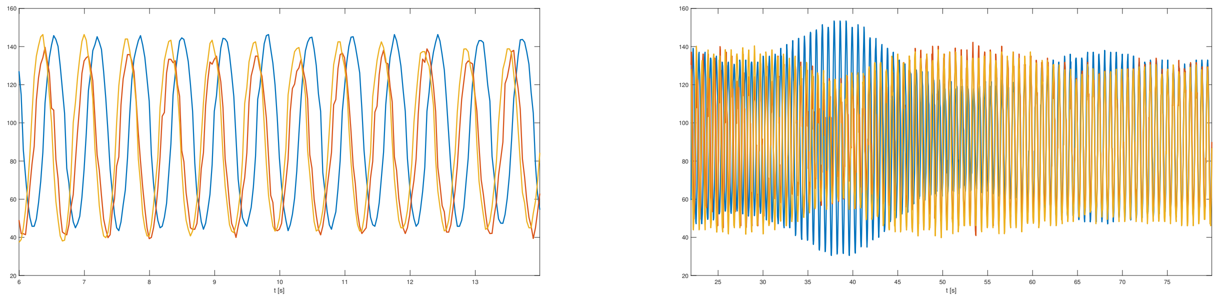
Kuramoto models are simple first order models that have been shown to capture a variety of synchronization phenomena. In their usual form, the Kuramoto oscillator model is given by

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_i - \theta_j), \ \forall i \in \{1, \ldots, N\}, \qquad (2)$$

where θ_i, ω_i model the phase and angular velocity of the i-th metronome respectively. Note that θ_i is not a physical angle like ϕ_i, but rather the phase of the oscillation of the bob. It has been shown that, via the application of averaging techniques [11], the behavior of the coupled metronome system can be indeed studied via a Kuramoto model. The coupling parameter K is a function of the mechanical properties of the system and will change as the mass of the platform increases.

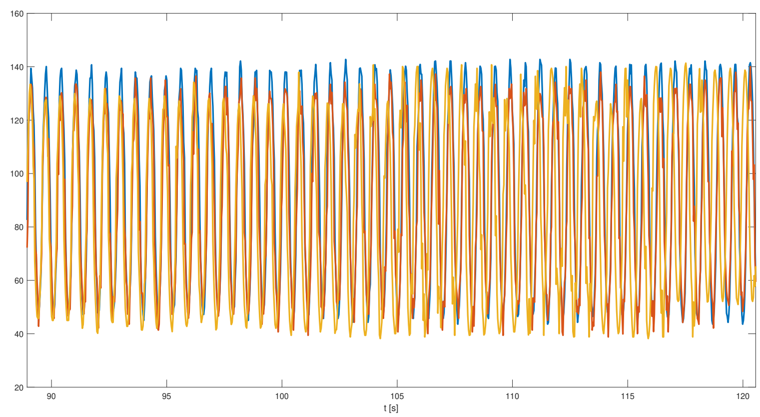
IV. Experimental Results

Initially, we conducted a simple trial that demonstrated a 3 phase synchronization/desynchronization process and was used to test and debug the tracking code. The metronomes were wound by hand before being placed on the platform. Afterwards, the platform with the oscillating metronomes was placed on the soda cans, effectively activating the coupling forces between them. In this phase of the run, the platform begun oscillating and perturbed the phases of the metronomes, forcing them to converge to a phase-locked state. In the third phase of the run the platform was moved off the soda cans. We observed that after removing the coupling, the metronomes desynchronized. Although the ideal dynamics would not predict desynchronization to take place, the movement of the platform and the disturbances due to air disturbances introduced forcing that caused this phenomenon to take place. Fig.2a we can see that after the coupling between the metronomes becomes active, there is a transience period during which the phase difference is nonzero. After sufficient time has passed, the metronomes get synchronized, and remain so. This holds because it has been shown that for a system of coupled Kuramoto oscillators, the state where the phase difference is an even multiple of 2π is asymptotically stable. However, in Fig.3 the metronomes are taken off the platform. Looking at the Kuramoto oscillator, in the ideal case, it should hold that $\dot{\theta}_i = \omega_i$, implying that the phases should continue to be synchronized (since the



(a) Metronome trajectories before coupling becomes active. (b) Metronome trajectories after coupling takes place: transience and eventual synchronization.

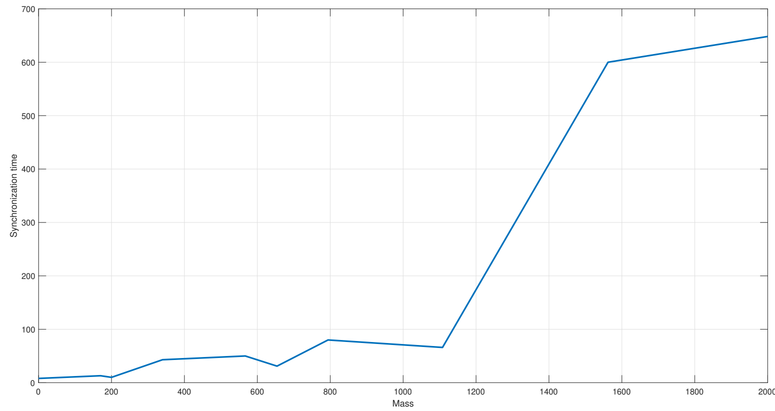
Fig.2: Synchronization of metronomes.



Metronome trajectories after the decoupling takes place, and subsequent desynchronization.

metronomes have the same eigenfrequency). However, due to disturbances from the movement of the platform, the metronomes do eventually desynchronize.

Subsequently, we performed a number of experimental runs that were designed to showcase the change in synchronization speed as the mass of the platform increased. The most challenging part in this problem was decoupling the different parameters in each problem. Stopping the metronomes, changing the mass and then restarting the experiment would mean that we would have no control over the initial conditions of the system; especially during the placement of the platform on top of the cans. Instead, the change of the mass was made once the metronomes had synchronized. Thus, on a the fully synchronized system, we placed a single weight on the platform, changing the coupling strength between the metronomes. Afterwards, to study the time of synchronization of the system in a systematic manner, we placed an obstacle on the path of one of the metronomes – specifically when $\phi_i = 90$ – for a specific amount of time. Since the frequency of the metronomes remains constant, the phase discrepancy created during this time period will be the same in all runs of the system. The metronomes were then allowed to reach synchronization again, and the time to synchronize was measured. The data gathered are shown in the table that follows.



Time to achieve synchronization as a function of the platform mass.

Synchronization time with different platform mass			
Foam platform		Tin platform	
Mass (kg)	Time (sec)	Mass (kg)	Time (sec)
~ 0	8	0.200	10
0.170	13	0.654	31
0.340	43	1.108	66
0.567	50	1.562	600
0.794	80	2.016	∞

It is interesting to see the trend that emerges as the platform mass increases, specifically, that the synchronization rate decreases. Looking at equation (2), we employ the results of [20], where the following Theorem is stated:

Theorem: The Kuramoto model given by (2) with $K > 0$, will synchronize to phase differences of an even multiple of 2π. The rate of approach to synchronization is no worse that $(2K/\pi N)\lambda_2(NI - \mathbf{1}_N\mathbf{1}_N^T/N)$, where I is the identity matrix of appropriate dimension, $\mathbf{1}_N$ a vector of ones and $\lambda_2(A)$ is the Fiedler eigenvalue of a matrix A.

Consequently, the linear dependence of the synchronization rate to the coupling parameter K implies that the increase in the mass has decreased K. To see the explicit dependence of K to M, we note that, due to [11], the coupling parameter is found to be

$$K = \sqrt{\left(3\frac{\beta}{\mu}\right)^2 + \left(\frac{\beta}{\gamma}\right)^2}\left(\frac{\gamma}{2N}\right),$$

where the only term related to the platform mass is $\beta = \left(\frac{m^2r^2}{(M+2m)I}\right)$. Therefore, increasing the platform mass, decreases β and, by extension, decreases the coupling strength K. In Fig. 4, we see the change of synchronization time as the mass changes.

V. Conclusion

In this project, we analyzed the behavior of a system of three metronomes, emulating the initial experiments by Huygens, and performed experiments to characterize the relationship between the mass of the moving platform and the time to synchronize. Initially, we developed software that was able to determine the time evolution of the angle of the metronomes via visual measurements. Specifically, the code decomposes the video into different partitions at each frame, depending on the position of the platform, and locally searches for color-coded pointers to determine the position of the metronome bob. The angle is found trivially as a result. After observing the importance of coupling strength through the platform's behavior, we considered a way that would allow us to measure the time of synchronization from almost identical initial conditions as the platform mass was gradually increased. The theoretical connection between platform mass and synchronization time agrees from a qualitative point of view with our results, but a more quantitative analysis would require a careful consideration and computation of the different parameters that affect the time of synchronization. This line of research would experimentally support the connection between the abstract Kuramoto oscillator model and the Euler-Lagrange based second-order mechanical model of metronome synchronization.

REFERENCES

[1] I. I. Blekhman, *Synchronization in science and technology*. ASME press, 1988.

[2] A. R. Willms, P. M. Kitanov, and W. F. Langford, "Huygens' clocks revisited," *Royal Society Open Science*, vol. 4, no. 9, p. 170777, 2017.

[3] S. H. Strogatz and I. Stewart, "Coupled oscillators and biological synchronization," *Scientific American*, vol. 269, no. 6, pp. 102–109, 1993.

[4] R. E. Mirollo and S. H. Strogatz, "Synchronization of pulse-coupled biological oscillators," *SIAM Journal on Applied Mathematics*, vol. 50, no. 6, pp. 1645–1662, 1990.

[5] S. H. Strogatz, "Norbert wiener's brain waves," in *Frontiers in mathematical biology*. Springer, 1994, pp. 122–138.

[6] E. M. Izhikevich, "Weakly pulse-coupled oscillators, fm interactions, synchronization, and oscillatory associative memory," *IEEE Transactions on Neural Networks*, vol. 10, no. 3, pp. 508–526, 1999.

[7] A. Arenas, A. Díaz-Guilera, J. Kurths, Y. Moreno, and C. Zhou, "Synchronization in complex networks," *Physics reports*, vol. 469, no. 3, pp. 93–153, 2008.

[8] A. Pluchino, V. Latora, and A. Rapisarda, "Changing opinions in a changing world: A new perspective in sociophysics," *International Journal of Modern Physics C*, vol. 16, no. 04, pp. 515–531, 2005.

[9] T. Miyano and T. Tsutsui, "Data synchronization in a network of coupled phase oscillators," *Physical review letters*, vol. 98, no. 2, p. 024102, 2007.

[10] G. Filatrella, A. H. Nielsen, and N. F. Pedersen, "Analysis of a power grid using a kuramoto-like model," *The European Physical Journal B*, vol. 61, no. 4, pp. 485–491, 2008.

[11] J. Pantaleone, "Synchronization of metronomes," *American Journal of Physics*, vol. 70, no. 10, pp. 992–1000, 2002.

[12] J. A. Acebrón, L. L. Bonilla, C. J. P. Vicente, F. Ritort, and R. Spigler, "The kuramoto model: A simple paradigm for synchronization phenomena," *Reviews of modern physics*, vol. 77, no. 1, p. 137, 2005.

[13] G. H. Goldsztein, A. N. Nadeau, and S. H. Strogatz, "Antiphase versus in-phase synchronization of coupled pendulum clocks and metronomes," *arXiv preprint arXiv:2008.02947*, 2020.

[14] E. A. Martens, S. Thutupalli, A. Fourrière, and O. Hallatschek, "Chimera states in mechanical oscillator networks," *Proceedings of the National Academy of Sciences*, vol. 110, no. 26, pp. 10 563–10 567, 2013.

[15] M. Francke, A. Pogromsky, and H. Nijmeijer, "Huygens' clocks:'sympathy'and resonance," *International Journal of Control*, vol. 93, no. 2, pp. 274–281, 2020.

[16] J. Jia, Z. Song, W. Liu, J. Kurths, and J. Xiao, "Experimental study of the triplet synchronization of coupled nonidentical mechanical metronomes," *Scientific reports*, vol. 5, no. 1, pp. 1–12, 2015.

[17] Y. Wu, N. Wang, L. Li, and J. Xiao, "Anti-phase synchronization of two coupled mechanical metronomes," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 22, no. 2, p. 023146, 2012.

[18] M. Bennett, M. F. Schatz, H. Rockwood, and K. Wiesenfeld, "Huygens's clocks," *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, vol. 458, no. 2019, pp. 563–579, 2002.

[19] M. Kapitaniak, K. Czolczynski, P. Perlikowski, A. Stefanski, and T. Kapitaniak, "Synchronization of clocks," *Physics Reports*, vol. 517, no. 1-2, pp. 1–69, 2012.

[20] A. Jadbabaie, N. Motee, and M. Barahona, "On the stability of the kuramoto model of coupled nonlinear oscillators," in *Proceedings of the 2004 American Control Conference*, vol. 5. IEEE, 2004, pp. 4296–4301.